

Binomial Hypothesis Testing (From OCR 4733)

Q1 (Jan 2006, Q3)

$H_0: p = 0.3$	B1	NH stated, must be this form (or π)
$H_1: p \neq 0.3$	B1	AH stated, must be this form (or π) [μ : B1 both]
$B(8, 0.3)$	M1	$B(8, 0.3)$ stated or implied
$P(\leq 4) = 0.9420;$ 0.0580	A1	Any one of these four probabilities seen
$P(\leq 5) = 0.9887;$ 0.0113	M1	Either compare $P(\geq 5) \& 0.025 / P(\leq 4) \& 0.975$
Compare 0.025 or critical value 6	M1	Or critical region ≥ 6 with 5
Do not reject H_0	A1√	H_0 not rejected, can be implied, needs essentially correct method
Insufficient evidence that manufacturer's claim is wrong	7	Correct conclusion in context [SR: Normal, Poisson: can get B2M1A0M0M1A1 $P(\leq 5)$: first 4 marks. $P(= 5)$: first 3 marks only.]

Q2, (Jun 2006, Q2)

(i) Find $P(R \geq 6)$ or $P(R < 6)$ $= 0.0083$ or 0.9917	M1	Find $P(= 6)$ from tables/calc, OR RH critical region
Compare with 0.025 [can be from N]	A1	$P(\geq 6)$ in range $[0.008, 0.0083]$ or $P(< 6) = 0.9917$
[0.05 if "empty LH tail stated]	B1	OR CR is 6 with probability 0.0083/0.9917
Reject H_0	A1√	Explicitly compare with 0.025 [or 0.975 if consistent] OR state that result is in critical region
(ii) $n = 9, P(\leq 1) = 0.0385$ [> 0.025]	M1	Correct comparison and conclusion, \sqrt on their p
$n = 10, P(\leq 1) = 0.0233$ [< 0.025]	A1	At least one, or $n = 8, P(\leq 1) = 0.0632$
Therefore $n = 9$	B1	Both of these probabilities seen, don't need 0.025
	3	Answer $n = 9$ only, indep't of M1A1, <i>not</i> from $P(= 1)$

Q3, (Jan 2007, Q7)

(i) $H_0: p = 0.35$ [or $p \geq 0.35$]	B1	Each hypothesis correct, B1+B1, allow $p \geq .35$ if .35 used
$H_1: p < 0.35$	B1	[Wrong or no symbol, B1, but r or x or \bar{x} : B0]
$B(14, 0.35)$	M1	Correct distribution stated or implied, can be implied by $N(4.9, \dots)$, but <i>not</i> $Po(4.9)$
$\alpha: P(\leq 2) = 0.0839 > 0.025$	A1	0.0839 seen, or $P(\leq 1) = 0.0205$ if clearly using CR
$\beta: CR \leq 1$, probability 0.0205	B1	Compare binomial tail with 0.025, or $R = 2$ binomial CR
Do not reject H_0 . Insufficient evidence that proportion that can receive Channel C is less than 35%	M1	Do not reject H_0 , \sqrt on their probability, <i>not</i> from N or Po
	A1√	or $P(< 2)$; Contextualised conclusion \sqrt
(ii) $B(8, 0.35): P(0) = 0.0319$	M1	Attempt to find $P(0)$ from $B(n, 0.35)$
$B(9, 0.35): P(0) = 0.0207$	A1	One correct probability [$P(\leq 2) = .0236, n = 18$: M1A1]
Hence largest value of n is 8	A1	Both probabilities correct
	A1	Answer 8 or ≤ 8 only, needs minimum M1A1
or $0.65^n > 0.025; n \ln 0.65 > \ln 0.025$	M1M1	$p^n > 0.025$, any relevant p ; take ln, or T&I to get 1 SF
$8.56;$ largest value of $n = 8$	A1A1	In range $[8.5, 8.6]$; answer 8 or ≤ 8 only

Q4, (Jun 2007, Q6)

