Binomial Hypothesis Testing (Sheet 2) Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Let C = the number of successful calls. $C \sim B\left(9, \frac{1}{6}\right)$	M1	3.3
	$P(C \geq 3) = 1 - P(C \leq 2) = 0.1782\ldots$ **awrt 0.178**	A1	1.1b
		(2)	
(b)	Let X = the number of occasions when at least 3 calls are successful. $P(X = 1) = 5 \times (\text{"0.1782..."}) \times (\text{"0.8217..."})^4$	M1	1.1b
	$= 0.4061\ldots$ **awrt 0.406**	A1	1.1b
		(2)	
(c)	$H_0 : p = \frac{1}{6}$ $H_1 : p > \frac{1}{6}$	B1	2.5
	Let R = the number of successful calls $R \sim B\left(35, \frac{1}{6}\right)$	M1	3.3
	$P(R \geq 11) = 1 - P(R \leq 10) = 0.02\ldots$	A1	3.4
	There is sufficient evidence to support that **Rowan** has more successful sales calls than Afrika.	A1	2.2b
		(4)	
			(8 marks)

Q2.

Question Number	Scheme	Marks
(a)	$X \sim B(20, 0.25)$ $P(X \geq 10) = 1 - 0.9861 = 0.0139$ $P(X \leq 1) = 0.0243$ $(0 \leq) X \leq 1 \cup 10 \leq X (\leq 20)$	M1 A1 A1 A1A1 (5)
(b)	$H_0: p = 0.25$ $H_1: p < 0.25$ $X \sim B(20, 0.25)$ $P(X \leq 3) = 0.2252$ or CR $X \leq 1$ Insufficient evidence to reject H_0, Accept H_0, Not significant. 3 does not lie in the Critical region. No evidence that the **changes** to the process have **reduced** the **percentage** of **defective articles (oe)**	 B1 M1A1 M1d A1cso (5) Total 10 marks

Q3.

Question Number	Scheme		Marks
	Allow any letter instead of X or c for this question		
(a)	$X \sim B(25, 0.2)$	**M1** Writing or using B(25,0.2)or B(25,1/5) [allow Po(5)] May be written in full or implied by a correct CR (allow written as a probability statement)	M1
	$[P(X \geq 9) =]0.0468$ $[P(X \leq 1) =]0.0274$	**1st A1** both awrt 0.0468 and awrt 0.0274 seen.	A1
	$X = [0 \leq]\ X \leq 1$	**2nd A1** $X \leq 1$ or $X < 2$ or $0 \leq X \leq 1$ or [0,1] or 0,1 or equivalent statements. $X \leq c$ and $c = 1$	A1
	$9 \leq X\ \ [\leq 25]$	**3rd A1d** dependent on seeing a probability from the B(25, 0.2) and $X \geq 9$ or $X > 8$ or $9 \leq X \leq 25$ or 9,10,11,12,13,14,15,16,17,18,19,20,21,22, 23,24,25 or [9,25] or equivalent statements. $X \geq c$ and $c = 9$	A1d
	NB These two final 2 A marks must be for statements with "X" only(or list) – not in probability statements **SC** If a probability from the B(25, 0.2) is seen **and** they either have both CR correct but written as probability statements **or** the CR is written as $1 \geq X \geq 9$ they get A1 A0 for final 2 marks		**(4)**
(b)	$H_0: p = 0.2$ $H_1: p < 0.2$	**B1** both hypotheses with p or π and clear which is H_0 and which is H_1	**B1**
	$P(X \leq 6) = 0.1034$ or CR $X \leq 5$	**1st M1** writing or using B(50, 0.2) and writing or using $P(X \leq 6)$ or $P(X \geq 7)$ on its own. May be implied by a correct CR	**M1**
		1st A1 awrt 0.103. Allow CR $X \leq 5$ or $X < 6$. or if not using CR allow awrt 0.897.	**A1**
	Insufficient evidence to reject H_0, Accept H_0, Not significant. 6 does not lie in the Critical region.	**2nd M1** dependent on previous M being awarded. A correct statement (do not allow if there are contradicting non-contextual statements). ft their Prob/CR compared with 0.05/6/(0.95 if using 0.8979). Do not follow through their hypotheses	**M1d**
	No evidence that increasing the batch size has **reduced** the **percentage** of broken **pots (oe)** **or** evidence that there is **no change** in the **percentage** of broken **pots (oe)**	**2nd A1cso** Conclusion must contain the words **reduced/ no change/not affect oe number/percentage/proportion/ probability oe**, and **pots**. All previous marks must be awarded for this mark to be awarded. Do **not** allow the potters claim /belief is wrong/true **NB** Correct contextual statement on its own scores M1A1	**A1cso** **(5)**
			(Total 9)

Q4.

