Question 1

Worked Solution

Part (a): Solve $\sin(x - 20°) = \frac{1}{\sqrt{2}}$ **for** $0° \le x < 360°$**.**

Let $u = x - 20°$. The range for u is $-20° \le u < 340°$.

$\sin u = \frac{1}{\sqrt{2}} \implies u = 45°$ or $u = 135°$.

Both are in range. So:

$$x = 45° + 20° = 65°, \qquad x = 135° + 20° = 155°$$

$x = 65°$ and $x = 155°$

Part (b): Solve $\cos 3x = -\frac{1}{2}$ **for** $0° \le x < 360°$**.**

Let $u = 3x$, so $u \in [0°, 1080°)$.

$\cos u = -\frac{1}{2} \implies u = 120°$ (principal), then using symmetry and period $360°$:

$$u = 120°,\ 240°,\ 480°,\ 600°,\ 840°,\ 960°$$

Divide by 3:

$$x = 40°,\ 80°,\ 160°,\ 200°,\ 280°,\ 320°$$

$x = 40°,\ 80°,\ 160°,\ 200°,\ 280°,\ 320°$

Question 2

Worked Solution

Part (a): Given $5\sin\theta = 2\cos\theta$, find $\tan\theta$.

Divide both sides by $\cos\theta$:

$$5\tan\theta = 2 \implies \tan\theta = \frac{2}{5}$$

$$\tan\theta = \frac{2}{5}$$

Part (b): Solve $5\sin 2x = 2\cos 2x$ for $0° \le x < 360°$.

Divide by $\cos 2x$:

$$\tan 2x = \frac{2}{5} = 0.4$$

Let $u = 2x$, $u \in [0°, 720°)$.

$\tan u = 0.4 \implies u = \arctan(0.4) = 21.8°$

All solutions (period 180°):

$$u = 21.8°,\ 201.8°,\ 381.8°,\ 561.8°$$

Divide by 2:

$$x = 10.9°,\ 100.9°,\ 190.9°,\ 280.9°$$

$x = 10.9°,\ 100.9°,\ 190.9°,\ 280.9°$ (to 1 d.p.)

Question 3

Worked Solution

Part (i): Solve $9\sin(\theta + 60°) = 4$ for $0° \leq \theta < 360°$.

Let $u = \theta + 60°$, so $u \in [60°, 420°)$.

$$\sin u = \frac{4}{9} \implies u = \arcsin\left(\tfrac{4}{9}\right) = 26.4°$$

This is outside the range $[60°, 420°)$. The solutions from sine symmetry are:

$$u = 180° - 26.4° = 153.6° \quad \text{and} \quad u = 360° + 26.4° = 386.4°$$

Both are in $[60°, 420°)$.

$\theta = u - 60°$:

$$\theta = 153.6° - 60° = 93.6°, \qquad \theta = 386.4° - 60° = 326.4°$$

$\theta = 93.6°$ and $\theta = 326.4°$ (to 1 d.p.)

Part (ii): Solve $2\tan x - 3\sin x = 0$ for $-180° \leq x < 180°$.

Write $\tan x = \dfrac{\sin x}{\cos x}$:

$$\frac{2\sin x}{\cos x} - 3\sin x = 0$$
$$\sin x\left(\frac{2}{\cos x} - 3\right) = 0$$

Either $\sin x = 0$ or $\cos x = \dfrac{2}{3}$.

$\sin x = 0 \implies x = -180°$ (not in half-open range), $x = 0°$.

Wait: range is $-180° \leq x < 180°$, so $x = -180°$ is included but $x = 180°$ is not.

$\sin x = 0 \implies x = -180°,\ 0°$.

$\cos x = \dfrac{2}{3} \implies x = \pm\arccos\left(\tfrac{2}{3}\right) = \pm 48.2°$.

$x = -180°,\ -48.19°,\ 0°,\ 48.19°$ (to 2 d.p.)

Question 4

Worked Solution

Part (a): Show $\cos^2 x = 8\sin^2 x - 6\sin x$ can be written as $(3\sin x - 1)^2 = 2$.