(i)	$H_0: p = 0.19, H_1: p < 0.19$ where p is population proportion $0.81^{20} + 20 \times 0.81^{19} \times 0.19$ $= 0.0841$ Compare 0.1	B2 M1 A1 A1 B1		Correct, B2. One error, B1, but x or \bar{x} or r : B0 Binomial probabilities, allow 1 term only Correct expression [0.0148 + 0.0693] Probability, a.r.t. 0.084 Explicit comparison of "like with like"
or	Add binomial probs until ans > 0.1 Critical region ≤ 1	A1 B1		[$P(\leq 2) = 0.239$]
	Reject H_0 Significant evidence that proportion of e 's in language is less than 0.19	M1 A1√	8	Correct deduction and method [needs $P(\leq 1)$] Correct conclusion in context [SR: N(3.8, 3.078): B2M1A0B1M0]
(ii)	Letters not independent	B1	1	Correct modelling assumption, stated in context Allow "random", "depends on message", etc

Q5, (Jan 2008, Q8)

(i)	$H_0: p = 0.65$ OR $p \geq 0.65$ $H_1: p < 0.65$ B(12, 0.65)	B2 M1		Both hypotheses correctly stated, in this form [One error (but not r, x or \bar{x}): B1] B(12, 0.65) stated or implied
	α : $P(\leq 6) = 0.2127$ Compare 0.10	A1 B1		Correct probability from tables, <i>not</i> $P(= 6)$ Explicit comparison with 0.10
	β : Critical region $\leq 5; 6 > 5$ Probability 0.0846	B1 A1		Critical region ≤ 5 or ≤ 6 or $\{\leq 4\} \cap \{\geq 11\}$ & compare 6 Correct probability
	Do not reject H_0 Insufficient evidence that proportion of population in favour is not at least 65%	M1√ A1√	7	Correct comparison and conclusion, needs correct distribution, correct tail, like-with-like Interpret in context, e.g. "consistent with claim" [SR: N(7.8, 2.73): can get B2M1A0B1M0: 4 ex 7]
(ii)	Insufficient evidence to reject claim; test and p/q symmetric	B1√ B1	2	Same conclusion as for part (i), don't need context Valid relevant reason, e.g. "same as (i)"
(iii)	$R \sim B(2n, 0.65), P(R \leq n) > 0.15$ B(18, 0.65), $p = 0.1391$ Therefore $n = 9$	M1 A1 A1 A1	4	B(2n, 0.65), $P(R \leq n) > 0.15$ stated or implied Any probability in list below seen $p = 0.1391$ picked out (i.e., not just in a list of > 2) Final answer $n = 9$ only [SR < n : M1A0, $n = 4, 0.1061$ A1A0] [SR 2-tail: M1A1A0A1 for 15 or 14] [SR: 9 only, no working: M1A1] [MR B(12, 0.35): M1A0, $n = 4, 0.1061$ A1A0] 3 0.3529 7 0.1836 12 0.0942 4 0.2936 8 0.1594 13 0.0832 5 0.2485 9 0.1391 14 0.0736 6 0.2127 10 0.1218 15 0.0652

Q6, (Jan 2011, Q9)

(i)	B(14, 0.7) CR is ≥ 13 with probability 0.0475	M1 A1 A1	3	B(14, 0.7) stated or implied, e.g. N(9.8, 2.94), can be recovered CV 13, or > 12 or {13, 14}, allow = but no other inequalities Exactly correct CR, and supporting prob .0475 or .9525 seen
(ii)	$H_0: p = 0.7, H_1: p > 0.7$ $12 < 13$ Do not reject H_0 . Insufficient evidence that proportion who show improvement is greater than 0.7	B2 B1 M1 A1 FT	5	Both, B2. Allow π . One error, B1, but r, x etc: B0 Compare CV from correct tail and inequality with 12, or $P(\geq 12) = 0.1608$ and > 0.05 or $P(< 12) = 0.8392$ and < 0.95 Correct method & conclusion, requires like-with-like; CV method needs ≥ 13 or < 12 ; p method needs ≥ 12 or < 12 Withhold if inconsistent Contextualised, acknowledge uncertainty [SR: Normal or Po: (i) M1, (ii) B2 maximum] [0.9932 or 0.0068 probably B2 maximum]
(iii)	B(14, 0.8) $P(\leq 12)$ from B(14, 0.8) 0.8021	M1 M1 A1	3	B(14, 0.8) stated or implied, allow from B(14, 0.75) Attempt prob of acceptance region, e.g. 0.8990, $\sqrt{\quad}$ on (i) Answer 0.802 or a.r.t. 0.8021