Question Number	Scheme		Marks
(a)		**notes**	
	$X \sim B(30, 0.25)$	B1: using B(30, 0.25)	B1
	$P(X \leq 10) - P(X \leq 4) = 0.8943 - 0.0979$	M1: using $P(X \leq 10) - P(X \leq 4)$ or $P(X \geq 5) - P(X \geq 11)$ oe	M1 A1
	$= 0.7964$	A1: awrt 0.796	
	NB a correct answer gains full marks		

(b)	$H_0 : p = 0.25 \quad H_1 : p < 0.25$	B1: Both hypotheses correct, labelled H_0 or NH or H_n and H_1or AH or H_a , must use p or $p(x)$ or π	B1
	B(15, 0.25)	M1: for using B(15, 0.25)	M1 A1
	$P(X \leq 1) = 0.0802$	A1: awrt 0.0802 or CR $X \leq 1$ (allow $P(X \geq 2) = 0.9198$)	
	NB: Allow M1 A1 for a correct CR with no incorrect working		
	Reject H_0 or Significant or 1 1ies in the critical region	M1: A correct statement – do not allow contradictory non contextual statements. Follow through their Probability/CR (for 1 or 2 tail test). If no H_1 given then M0. Ignore their comparison. For a probabillity < 0.5, statement must be correct compared to 0.1 for 1 tail test and 0.05 for 2 tailed test or if the probability > 0.5, statement must be correct compared to 0.9 for 1 tail test and 0.95 for 2 tailed test.	dM1 A1cso
	There is evidence that the radio **company's** claim is true. **Or** The new transmitter will reduce the proportion of houses unable to receive **radio**	A1: cso (all previous marks awarded) and a correct statement containing the word **company** if writing about the claim or **radio** if full context.	

Q5.

Question	Scheme	Marks	AOs
(a)	The alternative hypothesis should be $H_1: p > 0.15$	B1	2.5
	The calculation of the test statistic should be $P(X \geq 8)$ [= 0.0698]	B1	2.3
		(2)	
(b)	These will affect the conclusion (as the null hypothesis should not be rejected) since $P(X \geq 8)$ [= 0.0698] is greater than 0.05	B1	2.4
		(1)	
(c)	$P(X \leq 8) = 0.9722\ldots > 0.95$ or $P(X \geq 9) = 0.0277\ldots < 0.05$	M1	2.1
	CR: $\{X \geq 9\}$	A1	1.1b
		(2)	
(d)	awrt **0.0278**	B1ft	1.1b
		(1)	
			(6 marks)

Notes	
(a)	**B1:** Identifying that $\geq$ should be $>$ in the alternative hypothesis **B1:** Identifying that $P(X = 8)$ should be $P(X \geq 8)$ Stating $P(X = 8)$ is incorrect on its own is insufficient Check for errors identified and corrected next to the question
(b)	**B1:** Will affect conclusion **and** correct supporting reason
(c)	**M1:** For use of tables to find probability associated with critical value [$P(X \leq 8)$ or $P(X \geq 9)$ with B(30, 0.15) (may be implied by either correct probability awrt 0.97 or awrt 0.03) or by the correct CR] **A1:** $[30 \geq] X \geq 9$ o.e. e.g. $X > 8$ Allow '9 or more' or 'CR $\geq$ 9'
(d)	**B1ft:** awrt 0.0278 (allow awrt 2.78%) or correct ft their one-tailed upper CR from B(30, 0.15) to 3s.f.

Q6.

Question Number	Scheme	Marks
(a)	$X \sim B(15, 0.5)$	B1 B1 (2)
(b)	$P(X = 8) = P(X \leq 8) - P(X \leq 7)$ or $\left(\frac{15!}{8!7!}(p)^8(1-p)^7\right)$	M1
	$= 0.6964 - 0.5$	
	$= 0.1964$ awrt 0.196	A1 (2)
(c)	$P(X \geq 4) = 1 - P(X \leq 3)$	M1
	$= 1 - 0.0176$	
	$= 0.9824$	A1 (2)
(d)	$H_0 : p = 0.5$ $H_1 : p > 0.5$	B1 B1
	$X \sim B(15, 0.5)$	
	$P(X \geq 13) = 1 - P(X \leq 12)$ | $[P(X \geq 12) = 1 - 0.9824 = 0.0176]$ att $P(X \geq 13)$ $= 1 - 0.9963$ | $P(X \geq 13) = 1 - 0.9963 = 0.0037$ $= 0.0037$ | CR $X \geq 13$ awrt 0.0037/ CR $X \geq 13$	M1 A1
	$0.0037 < 0.01$ | $13 \geq 13$	
	Reject H_0 or it is significant or a correct statement in context from their values	M1
	There is sufficient evidence at the 1% significance level that the coin is biased in favour of heads *or* There is evidence that Sue's belief is correct	A1 (6)
		(12 marks)

Q7.