Use $\cos^2 x = 1 - \sin^2 x$:

$$1 - \sin^2 x = 8\sin^2 x - 6\sin x$$
$$9\sin^2 x - 6\sin x - 1 = 0$$
$$9\sin^2 x - 6\sin x + 1 = 2$$
$$(3\sin x - 1)^2 = 2 \qquad \checkmark$$

Part (b): Hence solve for $0° \leq x < 360°$.

$$3\sin x - 1 = \pm\sqrt{2}$$
$$\sin x = \frac{1 \pm \sqrt{2}}{3}$$

$\sin x = \dfrac{1+\sqrt{2}}{3} = \dfrac{1+1.4142}{3} = 0.8047\ldots$:

$$x = \arcsin(0.8047) = 53.58° \approx 53.58°$$

Second solution: $x = 180° - 53.58° = 126.42°$.

$\sin x = \dfrac{1-\sqrt{2}}{3} = \dfrac{1-1.4142}{3} = -0.1381\ldots$:

$$x = 180° - \arcsin(-0.1381) = 180° - (-7.94°) = 187.94°$$

Also $x = 360° + \arcsin(-0.1381) = 360° - 7.94° = 352.06°$.

$x = 53.58°,\ 126.42°,\ 187.94°,\ 352.06°$ (to 2 d.p.)

Question 5

Worked Solution

Part (i): Solve $\sin(2\theta - 30°) + 1 = 0.4$ **for** $0° \le \theta < 180°$**.**

$$\sin(2\theta - 30°) = -0.6$$

Let $u = 2\theta - 30°$. Range: $\theta \in [0°, 180°)$ so $u \in [-30°, 330°)$.

$\sin u = -0.6 \implies$ reference angle $= \arcsin(0.6) = 36.87°$.

In range $[-30°, 330°)$, $\sin u = -0.6$ gives:

$$u = -(180° - 36.87°) + 360° = 216.87° \quad \text{and} \quad u = 360° - 36.87° = 323.13°$$

(Note: $u = -36.87°$ is outside the range.)

$\theta = \dfrac{u + 30°}{2}$:

$$\theta = \frac{216.87 + 30}{2} = \frac{246.87}{2} = 123.4°$$
$$\theta = \frac{323.13 + 30}{2} = \frac{353.13}{2} = 176.6°$$

Both in $[0°, 180°)$. ✓

$\theta = 123.4°$ and $\theta = 176.6°$ (to 1 d.p.)

Part (ii): Solve $9\cos^2 x - 11\cos x + 3\sin^2 x = 0$ **for** $0° \le x < 360°$**.**

Use $\sin^2 x = 1 - \cos^2 x$:

$$9\cos^2 x - 11\cos x + 3(1 - \cos^2 x) = 0$$

$$6\cos^2 x - 11\cos x + 3 = 0$$
$$(3\cos x - 1)(2\cos x - 3) = 0$$

$\cos x = \frac{1}{3}$ or $\cos x = \frac{3}{2}$ (rejected).

$\cos x = \frac{1}{3} \implies x = \arccos\left(\frac{1}{3}\right) = 70.5°$ or $x = 360° - 70.5° = 289.5°$.

$x = 70.5°$ and $x = 289.5°$ (to 1 d.p.)

Question 6

Worked Solution

Part (a): Show that $\dfrac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$**.**

Use $\sin^2\theta = 1 - \cos^2\theta$:

$$\frac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3 + 2\cos\theta} = \frac{12 - 7\cos\theta - 10\cos^2\theta}{3 + 2\cos\theta}$$

Factorise the numerator:

$$12 - 7\cos\theta - 10\cos^2\theta = (3 + 2\cos\theta)(4 - 5\cos\theta)$$

Check: $(3)(4)+(3)(-5\cos\theta)+(2\cos\theta)(4)+(2\cos\theta)(-5\cos\theta) = 12-15\cos\theta+8\cos\theta-10\cos^2\theta = 12 - 7\cos\theta - 10\cos^2\theta$ ✓

Therefore:

$$\frac{(3 + 2\cos\theta)(4 - 5\cos\theta)}{3 + 2\cos\theta} = 4 - 5\cos\theta \qquad \checkmark$$

Part (b): Hence solve $\dfrac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$ **for** $0^\circ \le x \le 360^\circ$**.**

Using part (a):

$$4 - 5\cos x = 4 + 3\sin x$$
$$-5\cos x = 3\sin x$$
$$\tan x = -\frac{5}{3}$$

$x = 180^\circ + \arctan\left(-\frac{5}{3}\right) = 180^\circ - 59.0^\circ = 121.0^\circ$

$x = 360^\circ - 59.0^\circ = 301.0^\circ$.

