Trigonometric Equations and Identities Exam Questions MS (From OCR 4722)

Q1, (Jun 2012, Q7a)

(a)	(i)	$\cos\alpha = {}^{5}/_{\sqrt{29}}$	M1	Attempt $\cos\alpha$
			A1	Obtain ${}^{5}/_{\sqrt{29}}$
			[2]	
(a)	(ii)	$\cos\beta = {}^{-\sqrt{40}}/_{7}$	M1	Attempt $\cos\beta$
			A1	Obtain ${}^{\sqrt{40}}/_{7}$
			A1 FT	Obtain ${}^{-\sqrt{40}}/_{7}$, or –ve of their exact numerical value for $\cos\beta$
			[3]	

Q2, (OCR 4752, Jun 2006, Q3)

$1/\sqrt{15}$ i.s.w. not +/–	3	M2 for $\sqrt{15}$ seen M1 for rt angled triangle with side 1 and hyp 4, or $\cos^2\theta = 1 - 1/4^2$.	3

Q3, (OCR 4752, Jan 2007, Q3)

$\sqrt{8}$ or $2\sqrt{2}$ not $\pm\sqrt{8}$	3	M1 for use of $\sin^2\theta + (1/3)^2 = 1$ and M1for $\sin\theta = \sqrt{8}/3$ (ignore ±) Diag.: hypot = 3, one side =1 M1 3rd side $\sqrt{8}$ M1	3

Q4, (OCR 4752, Jan 2008, Q3)

right angled triangle with 1 and 2 on correct sides	M1	or M1 for $\sin\theta = \frac{1}{2}\cos\theta$ and M1 for substituting in $\sin^2\theta + \cos^2\theta = 1$	
Pythagoras used to obtain hyp = $\sqrt{5}$	M1	E1 for sufficient working	
$\cos\theta = \frac{a}{h} = \frac{2}{\sqrt{5}}$	A1		3

Q5, (Jan 2010, Q1)

(i) $2(1 - \cos^2 x) = 5\cos x - 1$
$2\cos^2 x + 5\cos x - 3 = 0$ **A.G.**

M1 Use $\sin^2 x = 1 - \cos^2 x$
A1 **2** Show given equation correctly

(ii) $(2\cos x - 1)(\cos x + 3) = 0$

$\cos x = ½$
$x = 60°$
$x = 300°$

M1 Recognise equation as quadratic in cos x and attempt recognisable method to solve
M1 Attempt to find x from root(s) of quadratic
A1 Obtain $60°$ or $\pi/3$ or 1.05 rad
A1√ **4** Obtain $300°$ only (or $360° -$ their x) and no extra in range
SR answer only is B1 B1

6

Q6, (Jun 2010, Q7)

(i) $\dfrac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = \dfrac{\sin^2 x - \cos^2 x}{\cos^2 x}$
$= \dfrac{\sin^2 x}{\cos^2 x} - \dfrac{\cos^2 x}{\cos^2 x}$
$= \tan^2 x - 1$

M1 Use either $\sin^2 x + \cos^2 x = 1$, or $\tan x = \sin x / \cos x$

A1 **2** Use other identity to obtain given answer convincingly.

(ii) $\tan^2 x - 1 = 5 - \tan x$
$\tan^2 x + \tan x - 6 = 0$
$(\tan x - 2)(\tan x + 3) = 0$
$\tan x = 2$, $\tan x = -3$
$x = 63.4°, 243°$ $x = 108°, 288°$

B1 State correct equation

M1 Attempt to solve three term quadratic in $\tan x$

A1 Obtain 2 and -3 as roots of their quadratic

M1 Attempt to solve $\tan x = k$ (at least one root)

A1ft Obtain at least 2 correct roots

A1 **6** Obtain all 4 correct roots

8

Q7, (Jun 2013, Q2)

(i)		$\frac{1}{2}x = 53.1°, 126.9°$	B1	Obtain 106°, or better
		$x = 106°, 254°$	M1	Attempt correct solution method to find second angle
			A1	Obtain 254°, or better
			[3]	
(ii)		$\tan x = 3$	B1	State $\tan x = 3$
		$x = 71.6°, 252°$	M1	Attempt to solve $\tan x = k$
			A1	Obtain 71.6° and 252°, or better
			[3]	

Q8, (Jun 2009, Q5)