Question Number	Scheme	Marks
(a)	$X \sim B(15, 0.5)$	B1 B1 (2)
(b)	$P(X = 8) = P(X \leq 8) - P(X \leq 7)$ or $\left(\frac{15!}{8!7!}(p)^8(1-p)^7\right)$	M1
	$= 0.6964 - 0.5$	
	$= 0.1964$ awrt 0.196	A1 (2)
(c)	$P(X \geq 4) = 1 - P(X \leq 3)$	M1
	$= 1 - 0.0176$	
	$= 0.9824$	A1 (2)
(d)	$H_0 : p = 0.5$	B1
	$H_1 : p > 0.5$	B1
	$X \sim B(15, 0.5)$	
	$P(X \geq 13) = 1 - P(X \leq 12)$ $= 1 - 0.9963$ $= 0.0037$ $0.0037 < 0.01$ | $[P(X \geq 12) = 1 - 0.9824 = 0.0176]$ att $P(X \geq 13)$ $P(X \geq 13) = 1 - 0.9963 = 0.0037$ CR $X \geq 13$ awrt 0.0037/ CR $X \geq 13$ $13 \geq 13$	M1 A1
	Reject H_0 or it is significant or a correct statement in context from their values	M1
	There is sufficient evidence at the 1% significance level that the coin is biased in favour of heads *or* There is evidence that Sue's belief is correct	A1 (6)
		(12 marks)

Q8.

Question Number	Scheme	Marks
	$H_0 : p = 0.2$ $H_1 : p > 0.2$	B1
	Under H_0, $X \sim \text{Bin}(10,0.2)$	B1
	$P(X \geq 4)$ $= 1 - P(X \leq 3)$ OR $P(X \leq 4) = 0.9672$	M1
	$= 1 - 0.8791$ $P(X \geq 5) = 0.0328$	
	$= 0.1209$ CR $X \geq 5$	A1
	0.1209>0.05. Insufficient evidence to reject H_0 so teacher's claim is supported.	M1A1ft
		[6]
	Notes	
	B1 for both H_0 and H_1 correct. Must use p or π (pi) B1 for writing or using Bin(10,0.2) M1 for finding or writing $1 - P(X \leq 3)$ or $P(X \leq 4) = 0.9672$ $P(X \geq 5) = 0.0328$ oe or a correct critical region A1 awrt 0.121 or CR $X \geq 5$ M1 need $p<0.5$ and: correct statement using their Probability and 0.05 if one tail test or correct statement using their Probability and 0.025 if two tail test (condone a comparison with 0.05 instead of 0.025 for a two tail test). Do not allow non-contextual conflicting statements eg "significant" and "accept H_0" A1ft correct contextual statement followed through from "their prob". Either a comment on whether the teacher's claim was correct or on whether the student was guessing the answers. NB if a correct contextual statement only is given for their probability then award M1 A1 If $p>0.5$ They may compare with 0.95 (one tail method) or 0.975 (two tail method) Probability is 0.8791.	

Q9.

Question Number	Scheme	Marks
(a)	$X \sim B(20, 0.3)$	M1
	$P(X \leq 2) = 0.0355$	
	$P(X \geq 11) = 1 - 0.9829 = 0.0171$	
	Critical region is $(X \leq 2) \cup (X \geq 11)$	A1 A1 (3)
(b)	Significance level = 0.0355 + 0.0171, = 0.0526 or 5.26%	M1 A1 (2)
(c)	Insufficient evidence to reject H_0 **Or** sufficient evidence to accept H_0 /not significant	B1 ft
	$x = 3$ (or the value) is not in the critical region or 0.1071> 0.025	B1 ft (2)
	Do not allow inconsistent comments	

Q10.

Question	Scheme	Marks	AOs
(a)	$P(X \geqslant 16) = 1 - P(X \leqslant 15)$	M1	1.1b
	$= 1 - 0.949077\ldots$ = awrt **0.0509**	A1	1.1b
		(2)	
(b)	$H_0 : p = 0.3$ $H_1 : p \neq 0.3$ (Both correct in terms of p or π)	B1	2.5
		(1)	
(c)	$[Y \sim B(20, 0.3)]$ sight of $P(Y \leqslant 2) = 0.0355$ or $P(Y \leqslant 9) = 0.9520$	M1	2.1
	Critical region is $\{Y \leqslant 2\}$ or (o.e.)	A1	1.1b
	$\{Y \geqslant 10\}$ (o.e.)	A1	1.1b
		(3)	
(d)	$[0.0355 + (1 - 0.9520)] = 0.0835$ or **8.35%**	B1ft	1.1b
		(1)	
(e)	(Assuming that the 20 customers represent a random sample then) 12 is in the CR so the manager's suspicion is supported	B1ft	3.2a
		(1)	
(f)	e.g. (e) requires the 20 customers to be a random sample or independent and the members of the scout group may invalidate this so binomial distribution would not be valid (and conclusion in (e) is probably not valid)	B1	3.5a
		(1)	
			(9 marks)

