Question 1 (Jun 2012, Q7a)

Worked Solution

Part (i): Given $\tan\alpha = \frac{2}{5}$ with α acute, find $\cos\alpha$.

Draw a right-angled triangle with opposite = 2, adjacent = 5.

$$\text{hypotenuse} = \sqrt{2^2 + 5^2} = \sqrt{29}$$

$$\cos\alpha = \frac{5}{\sqrt{29}}$$

Part (ii): Given $\sin\beta = \frac{3}{7}$ with β obtuse, find $\cos\beta$.

Using $\sin^2\beta + \cos^2\beta = 1$:

$$\cos^2\beta = 1 - \left(\frac{3}{7}\right)^2 = 1 - \frac{9}{49} = \frac{40}{49}$$

Since β is obtuse, $\cos\beta < 0$:

$$\cos\beta = -\frac{\sqrt{40}}{7} = -\frac{2\sqrt{10}}{7}$$

Question 2 (OCR 4752, Jun 2006, Q3)

Worked Solution

θ is acute and $\sin\theta = \frac{1}{4}$. Find the exact value of $\tan\theta$.

Using $\sin^2\theta + \cos^2\theta = 1$:

$$\cos^2\theta = 1 - \frac{1}{16} = \frac{15}{16} \implies \cos\theta = \frac{\sqrt{15}}{4} \quad (\theta \text{ acute})$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/4}{\sqrt{15}/4} = \frac{1}{\sqrt{15}}$$

$$\tan\theta = \frac{1}{\sqrt{15}}$$

Question 3 (OCR 4752, Jan 2007, Q3)

Worked Solution

$\cos\theta = \frac{1}{3}$, θ acute. Find the exact value of $\tan\theta$.

Using $\sin^2\theta + \cos^2\theta = 1$:

$$\sin^2\theta = 1 - \frac{1}{9} = \frac{8}{9} \implies \sin\theta = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}/3}{1/3} = 2\sqrt{2}$$

$\tan\theta = 2\sqrt{2}$

Question 4 (OCR 4752, Jan 2008, Q3)

Worked Solution

$\tan\theta = \frac{1}{2}$, θ acute. Show that $\cos^2\theta = \frac{4}{5}$.

Draw a right-angled triangle with opposite = 1, adjacent = 2:

$$\text{hypotenuse} = \sqrt{1^2 + 2^2} = \sqrt{5}$$

Therefore:

$$\cos\theta = \frac{2}{\sqrt{5}}$$

$$\cos^2\theta = \frac{4}{5} \qquad \checkmark$$

Alternatively, using $\tan^2\theta + 1 = \sec^2\theta$:

$$\sec^2\theta = \frac{1}{4} + 1 = \frac{5}{4} \implies \cos^2\theta = \frac{4}{5} \qquad \checkmark$$

Question 5 (Jan 2010, Q1)

Worked Solution

Part (i): Show $2\sin^2 x = 5\cos x - 1$ can be written as $2\cos^2 x + 5\cos x - 3 = 0$.

Use the identity $\sin^2 x = 1 - \cos^2 x$:

$$2(1 - \cos^2 x) = 5\cos x - 1$$

$$2 - 2\cos^2 x = 5\cos x - 1$$

$$0 = 2\cos^2 x + 5\cos x - 3 \qquad \checkmark$$

Part (ii): Hence solve $2\sin^2 x = 5\cos x - 1$ for $0° \leq x \leq 360°$.

Factorise $2\cos^2 x + 5\cos x - 3 = 0$:

$$(2\cos x - 1)(\cos x + 3) = 0$$

$\cos x = \frac{1}{2}$ or $\cos x = -3$ (rejected, since $|\cos x| \leq 1$).

$\cos x = \frac{1}{2} \implies x = 60°$ or $x = 300°$.

$x = 60°$ and $x = 300°$

Question 6 (Jun 2010, Q7)

Worked Solution

Part (i): Show that $\dfrac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} \equiv \tan^2 x - 1$**.**

Since $1 - \sin^2 x = \cos^2 x$:

$$\frac{\sin^2 x - \cos^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} = \tan^2 x - 1 \qquad \checkmark$$

Part (ii): Hence solve $\dfrac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x$ **for** $0^\circ \le x \le 360^\circ$**.**

Using part (i):

$$\tan^2 x - 1 = 5 - \tan x$$
$$\tan^2 x + \tan x - 6 = 0$$
$$(\tan x - 2)(\tan x + 3) = 0$$

$\tan x = 2$ or $\tan x = -3$.

