Question 1

Worked Solution

Part (i): Name distribution, parameters and assumptions

Let X = number of seeds that germinate out of 10.

Distribution: Binomial, $X \sim B(10, 0.9)$.

Parameters: $n = 10$, $p = 0.9$.

Assumptions required for the model to be valid:

- Each seed is equally likely to germinate (probability of germinating is constant at 0.9 for each seed).
- Seeds germinate independently of each other.

> $X \sim B(10, 0.9)$. Assumptions: constant probability (each seed equally likely to germinate); seeds germinate independently.

Part (ii): $P(X < 8)$

$$P(X < 8) = P(X \leq 7)$$

Using the calculator: B(10, 0.9), cumulative up to 7:

$$P(X \leq 7) = 0.0702$$

> $P(X < 8) = 0.0702 \quad$ (3 s.f.)

Part (iii): 20 trays, at least 19 in which at least 8 seeds germinate

Let Y = number of trays (out of 20) in which at least 8 seeds germinate.

In part (ii) we calculated $P(X \leq 7)$. Here a success is classed as

$$\text{P(at least 8 seeds germinate in a tray)} = P(X \geq 8) = 1 - P(X \leq 7) = 0.9298.$$

So $Y \sim B(20, 0.9298)$.

$$P(Y \geq 19) = 1 - P(Y \leq 18)$$

Using the calculator: B(20, 0.9298), cumulative up to 18:

$$P(Y \leq 18) = 0.4253\ldots$$

$$P(Y \geq 19) = 1 - 0.4253 = 0.5747\ldots \approx 0.575$$

> $P(Y \geq 19) \approx 0.575 \quad$ (3 s.f.)

Question 2

Worked Solution

Part (i)(a): $W \sim \mathrm{B}(10, \frac{1}{3})$**, find** $\mathrm{P}(W \leq 2)$

Using tables (or calculator) for $\mathrm{B}(10, \frac{1}{3})$:

$$\mathrm{P}(W \leq 2) = 0.299 \quad (3 \text{ s.f.})$$

$\mathrm{P}(W \leq 2) = 0.299$

Part (i)(b): $\mathrm{P}(W = 2)$

Using the calculator: $\mathrm{B}(10, \frac{1}{3})$, $\mathrm{P}(W = 2)$:

$$\mathrm{P}(W = 2) = 0.1951\ldots \approx 0.195$$

$\mathrm{P}(W = 2) = 0.195$ (3 s.f.)

Part (ii)(a): $X \sim \mathrm{B}(15, 0.22)$**, find** $\mathrm{P}(X = 4)$

Using the calculator: $\mathrm{B}(15, 0.22)$, $\mathrm{P}(X = 4)$:

$$\mathrm{P}(X = 4) = 0.2082\ldots \approx 0.208$$

$\mathrm{P}(X = 4) = 0.208$ (3 s.f.)

Question 3

Worked Solution

Part (i)(a): $X \sim \mathrm{B}(12, 0.85)$, **find** $\mathrm{P}(X > 10)$

$$\mathrm{P}(X > 10) = 1 - \mathrm{P}(X \leq 10)$$

Using the calculator: B(12, 0.85), cumulative up to 10:

$$\mathrm{P}(X \leq 10) = 0.5565\ldots$$

$$\mathrm{P}(X > 10) = 1 - 0.5565 = 0.4435\ldots \approx 0.444$$

$\mathrm{P}(X > 10) = 0.4435$ (awrt 0.443 or 0.444)

Part (i)(b): $\mathrm{P}(X = 10)$

Using the calculator: B(12, 0.85), $\mathrm{P}(X = 10)$:

$$\mathrm{P}(X = 10) = 0.2924\ldots \approx 0.292$$

$\mathrm{P}(X = 10) = 0.292$ (awrt 0.292 or 0.293)

Part (ii): $Y \sim \mathrm{B}(2, \frac{1}{4})$, **two independent values,** $\mathrm{P}(Y_1 + Y_2 = 1)$

The sum $Y_1 + Y_2 = 1$ can happen in two ways: $(Y_1 = 0, Y_2 = 1)$ or $(Y_1 = 1, Y_2 = 0)$.

Find the distribution of Y: $\mathrm{P}(Y = 0) = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$, $\mathrm{P}(Y = 1) = 2 \times \frac{1}{4} \times \frac{3}{4} = \frac{6}{16} = \frac{3}{8}$, $\mathrm{P}(Y = 2) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$.

$$\mathrm{P}(Y_1 + Y_2 = 1) = \mathrm{P}(Y_1 = 0)\mathrm{P}(Y_2 = 1) + \mathrm{P}(Y_1 = 1)\mathrm{P}(Y_2 = 0)$$
$$= 2 \times \frac{9}{16} \times \frac{6}{16} = 2 \times \frac{54}{256} = \frac{108}{256} = \frac{27}{64} = 0.421875$$

$\mathrm{P}(Y_1 + Y_2 = 1) = \dfrac{27}{64} \approx 0.422$

Question 4

Worked Solution

Let $X \sim \mathrm{B}(30, 0.05)$ be the number of defective mugs in the first sample of 30.