Part	Notes
(a)	M1 for dealing with $P(X \geqslant 16)$ – they need to use cumulative prob. function on calc.
	A1 awrt 0.0509 (from calculator)
(b)	B1 for both hypotheses in terms of p or π and H_1 must be 2-tail
(c)	M1 for correct use of tables to find probability associated with critical value.
	1st A1 for the correct lower limit of the CR. Do not award for $P(Y \leqslant 2)$
	2nd A1 for the correct upper limit.
(d)	B1ft ft on their 0.0355 and (1 – their 0.9520) provided each probability is less than 0.05
(e)	B1ft for a comment that relates 12 to their CR and makes a consistent comment relating this to the manager's suspicion
(f)	B1 for a comment that: gives a suitable reason based on lack of independence or the sample not being random so the binomial model is not valid

Q11.

Question Number	Scheme	Marks
(a)	$X \sim B(20, 0.3)$ $P(X \leq 9) = 0.9520$ so $P(X \leq 2) = 0.0355$ $P(X \geq 10) = 0.0480$ Therefore the critical region is $\{X \leq 2\} \cup \{X \geq 10\}$	M1 A1 A1 A1A1 (5)
(b)	$0.0355 + 0.0480 = 0.0835$ awrt (0.083 or 0.084)	B1 (1)
(c)	11 is in the critical region there is evidence of a change/ increase in the proportion/number of customers buying single tins	B1ft B1ft (2) [8]
(a)	M1 for B(20,0.3) seen or used 1st A1 for 0.0355 2nd A1 for 0.048 3rd A1 for $(X) \leq 2$ or $(X) < 3$ or [0,2] They get **A0** if they write $P(X \leq 2 / X < 3)$ 4th A1 $(X) \geq 10$ or $(X) > 9$ or [10,20] They get **A0** if they write $P(X \geq 10 / X > 9)$ **10** $\leq X \leq 2$ etc is accepted To describe the critical regions they can use any letter or no letter at all. It does not have to be X.	
(b)	B1 correct answer only	
(c)	1st B1 for a correct statement about 11 and their critical region. 2nd B1 for a correct comment in context consistent with their CR and the value 11 Alternative solution 1st B0 $P(X \geq 11) = 1 - 0.9829 = 0.0171$ since no comment about the critical region 2nd B1 a correct contextual statement.	

Q12.

Question Number	Scheme	Marks
(a)	The set of values of the test statistic for which the null hypothesis is rejected in a hypothesis test.	B1 B1 (2)
(b)	X~B(30,0.3) $P(X \leq 3) = 0.0093$ $P(X \leq 2) = 0.0021$ $P(X \geq 16) = 1 - 0.9936 = 0.0064$ $P(X \geq 17) = 1 - 0.9979 = 0.0021$ Critical region is $(0 \leq) x \leq 2$ or $16 \leq x (\leq 30)$	M1 A1 A1 A1A1 (5)
(c)	Actual significance level 0.0021+0.0064=0.0085 or 0.85%	B1 (1)
(d)	15 (it) is not in the critical region not significant No significant evidence of a change in $p = 0.3$ accept H_0, (reject H_1) $P(x \geq 15) = 0.0169$	Bft 2, 1, 0 (2) **Total [10]**
	Notes	
(a)	**1st B1** for "values/ numbers" **2nd B1** for "reject the null hypothesis" o.e or the test is significant	
(b)	**M1** for using B(30,0.3) **1st A1** $P(x \leq 2) = 0.0021$ **2nd A1** 0.0064 **3rd A1** for $(X) \leq 2$ or $(X) < 3$ They get **A0** if they write $P(X \leq 2 / X < 3)$ **4th A1** $(X) \geq 16$ or $(X) > 15$ They get **A0** if they write $P(X \geq 16\, X > 15$ **NB these are B1 B1 but mark as A1 A1** $16 \leq X \leq 2$ etc is accepted To describe the critical regions they can use any letter or no letter at all. It does not have to be X.	
(c)	**B1** correct answer only	
(d)	Follow through 15 and their critical region **B1** for any one of the 5 correct statements up to a maximum of B2 – **B1** for any incorrect statements	