For $\tan x = 2$:

$$x = \arctan 2 = 63.4^\circ, \quad x = 63.4^\circ + 180^\circ = 243.4^\circ$$

For $\tan x = -3$:

$$x = \arctan(-3) + 180^\circ = 108.4^\circ, \quad x = 108.4^\circ + 180^\circ = 288.4^\circ$$

$x = 63.4^\circ,\ 108.4^\circ,\ 243.4^\circ,\ 288.4^\circ$ (to 1 d.p.)

Question 7 (Jun 2013, Q2)

Worked Solution

Part (i): Solve $\sin \frac{1}{2}x = 0.8$ **for** $0° \leq x \leq 360°$**.**

Let $u = \frac{1}{2}x$, so $u \in [0°, 180°]$:

$$u = \arcsin(0.8) = 53.1°, \quad u = 180° - 53.1° = 126.9°$$

$$x = 2u : \quad x = 106.2°, \quad x = 253.8°$$

$x = 106°$ and $x = 254°$ (to nearest degree)

Part (ii): Solve $\sin x = 3\cos x$ **for** $0° \leq x \leq 360°$**.**

Divide by $\cos x$: $\tan x = 3$.

$$x = \arctan 3 = 71.6°, \quad x = 71.6° + 180° = 251.6°$$

$x = 71.6°$ and $x = 251.6°$

Question 8 (Jun 2009, Q5)

Worked Solution

Part (i): Solve $\sin 2x = 0.5$ **for** $0^\circ \leq x \leq 180^\circ$**.**

Let $u = 2x$, so $u \in [0^\circ, 360^\circ]$:

$$\sin u = 0.5 \implies u = 30^\circ,\ 150^\circ$$

$$x = 15^\circ,\ 75^\circ$$

$x = 15^\circ$ and $x = 75^\circ$

Part (ii): Solve $2\sin^2 x = 2 - \sqrt{3}\cos x$ **for** $0^\circ \leq x \leq 180^\circ$**.**

Use $\sin^2 x = 1 - \cos^2 x$:

$$2(1 - \cos^2 x) = 2 - \sqrt{3}\cos x$$
$$2 - 2\cos^2 x = 2 - \sqrt{3}\cos x$$
$$2\cos^2 x - \sqrt{3}\cos x = 0$$
$$\cos x(2\cos x - \sqrt{3}) = 0$$

$\cos x = 0$ or $\cos x = \frac{\sqrt{3}}{2}$.

$\cos x = 0 \implies x = 90^\circ$.

$\cos x = \frac{\sqrt{3}}{2} \implies x = 30^\circ$.

$x = 30^\circ$ and $x = 90^\circ$

Question 9 (Jun 2014, Q4)

Worked Solution

Part (i): Show $\sin x - \cos x = \dfrac{6\cos x}{\tan x}$ **can be written as** $\tan^2 x - \tan x - 6 = 0$**.**

Multiply both sides by $\tan x$ (and use $\tan x = \sin x / \cos x$):

$$\tan x(\sin x - \cos x) = 6\cos x$$

$$\frac{\sin x}{\cos x}(\sin x - \cos x) = 6\cos x$$

$$\sin x\left(\frac{\sin x}{\cos x} - 1\right) = 6\cos x$$

Instead, multiply the original equation by $\tan x$:

$$\tan x \cdot \sin x - \tan x \cdot \cos x = 6\cos x$$

$$\sin x \cdot \frac{\sin x}{\cos x} - \sin x = 6\cos x$$

Multiply through by $\frac{1}{\cos x}$...

More directly: multiply the original by $\tan x / \cos x$...

Cleanest approach: multiply both sides by $\tan x$:

$$\tan x(\sin x - \cos x) = 6\cos x$$

$$\sin x \tan x - \cos x \tan x = 6\cos x$$

$$\sin x \cdot \frac{\sin x}{\cos x} - \sin x = 6\cos x$$

Divide through by $\cos x$:

$$\tan^2 x - \tan x = 6$$

$$\tan^2 x - \tan x - 6 = 0 \qquad \checkmark$$

Part (ii): Hence solve for $0° \le x \le 360°$**.**

$$(\tan x - 3)(\tan x + 2) = 0$$

$\tan x = 3$ or $\tan x = -2$.

For $\tan x = 3$: $x = 71.6°,\ 251.6°$.