Part (i)(a): Probability batch is rejected after first sample

The batch is rejected after the first sample if $X > 2$:

$$\mathrm{P}(X > 2) = 1 - \mathrm{P}(X \leq 2)$$

Using the calculator: B(30, 0.05), cumulative up to 2:

$$\mathrm{P}(X \leq 2) = 0.81268\ldots$$

$$\mathrm{P}(X > 2) = 1 - 0.81268 = 0.18732\ldots \approx 0.187 \text{ (3 s.f.)}$$

P(rejected after first sample) = 0.187 (3 s.f.)

Part (i)(b): Show total probability of rejection $= 0.327$

Let $Y \sim \mathrm{B}(15, 0.05)$ be the number of defective mugs in the second sample.

The batch is rejected overall if:

- $X > 2$ (rejected on first sample), **or**
- $X = 2$ **and** $Y \geq 1$ (referred to second sample, then rejected)

$$\mathrm{P}(\text{rejected}) = \mathrm{P}(X > 2) + \mathrm{P}(X = 2) \times \mathrm{P}(Y \geq 1)$$

$$\mathrm{P}(X = 2) = \mathrm{P}(X \leq 2) - \mathrm{P}(X \leq 1) = 0.81268 - 0.55405 = 0.25863\ldots$$

$$\mathrm{P}(Y \geq 1) = 1 - \mathrm{P}(Y = 0) = 1 - 0.46329 = 0.53671\ldots$$

$$\mathrm{P}(\text{rejected}) = 0.18728 + 0.25862 \times 0.53671 = 0.18728 + 0.13882 = 0.32610\ldots$$

$\mathrm{P}(\text{rejected}) = 0.326\ldots \approx \mathbf{0.327}$ (3 s.f.) ✓

Part (ii): First batch rejected is 4th or 5th

Each batch is independently rejected with probability $p = 0.327$.

$$\begin{aligned} \text{P(1st rejection is 4th)} &= (1 - 0.327)^3 \times 0.327 = (0.673)^3 \times 0.327 \\ &= 0.30489 \times 0.327 = 0.09970\ldots \end{aligned}$$

$$\text{P(1st rejection is 5th)} = (0.673)^4 \times 0.327 = 0.20519 \times 0.327 = 0.06710\ldots$$

$$\text{P(4th or 5th)} = 0.09970 + 0.06710 = 0.1668\ldots \approx 0.167$$

P(first rejection is 4th or 5th) = 0.167 (3 s.f.)

Question 5

Worked Solution

$X \sim B(9, \frac{2}{3})$ is the number of successes in 9 crossword attempts.

Part (i)(a): $P(X = 6)$

Using the calculator: $B(9, \frac{2}{3})$, $P(X = 6)$:

$$P(X = 6) = 0.2731\ldots$$

$P(X = 6) = 0.273$ (3 s.f.)

Part (i)(b): $P(X < 6)$

$$P(X < 6) = P(X \leq 5)$$

Using the calculator: $B(9, \frac{2}{3})$, cumulative up to 5:

$$P(X \leq 5) = 0.3497\ldots \approx 0.350$$

$P(X < 6) = 0.350$ (3 s.f.) [accept 0.349]

Part (ii): $X_1 + X_2 + X_3 = 18$ **(three sets of 9 attempts)**

Total attempts $= 3 \times 9 = 27$. So $X_1 + X_2 + X_3 \sim B(27, \frac{2}{3})$.

Using the calculator: $B(27, \frac{2}{3})$, $P(X_1 + X_2 + X_3 = 18)$:

$$P(X_1 + X_2 + X_3 = 18) = 0.1612\ldots$$

$P(X_1 + X_2 + X_3 = 18) = 0.161$ (3 s.f.)

Question 6

Worked Solution

$p = 0.27$ is the probability Jack wins a ticket in any given year. Attempts are independent.

Part (ii): Probability of winning exactly 2 of first 8 attempts

Let $X \sim \mathrm{B}(8, 0.27)$.

Using the calculator: $\mathrm{B}(8, 0.27)$, $\mathrm{P}(X = 2)$:

$$\mathrm{P}(X = 2) = 0.32567\ldots \approx 0.326$$

$\mathrm{P}(X = 2) = 0.326$ (3 s.f.)

Part (iii): 3rd win on 9th attempt AND 4th win on 12th attempt

Step 1: For the 3rd win to be on the 9th attempt: exactly 2 wins in first 8 attempts, then a win on attempt 9.

$$\mathrm{P}(\text{3rd win on 9th}) = \mathrm{P}(X = 2) \times 0.27 = 0.32567 \times 0.27 = 0.08793\ldots$$

Step 2: For the 4th win to be on the 12th attempt (given 3rd win was on 9th): exactly 0 wins in attempts 10 and 11, then a win on attempt 12.

