Binomial Hypothesis Testing (From OCR 4733)

Q1 (Jan 2006, Q3)

The manufacturers of a brand of chocolates claim that, on average, 30% of their chocolates have hard centres. In a random sample of 8 chocolates from this manufacturer, 5 had hard centres. Test, at the 5% significance level, whether there is evidence that the population proportion of chocolates with hard centres is not 30%, stating your hypotheses clearly. Show the values of any relevant probabilities. [7]

Q2, (Jun 2006, Q2)

(i) The random variable R has the distribution B(6, p). A random observation of R is found to be 6. Carry out a 5% significance test of the null hypothesis H_0: $p = 0.45$ against the alternative hypothesis H_1: $p \neq 0.45$, showing all necessary details of your calculation. [4]

(ii) The random variable S has the distribution B(n, p). H_0 and H_1 are as in part **(i)**. A random observation of S is found to be 1. Use tables to find the largest value of n for which H_0 is not rejected. Show the values of any relevant probabilities. [3]

Q3, (Jan 2007, Q7)

A television company believes that the proportion of households that can receive Channel C is 0.35.

(i) In a random sample of 14 households it is found that 2 can receive Channel C. Test, at the 2.5% significance level, whether there is evidence that the proportion of households that can receive Channel C is less than 0.35. [7]

(ii) On another occasion the test is carried out again, with the same hypotheses and significance level as in part **(i)**, but using a new sample, of size n. It is found that no members of the sample can receive Channel C. Find the largest value of n for which the null hypothesis is not rejected. Show all relevant working. [4]

Q4, (Jun 2007, Q6)

In a rearrangement code, the letters of a message are rearranged so that the frequency with which any particular letter appears is the same as in the original message. In ordinary German the letter e appears 19% of the time. A certain encoded message of 20 letters contains one letter e.

(i) Using an exact binomial distribution, test at the 10% significance level whether there is evidence that the proportion of the letter e in the language from which this message is a sample is less than in German, i.e., less than 19%. [8]

(ii) Give a reason why a binomial distribution might not be an appropriate model in this context. [1]

Q5, (Jan 2008, Q8)

Consultations are taking place as to whether a site currently in use as a car park should be developed as a shopping mall. An agency acting on behalf of a firm of developers claims that at least 65% of the local population are in favour of the development. In a survey of a random sample of 12 members of the local population, 6 are in favour of the development.

(i) Carry out a test, at the 10% significance level, to determine whether the result of the survey is consistent with the claim of the agency. [7]

(ii) A local residents' group claims that no more than 35% of the local population are in favour of the development. Without further calculations, state with a reason what can be said about the claim of the local residents' group. [2]

(iii) A test is carried out, at the 15% significance level, of the agency's claim. The test is based on a random sample of size $2n$, and exactly n of the sample are in favour of the development. Find the smallest possible value of n for which the outcome of the test is to reject the agency's claim. [4]

Q6, (Jan 2011, Q9)

A pharmaceutical company is developing a new drug to treat a certain disease. The company will continue to develop the drug if the proportion p of those who have the disease and show a substantial improvement after treatment is greater than 0.7. The company carries out a test, at the 5% significance level, on a random sample of 14 patients who suffer from the disease.

(i) Find the critical region for the test. **[3]**

(ii) Given that 12 of the 14 patients in the sample show a substantial improvement, carry out the test. **[5]**

Q7, (Jun 2014, Q6)

In a city the proportion of inhabitants from ethnic group Z is known to be 0.4. A sample of 12 employees of a large company in this city is obtained and it is found that 2 of them are from ethnic group Z. A test is carried out, at the 5% significance level, of whether the proportion of employees in this company from ethnic group Z is less than in the city as a whole.

(i) State an assumption that must be made about the sample for a significance test to be valid. **[1]**

(ii) Describe briefly an appropriate way of obtaining the sample. **[2]**

(iii) Carry out the test. **[7]**

(iv) A manager believes that the company discriminates against ethnic group Z. Explain whether carrying out the test at the 10% significance level would be more supportive or less supportive of the manager's belief. **[2]**

Q8, (Jun 2016, Q5)

55% of the pupils in a large school are girls. A member of the student council claims that the probability that a girl rather than a boy becomes Head Student is greater than 0.55. As evidence for his claim he says that 6 of the last 8 Head Students have been girls.

(i) Use an exact binomial distribution to test the claim at the 10% significance level. **[7]**

(ii) A statistics teacher says that considering only the last 8 Head Students may not be satisfactory. Explain what needs to be assumed about the data for the test to be valid. **[1]**