For $\tan x = -2$: $x = 180° - 63.4° = 116.6°,\ 296.6°$.

$x = 71.6°,\ 116.6°,\ 251.6°,\ 296.6°$ (to 1 d.p.)

Question 10 (Jun 2017, Q9) [Modified]

Worked Solution

$f(x) = 4x^3 + 9x - 5$.

Part (i): Show $(2x-1)$ is a factor and express $f(x)$ as a product of linear and quadratic factors.

Test $x = \frac{1}{2}$:

$$f\left(\tfrac{1}{2}\right) = 4 \cdot \tfrac{1}{8} + 9 \cdot \tfrac{1}{2} - 5 = \tfrac{1}{2} + \tfrac{9}{2} - 5 = 5 - 5 = 0 \qquad \checkmark$$

So $(2x-1)$ is a factor. Divide $4x^3 + 9x - 5$ by $(2x-1)$:

$$4x^3 + 0x^2 + 9x - 5 = (2x-1)(2x^2 + x + 5)$$

Verify: $(2x-1)(2x^2+x+5) = 4x^3 + 2x^2 + 10x - 2x^2 - x - 5 = 4x^3 + 9x - 5$ ✓

$f(x) = (2x-1)(2x^2 + x + 5)$

Part (ii)(a): Show $4\sin 2\theta \cos 2\theta + \dfrac{5}{\cos 2\theta} = 13\tan 2\theta$ can be written as $4\sin^3 2\theta + 9\sin 2\theta - 5 = 0$.

Write $\tan 2\theta = \dfrac{\sin 2\theta}{\cos 2\theta}$, then multiply through by $\cos 2\theta$:

$$4\sin 2\theta \cos^2 2\theta + 5 = 13\sin 2\theta$$

Use $\cos^2 2\theta = 1 - \sin^2 2\theta$:

$$4\sin 2\theta(1 - \sin^2 2\theta) + 5 = 13\sin 2\theta$$

$$4\sin 2\theta - 4\sin^3 2\theta + 5 = 13\sin 2\theta$$

$$4\sin^3 2\theta + 9\sin 2\theta - 5 = 0 \qquad \checkmark$$

Part (ii)(b): Hence solve for $0° \le \theta \le 360°$ (exact answers).

Let $s = \sin 2\theta$. From part (i) with $x = s$: $4s^3 + 9s - 5 = (2s-1)(2s^2+s+5)$.

$2s - 1 = 0 \implies s = \frac{1}{2}$; the quadratic $2s^2 + s + 5 = 0$ has discriminant $1 - 40 < 0$ (no real roots).

So $\sin 2\theta = \frac{1}{2}$.

Let $u = 2\theta$, $u \in [0°, 720°]$:

$$u = 30°,\ 150°,\ 390°,\ 510°$$

$$\theta = 15^\circ,\ 75^\circ,\ 195^\circ,\ 255^\circ$$

$\theta = 15^\circ,\ 75^\circ,\ 195^\circ,\ 255^\circ$

Question 11 (OCR 4752, Jun 2009, Q7)

Worked Solution

Show $4\cos^2\theta = 4 - \sin\theta$ may be written as $4\sin^2\theta - \sin\theta = 0$, then solve for $0° \leq \theta \leq 180°$.

Use $\cos^2\theta = 1 - \sin^2\theta$:

$$4(1 - \sin^2\theta) = 4 - \sin\theta$$
$$4 - 4\sin^2\theta = 4 - \sin\theta$$
$$4\sin^2\theta - \sin\theta = 0 \qquad \checkmark$$

Factorise:

$$\sin\theta(4\sin\theta - 1) = 0$$

$\sin\theta = 0 \implies \theta = 0°,\ 180°.$

$\sin\theta = \frac{1}{4} \implies \theta = \arcsin\left(\frac{1}{4}\right) = 14.5°$ or $\theta = 165.5°$.

$\theta = 0°,\ 14.5°,\ 165.5°,\ 180°$

Question 12 (OCR 4752, Jun 2011, Q7)

Worked Solution

Solve $\tan\theta = 2\sin\theta$ for $0° \le \theta \le 360°$.

Write $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$:

$$\frac{\sin\theta}{\cos\theta} = 2\sin\theta$$

$$\sin\theta = 2\sin\theta\cos\theta$$

$$\sin\theta(2\cos\theta - 1) = 0$$

$\sin\theta = 0 \implies \theta = 0°,\ 180°,\ 360°.$

$2\cos\theta - 1 = 0 \implies \cos\theta = \frac{1}{2} \implies \theta = 60°,\ 300°.$

$\theta = 0°,\ 60°,\ 180°,\ 300°,\ 360°$

Question 13 (Jan 2008, Q9)

Worked Solution

Part (i): State the coordinates of the maximum and minimum points of $y = 2\sin x$ for $-180° \le x \le 180°$.