Using the calculator: $\mathrm{B}(2, 0.27)$, $\mathrm{P}(W = 0) = 0.5329$.

$$\mathrm{P}(\text{4th win on 12th} \mid \text{3rd on 9th}) = 0.5329 \times 0.27 = 0.14388\ldots$$

Step 3: Multiply (independent events):

$$\mathrm{P} = 0.08793 \times 0.14388 = 0.01265\ldots \approx 0.0120 \text{ (3 s.f.)}$$

$\mathrm{P} = 0.0120$ (3 s.f.) [allow 0.012]

Question 7

Worked Solution

Part (i)(a): $X \sim \mathrm{B}(25, 0.6)$, $\mathrm{P}(X \leq 14)$

From tables: $\mathrm{P}(X \leq 14) = 0.414$ (3 s.f.).

$\mathrm{P}(X \leq 14) = 0.414$

Part (i)(b): $\mathrm{P}(X = 14)$

Using the calculator: B(25, 0.6), $\mathrm{P}(X = 14)$:

$$\mathrm{P}(X = 14) = 0.1465\ldots \approx 0.147$$

$\mathrm{P}(X = 14) = 0.147$ (3 s.f.)

Part (ii): $Y \sim \mathrm{B}(24, 0.3)$**, write expression for** $\mathrm{P}(Y = y)$**, evaluate at** $y = 8$

$$\mathrm{P}(Y = y) = \binom{24}{y}(0.3)^y(0.7)^{24-y}$$

For $y = 8$, using the calculator: $\mathrm{B}(24, 0.3)$, $\mathrm{P}(Y = 8)$:

$$\mathrm{P}(Y = 8) = 0.1600\ldots \approx 0.160$$

$\mathrm{P}(Y = y) = \binom{24}{y}(0.3)^y(0.7)^{24-y}$; at $y = 8$: $\mathrm{P}(Y = 8) = 0.160$ (3 s.f.)

Part (iii): $Z \sim \mathrm{B}(2, 0.2)$**, two randomly chosen values of** Z **are equal**

Z can take values 0, 1, or 2.

$$\mathrm{P}(Z = 0) = (0.8)^2 = 0.64, \quad \mathrm{P}(Z = 1) = 2(0.2)(0.8) = 0.32, \quad \mathrm{P}(Z = 2) = (0.2)^2 = 0.04$$

Two independent values Z_1, Z_2 are equal:

$$\mathrm{P}(Z_1 = Z_2) = (0.64)^2 + (0.32)^2 + (0.04)^2 = 0.4096 + 0.1024 + 0.0016 = 0.5136$$

$\mathrm{P}(Z_1 = Z_2) = \frac{321}{625} = 0.5136 \approx 0.514$ (3 s.f.)

Question 8

Worked Solution

Part (i): Distribution and assumption

Let Y = number of parcels that arrive in a particular month out of 10.

Distribution: $Y \sim \mathrm{B}\left(10, \frac{7}{8}\right)$.

Assumption: Each parcel arrives independently of the others (or: the probability that each parcel arrives is constant at $\frac{7}{8}$, independent of the others).

> $Y \sim \mathrm{B}\left(10, \frac{7}{8}\right)$. Assumption: parcels arrive independently (arrival of one parcel is not affected by the others).

Part (ii)(a): All 10 parcels arrive

Using the calculator: $\mathrm{B}(10, \frac{7}{8})$, $\mathrm{P}(Y = 10)$:

$$\mathrm{P}(Y = 10) = 0.2631\ldots \approx 0.263$$

> $\mathrm{P}(Y = 10) = 0.263$ (3 s.f.)

Part (ii)(b): At least 9 parcels arrive

$$\mathrm{P}(Y \geq 9) = 1 - \mathrm{P}(Y \leq 8)$$

Using the calculator: $\mathrm{B}(10, \frac{7}{8})$, cumulative up to 8:

$$\mathrm{P}(Y \leq 8) = 0.36109\ldots$$

$$\mathrm{P}(Y \geq 9) = 1 - 0.36109 = 0.63891\ldots \approx 0.639$$

> $\mathrm{P}(Y \geq 9) = 0.639$ (3 s.f.)

Part (iii): In at least 4 of 5 months, all 10 parcels arrive

Let W = number of months (out of 5) in which all 10 parcels arrive.

$W \sim \mathrm{B}(5, 0.263)$ (using the answer from part (ii)(a)).

$$\mathrm{P}(W \geq 4) = 1 - \mathrm{P}(W \leq 3)$$

Using the calculator: $\mathrm{B}(5, 0.263)$, cumulative up to 3:

$$\mathrm{P}(W \leq 3) = 0.98103\ldots$$

$$\mathrm{P}(W \geq 4) = 1 - 0.98103 = 0.01897\ldots \approx 0.0190 \text{ (3 s.f.)}$$

$P(W \geq 4) = 0.0190$ (3 s.f.)

End of Worked Solutions