5	**(i)** $2x = 30^\circ, 150^\circ$	M1		Attempt $\sin^{-1} 0.5$, then divide or multiply by 2
	$x = 15^\circ, 75^\circ$	A1		Obtain 15° (allow $\pi/12$ or 0.262)
		A1	**3**	Obtain 75° (not radians), and no extra solutions in range
	(ii) $2(1 - \cos^2 x) = 2 - \sqrt{3}\cos x$	M1		Use $\sin^2 x = 1 - \cos^2 x$
	$2\cos^2 x - \sqrt{3}\cos x = 0$	A1		Obtain $2\cos^2 x - \sqrt{3}\cos x = 0$ or equiv (no constant terms)
	$\cos x\,(2\cos x - \sqrt{3}) = 0$	M1		Attempt to solve quadratic in $\cos x$
	$\cos x = 0$, $\cos x = \frac{1}{2}\sqrt{3}$	A1		Obtain 30° (allow $\pi/6$ or 0524), and no extra solns in range
	$x = 90^\circ$, $x = 30^\circ$	B1	**5**	Obtain 90° (allow $\pi/2$ or 1.57), from correct quadratic only
			SR	answer only B1 one correct solution
				B1 second correct solution, and no others

8

Q9, (Jun 2014, Q4)

(i)	$\tan x(\sin x - \cos x) = 6\cos x$	M1		Use $\tan x = \frac{\sin x}{\cos x}$ correctly once
	$\tan x(\sin x/\cos x - 1) = 6$			
	$\tan x(\tan x - 1) = 6$			
	$\tan^2 x - \tan x = 6$			
	$\tan^2 x - \tan x - 6 = 0$ **AG**	A1		Obtain $\tan^2 x - \tan x - 6 = 0$
		[2]		
(ii)	$(\tan x - 3)(\tan x + 2) = 0$	M1		Attempt to solve quadratic in tan x
	$\tan x = 3$, $\tan x = -2$			
	$x = \tan^{-1}(3)$, $x = \tan^{-1}(-2)$	M1		Attempt to solve $\tan x = k$ at least once
	$x = 71.6°, 252°, 117°, 297°$	A1		Obtain two correct solutions
		A1		Obtain all 4 correct solutions, and no others in range

Q10, (Jun 2017, Q9) [Modified]

(i)		$f(1/2) = 1/2 + 9/2 - 5 = 0$ $f(x) = (2x - 1)(2x^2 + x + 5)$	B1	Confirm $f(1/2) = 0$, with detail shown
			M1	Attempt complete division or equiv
			A1	Obtain correct quotient
			A1	Obtain $(2x - 1)(2x^2 + x + 5)$
(ii)	**(a)**	$4\sin 2\theta \cos 2\theta + \frac{5}{\cos 2\theta} = \frac{13\sin 2\theta}{\cos 2\theta}$	B1	Use $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$ or $\tan 2\theta \cos 2\theta = \sin 2\theta$
		$4\sin 2\theta \cos^2 2\theta + 5 = 13\sin 2\theta$	B1	Correct method to remove fraction(s)
		$4\sin 2\theta(1 - \sin^2 2\theta) + 5 = 13\sin 2\theta$	B1	Use $\cos^2 2\theta = 1 - \sin^2 2\theta$
		$4\sin 2\theta - 4\sin^3 2\theta + 5 = 13\sin 2\theta$	B1	Obtain correct equation, from correct working
		$4\sin^3 2\theta + 9\sin 2\theta - 5 = 0$	**[4]**	
		$(2\sin 2\theta - 1)(2\sin^2 2\theta + \sin 2\theta + 5) = 0$ $\sin 2\theta = 1/2$	B1	State that $\sin 2\theta = 1/2$ oe
		$2\theta =$ 30, 150, 390, 510	M1	Attempt to solve $\sin 2\theta = \pm 1/2$ to find at least one root
		$\theta =$ 15, 75, 195, 255	A1	Obtain at least 2 correct roots
			A1	Obtain 4 correct roots

Q11, (OCR 4752, Jun 2009, Q7)

use of $\cos^2\theta = 1 - \sin^2\theta$	M1		
at least one correct interim step in obtaining $4\sin^2\theta - \sin\theta = 0$.	M1	NB answer given	
$\theta = 0$ and 180,	B1		
14.(47…)	B1	r.o.t to nearest degree or better	
165 - 166	B1	-1 for extras in range	5

Q12, (OCR 4752, Jun 2011, Q7)

$\frac{\sin\theta}{\cos\theta} = 2\sin\theta$ $2\cos\theta - 1 = 0$ *and* $\sin\theta = 0$ $[\theta =]$ 0, 180, 360, $[\theta =]$ 60, 300 if 4 marks awarded, lose 1 mark for extra values in the range, ignore extra values outside the range	**M1** **A1** **B1** **B1**	*may* be implied by $2\cos\theta - 1 = 0$ or better	or, if to advantage of candidate **B4** for all 5 correct **B3** for 4 correct **B2** for 3 correct **B1** for 2 correct if extra value(s) in range, deduct one mark from total do not award if values embedded in trial and improvement approach