The maximum of $\sin x$ is 1 at $x = 90°$; the minimum is -1 at $x = -90°$.

Maximum: $(90°,\ 2)$; Minimum: $(-90°,\ -2)$

Part (ii): For $y = 2\sin x$ and $y = k$ (with smallest positive solution α):

(a) Another solution of $2\sin x = k$ in $[-180°, 180°]$:

The sine curve is symmetric about $x = 90°$, so the other solution is $180° - \alpha$.

$x = 180° - \alpha$

(b) One solution of $2\sin x = -k$ in $[-180°, 180°]$:

$-k$ corresponds to the negative of k; by odd symmetry of sine, one solution is $x = -\alpha$.

$x = -\alpha$ (or $x = \alpha - 180°$)

Part (iii): Solve $2\sin x = 2 - 3\cos^2 x$ for $-180° \le x \le 180°$.

Use $\cos^2 x = 1 - \sin^2 x$:

$$2\sin x = 2 - 3(1 - \sin^2 x) = -1 + 3\sin^2 x$$

$$3\sin^2 x - 2\sin x - 1 = 0$$

$$(3\sin x + 1)(\sin x - 1) = 0$$

$\sin x = 1 \implies x = 90°$.

$\sin x = -\frac{1}{3} \implies x = \arcsin(-\frac{1}{3}) = -19.5°$ or $x = -180° - (-19.5°) = -160.5°$ (i.e. $180° + 19.5°$ below, reflected).

Second solution: $x = -(180° - 19.5°) = -160.5°$.

Wait, more carefully: $\sin x = -\frac{1}{3}$ in $[-180°, 180°]$: principal value $= -19.5°$; also $x = -(180° - 19.5°) = -160.5°$.

$x = -160.5°,\ -19.5°,\ 90°$

Question 14 (OCR 4752, Jun 2013, Q9)

Worked Solution

Part (i): Show $\frac{\tan\theta}{\cos\theta} = 1$ **may be rewritten as** $\sin\theta = 1 - \sin^2\theta$**.**

$$\frac{\tan\theta}{\cos\theta} = \frac{\sin\theta/\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos^2\theta} = 1$$

So $\sin\theta = \cos^2\theta = 1 - \sin^2\theta$ ✓

Part (ii): Hence solve $\frac{\tan\theta}{\cos\theta} = 1$ **for** $0° \le \theta \le 360°$**.**

From part (i): $\sin^2\theta + \sin\theta - 1 = 0$.

$$\sin\theta = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$\sin\theta = \frac{-1+\sqrt{5}}{2} \approx 0.618$ (valid) or $\sin\theta = \frac{-1-\sqrt{5}}{2} \approx -1.618$ (rejected).
$\sin\theta = 0.618\ldots$:

$$\theta = \arcsin(0.618) = 38.2° \quad \text{or} \quad \theta = 180° - 38.2° = 141.8°$$

$\theta = 38.2°$ and $\theta = 141.8°$

Question 15 (Jun 2014, Q4)

Worked Solution

Part (i): Show $\sin x - \cos x = \dfrac{6\cos x}{\tan x}$ **can be written as** $\tan^2 x - \tan x - 6 = 0$**.**

This is identical to Question 9. Multiply both sides by $\tan x$:

$$\tan x(\sin x - \cos x) = 6\cos x$$

$$\frac{\sin x}{\cos x} \cdot \sin x - \sin x = 6\cos x$$

$$\frac{\sin^2 x}{\cos x} - \sin x = 6\cos x$$

Divide through by $\cos x$:

$$\tan^2 x - \tan x = 6$$

$$\tan^2 x - \tan x - 6 = 0 \qquad \checkmark$$

Part (ii): Hence solve for $0° \leq x \leq 360°$**.**

$$(\tan x - 3)(\tan x + 2) = 0$$

$\tan x = 3$: $x = 71.6°,\ 251.6°$.

$\tan x = -2$: $x = 180° - 63.4° = 116.6°,\ 296.6°$.

$x = 71.6°,\ 116.6°,\ 251.6°,\ 296.6°$ (to 1 d.p.)