$x = 121^\circ$ and $x = 301^\circ$ (to nearest degree)

Question 7

Worked Solution

Part (i): Solve $\tan(x - 40°) = 1.5$ **for** $-180° \leq x < 180°$**.**

Let $u = x - 40°$, $u \in [-220°, 140°)$.

$\tan u = 1.5 \implies u = \arctan(1.5) = 56.3°$.

Second solution (subtract 180°): $u = 56.3° - 180° = -123.7°$ (in range).

$x = u + 40°$:

$$x = 56.3° + 40° = 96.3°, \qquad x = -123.7° + 40° = -83.7°$$

$x = 96.3°$ and $x = -83.7°$ (to 1 d.p.)

Part (ii)(a): Show $\sin\theta \tan\theta = 3\cos\theta + 2$ **can be written as** $4\cos^2\theta + 2\cos\theta - 1 = 0$**.**

$$\sin\theta \cdot \frac{\sin\theta}{\cos\theta} = 3\cos\theta + 2$$
$$\frac{\sin^2\theta}{\cos\theta} = 3\cos\theta + 2$$
$$\frac{1 - \cos^2\theta}{\cos\theta} = 3\cos\theta + 2$$
$$1 - \cos^2\theta = 3\cos^2\theta + 2\cos\theta$$
$$0 = 4\cos^2\theta + 2\cos\theta - 1 \qquad \checkmark$$

Part (ii)(b): Hence solve $\sin\theta \tan\theta = 3\cos\theta + 2$ **for** $0° \leq \theta < 360°$**.**

Using the quadratic formula:

$$\cos\theta = \frac{-2 \pm \sqrt{4 + 16}}{8} = \frac{-2 \pm \sqrt{20}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

$\cos\theta = \dfrac{-1 + \sqrt{5}}{4} = \dfrac{-1 + 2.2361}{4} = 0.3090\ldots$:

$$\theta = \arccos(0.3090) = 72° \quad \text{or} \quad \theta = 360° - 72° = 288°$$

$\cos\theta = \dfrac{-1 - \sqrt{5}}{4} = -0.8090\ldots$:

$$\theta = \arccos(-0.8090) = 144° \quad \text{or} \quad \theta = 360° - 144° = 216°$$

$\theta = 72°,\ 144°,\ 216°,\ 288°$

Question 8

Worked Solution

Part (a): Show $\tan 2x = 5\sin 2x$ **can be written as** $(1 - 5\cos 2x)\sin 2x = 0$**.**

Write $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$:

$$\frac{\sin 2x}{\cos 2x} = 5\sin 2x$$

$$\frac{\sin 2x}{\cos 2x} - 5\sin 2x = 0$$

$$\sin 2x\left(\frac{1}{\cos 2x} - 5\right) = 0$$

$$\sin 2x \cdot \frac{1 - 5\cos 2x}{\cos 2x} = 0$$

So $(1 - 5\cos 2x)\sin 2x = 0$ ✓ (since $\cos 2x \neq 0$ provided $\tan 2x$ is defined)

Part (b): Hence solve $\tan 2x = 5\sin 2x$ **for** $0° \leq x \leq 180°$**.**

Let $u = 2x$, $u \in [0°, 360°]$.

Case 1: $\sin 2x = 0 \implies 2x = 0°,\ 180°,\ 360° \implies x = 0°,\ 90°,\ 180°$.

Case 2: $1 - 5\cos 2x = 0 \implies \cos 2x = \frac{1}{5}$:

$$2x = \arccos(0.2) = 78.5° \quad \text{or} \quad 2x = 360° - 78.5° = 281.5°$$

$$x = 39.2° \quad \text{or} \quad x = 140.8°$$

(Check: $\tan 2x$ must be defined, i.e. $2x \neq 90°, 270°$; $x = 90°$ gives $2x = 180°$ where $\sin 180° = 0$ and $\tan 180° = 0$, consistent. So $x = 90°$ is valid.)

$x = 0°,\ 39.2°,\ 90°,\ 140.8°,\ 180°$