



Binomial Hypothesis Testing (Sheet 2)

Q1.

Afrika works in a call centre.

She assumes that calls are independent and knows, from past experience, that on each sales call

that she makes there is a probability of $\frac{1}{6}$ that it is successful.

Afrika makes 9 sales calls.

(a) Calculate the probability that at least 3 of these sales calls will be successful.

(2)

The probability of Afrika making a successful sales call is the same each day.

Afrika makes 9 sales calls on each of 5 different days.

(b) Calculate the probability that at least 3 of the sales calls will be successful on exactly 1 of these days.

(2)

Rowan works in the same call centre as Afrika and believes he is a more successful salesperson.

To check Rowan's belief, Afrika monitors the next 35 sales calls Rowan makes and finds that 11 of the sales calls are successful.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence to support Rowan's belief.

(4)

(Total for question = 8 marks)

Q2.

In a manufacturing process 25% of articles are thought to be defective. Articles are produced in batches of 20

(a) A batch is selected at random. Using a 5% significance level, find the critical region for a two tailed test that the probability of an article chosen at random being defective is 0.25

You should state the probability in each tail which should be as close as possible to 0.025

(5)

The manufacturer changes the production process to try to reduce the number of defective articles. She then chooses a batch at random and discovers there are 3 defective articles.

(b) Test at the 5% level of significance whether or not there is evidence that the changes to the process have reduced the percentage of defective articles. State your hypotheses clearly.

(5)

(Total 10 marks)



Q3.

A potter believes that 20% of pots break whilst being fired in a kiln. Pots are fired in batches of 25.

(a) Let X denote the number of broken pots in a batch. A batch is selected at random. Using a 10% significance level, find the critical region for a two tailed test of the potter's belief. You should state the probability in each tail of your critical region.

(4)

The potter aims to reduce the proportion of pots which break in the kiln by increasing the size of the batch fired. He now fires pots in batches of 50. He then chooses a batch at random and discovers there are 6 pots which broke whilst being fired in the kiln.

(b) Test, at the 5% level of significance, whether or not there is evidence that increasing the number of pots in a batch has reduced the percentage of pots that break whilst being fired in the kiln. State your hypotheses clearly.

(5)

(Total for question = 9 marks)

Q4.

The proportion of houses in Radville which are unable to receive digital radio is 25%. In a survey of a random sample of 30 houses taken from Radville, the number, X , of houses which are unable to receive digital radio is recorded.

(a) Find $P(5 \leq X < 11)$

(3)

A radio company claims that a new transmitter set up in Radville will reduce the proportion of houses which are unable to receive digital radio. After the new transmitter has been set up, a random sample of 15 houses is taken, of which 1 house is unable to receive digital radio.

(b) Test, at the 10% level of significance, the radio company's claim. State your hypotheses clearly.

(5)

(Total for question = 8 marks)



Q5.

Past records show that 15% of customers at a shop buy chocolate. The shopkeeper believes that moving the chocolate closer to the till will increase the proportion of customers buying chocolate.

After moving the chocolate closer to the till, a random sample of 30 customers is taken and 8 of them are found to have bought chocolate.

Julie carries out a hypothesis test, at the 5% level of significance, to test the shopkeeper's belief.

Julie's hypothesis test is shown below.

$$H_0 : p = 0.15$$

$$H_1 : p \geq 0.15$$

Let X = the number of customers who buy chocolate.

$$X \sim B(30, 0.15)$$

$$P(X = 8) = 0.0420$$

$$0.0420 < 0.05 \text{ so reject } H_0$$

There is sufficient evidence to suggest that the proportion of customers buying chocolate has increased.

- (a) Identify the first two errors that Julie has made in her hypothesis test. (2)
- (b) Explain whether or not these errors will affect the conclusion of her hypothesis test. Give a reason for your answer. (1)
- (c) Find, using a 5% level of significance, the critical region for a one-tailed test of the shopkeeper's belief. The probability in the tail should be less than 0.05 (2)
- (d) Find the actual level of significance of this test. (1)

(Total for question = 6 marks)

Q6.

Sue throws a fair coin 15 times and records the number of times it shows a head.

- (a) State the distribution to model the number of times the coin shows a head. (2)

Find the probability that Sue records

- (b) exactly 8 heads,
(c) at least 4 heads. (2)

Sue has a different coin which she believes is biased in favour of heads. She throws the coin 15 times and obtains 13 heads.

- (d) Test Sue's belief at the 1% level of significance. State your hypotheses clearly. (6)

(Total for question = 10 marks)

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Q7.

Sue throws a fair coin 15 times and records the number of times it shows a head.

(a) State the distribution to model the number of times the coin shows a head.

(2)

Find the probability that Sue records

(b) exactly 8 heads,

(c) at least 4 heads.

(2)

Sue has a different coin which she believes is biased in favour of heads. She throws the coin 15 times and obtains 13 heads.

(d) Test Sue's belief at the 1% level of significance. State your hypotheses clearly.

(6)

(Total for question = 10 marks)

Q8.

A student takes a multiple choice test. The test is made up of 10 questions each with 5 possible answers. The student gets 4 questions correct. Her teacher claims she was guessing the answers. Using a one tailed test, at the 5% level of significance, test whether or not there is evidence to reject the teacher's claim. State your hypotheses clearly.

(6)

(Total 6 marks)

Q9.

A single observation x is to be taken from a Binomial distribution $B(20, p)$.

This observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(a) Using a 5% level of significance, find the critical region for this test. The probability of rejecting either tail should be as

close as possible to 2.5%.

(3)

(b) State the actual significance level of this test.

(2)

The actual value of x obtained is 3.

(c) State a conclusion that can be drawn based on this value giving a reason for your answer.

(2)

(Total 7 marks)



Q10.

- (a) The discrete random variable $X \sim B(40, 0.27)$

Find $P(X \geq 16)$

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

- (b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(3)

- (d) Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

- (e) Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

- (f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

(Total for question = 9 marks)

Q11.

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager questions whether or not there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

- (a) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's question. You should state the probability of rejection in each tail which should be less than 0.05.

(5)

- (b) Write down the actual significance level of a test based on your critical region from part (a).

(1)

The manager found that 11 customers from the sample of 20 had bought baked beans in single tins.

- (c) Comment on this finding in the light of your critical region found in part (a).

(2)

(Total 8 marks)

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Q12.

(a) Define the critical region of a test statistic.

(2)

A discrete random variable X has a Binomial distribution $B(30, p)$. A single observation is used to test

$$H_0 : p = 0.3 \text{ against } H_1 : p \neq 0.3$$

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each

tail which should be as close as possible to 0.005

(5)

(c) Write down the actual significance level of the test.

(1)

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region.

(2)

(Total 10 marks)