Q13, (Jan 2008, Q9)

(i)	$(90^o, 2), (-90^o, -2)$	B1 B1	**2**	State at least 2 correct values State all 4 correct values (radians is B1 B0)
(ii)	**(a)** $180 - \alpha$	B1	**1**	State $180 - \alpha$
	(b) $-\alpha$ or $\alpha - 180$	B1	**1**	State $-\alpha$ or $\alpha - 180$ (radians or unsimplified is B1B0)
(iii)	$2\sin x = 2 - 3\cos^2 x$ $2\sin x = 2 - 3(1 - \sin^2 x)$ $3\sin^2 x - 2\sin x - 1 = 0$ $(3\sin x + 1)(\sin x - 1) = 0$ $\sin x = -{}^1/_3$, $\sin x = 1$ $x = -19.5^o, -161^o, 90^o$	M1 A1 M1 A1 A1√ A1	**6**	Attempt use of $\cos^2 x = 1 - \sin^2 x$ Obtain $3\sin^2 x - 2\sin x - 1 = 0$ aef with no brackets Attempt to solve 3 term quadratic in $\sin x$ Obtain $x = -19.5^o$ Obtain second correct answer in range, following their x Obtain 90^o (radians or extra answers is max 5 out of 6) SR: answer only (and no extras) is B1 B1√ B1
			10	

Q14, (OCR 4752, Jun 2013, Q9)

(i)		$(\frac{\sin\theta}{\cos\theta})\div\cos\theta = 1$ oe $\sin\theta = \cos^2\theta$ and completion to given result	M1 A1 **[2]**	www	
(ii)		$\sin^2\theta + \sin\theta - 1[=0]$ $[\sin\theta =]\frac{-1\pm\sqrt{5}}{2}$ oe may be implied by correct answers $[\theta =]$ 38.17... ,or 38.2 and 141.83..., 141.8 or 142	M1 A1 A1 **[3]**	allow 1 on RHS if attempt to complete square may be implied by correct answers ignore extra values outside range, **A0** if extra values in range or in radians **NB** 0.6662 and 2.4754 if working in radian mode earns **M1A1A0**	condone $y^2 + y - 1 = 0$ mark to benefit of candidate ignore any work with negative root & condone omission of negative root with no comment eg **M1** for 0.618… if unsupported, **B1** for one of these, **B2** for both. If both values correct with extra values in range, then **B1**. **NB** 0.6662 and 2.4754 to 3sf or more

Q15, (Jun 2014, Q4)

(i)		$\tan x(\sin x - \cos x) = 6\cos x$ $\tan x\left(\frac{\sin x}{\cos x} - 1\right) = 6$ $\tan x(\tan x - 1) = 6$ $\tan^2 x - \tan x = 6$ $\tan^2 x - \tan x - 6 = 0$ **AG**	M1 A1 **[2]**	Use $\tan x = \frac{\sin x}{\cos x}$ correctly once Obtain $\tan^2 x - \tan x - 6 = 0$	Must be used clearly at least once - either explicitly or by writing eg 'divide by $\cos x$' at side of solution Allow M1 for any equiv eg $\sin x = \cos x \tan x$ Allow poor notation eg writing just tan rather than $\tan x$ Correct equation in given form, including $= 0$ Correct notation throughout so A0 if eg tan rather than $\tan x$ seen in solution
(ii)		$(\tan x - 3)(\tan x + 2) = 0$ $\tan x = 3, \tan x = -2$ $x = \tan^{-1}(3),\ x = \tan^{-1}(-2)$ $x = 71.6°, 252°, 117°, 297°$	M1 M1 A1 A1 **[4]**	Attempt to solve quadratic in tan x Attempt to solve $\tan x = k$ at least once Obtain two correct solutions Obtain all 4 correct solutions, and no others in range	This M mark is just for solving a 3 term quadratic (see guidance sheet for acceptable methods) Condone any substitution used, inc $x = \tan x$ Attempt $\tan^{-1} k$ at least once Not dependent on previous mark so M0M1 possible If going straight from $\tan x = k$ to $x = \ldots$, then award M1 only if their angle is consistent with their k Allow 3sf or better Must come from a correct method to solve the quadratic (as far as correct factorisation or substitution into formula) Allow radian equivs ie 1.25 / 4.39 / 2.03 / 5.18 Must now all be in degrees Allow 3sf or better A0 if other incorrect solutions in range $0° - 360°$ (but ignore any outside this range) **SR** If no working shown then allow **B1** for each correct solution (max of **B3** if in radians, or if extra solns in range).