Q7, (Jun 2014, Q6)

(i)	Sample is random	B1 [1]	Indicate random sample. Allow “unbiased sample” or “randomly selected” or “all equally likely”. Allow “representative” provided it’s clearly “of company” (not city) Not just “independent”. Withhold if extra wrong bits.
(ii)	List population, number sequentially Select using random numbers	B1 B1 [2]	List can be implied; must imply employees or people. “Sequential” can be assumed. Not “select numbers randomly”, Don’t need “ignore outside range” etc. Number randomly <i>and</i> select randomly, B1, but “assign random nos & arrange”, B2 SC: Put names into hat/lottery machine and take them out: B2 SC: Systematic: B1 for list, can get second B1 if starting-point random
(iii)	$H_0: p = 0.4; H_1: p < 0.4$ B(12, 0.4)	B2 M1	Both correct, B2. Allow π . One error, e.g. μ or no symbol, B1, but \bar{x} , z etc: B0. B(12, 0.4) stated or implied. Can be implied by N(4.8, 2.88) but no further marks. 0.1673, 0.0398, 0.1513, 0.0421: M1A0(A1M1A1)
α :	$P(\leq 2) = 0.0834$ > 0.05	A1 A1	$P(\leq 2) = 0.0834$, or $P(> 2) = 0.9166$. Compare numerical $P(\leq 2)$ with 0.05, or $P(> 2)$ with 0.95
β :	CR is ≤ 1 0.0196 seen and compare 2 with ≤ 1	A1 A1	CR is ≤ 1 stated. Explicitly compare 2 with CR, probability 0.0196 must be seen
	Do not reject H_0 . Insufficient evidence that proportion of employees from group Z is less.	M1 A1ft [7]	Correct first conclusion, needs $P(\leq 2 p = 0.4)$ or fully consistent equivalent In context (mention “employees”, “city” etc), acknowledge uncertainty (“evidence”) <i>Not</i> “there is evidence that the proportion of employees is 0.4” FT on wrong p -value or wrong critical value if previous mark gained SC: Normal: B2 M1 max SC: $P(= 2)$ or $P(\geq 2)$ or $P(< 2)$: B2 M1 max SC: two-tailed: can get B1B0 M1A1A0 M1A1 (don’t give second A1 for 0.05)
(iv)	Yes as H_0 is rejected	M1 A1 [2]	Realise this changes conclusion (FT!), or “more likely to reject H_0 ”, “larger CR” More supportive [just “more supportive” without evidence is M0A0]

Q8, (Jun 2016, Q5)

(i)	$H_0: p = 0.55, H_1: p > 0.55$ $R \sim B(8, 0.55)$ where R is the number of girls $\alpha: P(R \geq 6) = 1 - 0.7799 = 0.2201 > 0.1$	B2 M1 A1 B1	All correct, B2. One error (e.g. \neq , wrong or no letter) B1, but r, x etc: B0 $B(8, 0.55)$ stated or implied, e.g. $N(4.4, 1.98)$ $P(\geq 6) = 0.2201$, or $P(< 6) = 0.7799$ Compare $P(\geq 6)$ with 0.1 or $P(< 6)$ with 0.9
	$\beta: CR \text{ is } \geq 7 \text{ and } 6 < 7$ $p = 0.0632$	B1 A1	Correct CR stated and explicit comparison with 6 This probability seen, a.r.t. 0.0632. Award if 0.9368 seen and CR is correct. If CR not clearly stated, cannot get last M1A1
	Do not reject H_0 . There is insufficient evidence that the girls are proportionately more likely to become Head Student.	M1 A1 7	Correct first conclusion, requires $B(8, 0.55)$, <i>not</i> $P(> 6) [= 0.0632]$ or $P(\leq 6) [= 0.9368]$ or $P(= 6) [= 0.1569]$. Allow 0.7799 if compared with 0.9 Interpreted, in context, acknowledge uncertainty, double negative. SC: Normal: max B2 M1 SC: Two different attempts: max B2 M1 unless both correct
(ii)	Assume that the last 8 years are a random sample of years when Head Student has been chosen	B1 1	Refer to random sample, allow implied by any method described Must be choosing <i>years</i> , not <i>students</i> <i>Not</i> quote conditions for random sample unless explicitly “years” Extras: ignore unless clearly wrong, in which case B0