Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	45 (α)	B1
	$180-\alpha$, Add 20 (for at least one angle)	M1 M1
	65 155	A1 (4)
(b)	120 or 240 (β):	B1
	$360-\beta$, $360+\beta$	M1 M1
	Dividing by 3 (for at least one angle)	M1
	40 80 160 200 280 320	A1 A1 (6)
		(10 marks)

Q2.

Question Number	Scheme	Marks
	(a) $\tan\theta = \frac{2}{5}$ (or 0.4) (i.s.w. if a value of θ is subsequently found) Requires the correct value with no incorrect working seen.	B1 (1)
	(b) awrt 21.8 (α) (Also allow awrt 68.2, ft from $\tan\theta = \frac{5}{2}$ in (a), but no other ft) (This value must be seen in part (b). It may be implied by a correct solution, e.g. 10.9)	B1
	$180+\alpha$ (= 201.8), or $90+(\alpha/2)$ (if division by 2 has already occurred) (α found from $\tan 2x = ...$ or $\tan x = ...$ or $\sin 2x = \pm...$ or $\cos 2x = \pm...$)	M1
	$360+\alpha$ (= 381.8), or $180+(\alpha/2)$ (α found from $\tan 2x = ...$ or $\sin 2x = ...$ or $\cos 2x = ...$) OR $540+\alpha$ (= 561.8), or $270+(\alpha/2)$ (α found from $\tan 2x = ...$)	M1
	Dividing at least one of the angles by 2 (α found from $\tan 2x = ...$ or $\sin 2x = ...$ or $\cos 2x = ...$)	M1
	$x = 10.9, 100.9, 190.9, 280.9$ (Allow awrt)	A1 (5)
		6

Q3.

Question Number	Scheme		Marks
	(i) $9\sin(\theta+60^\circ)=4;\ 0\le\theta<360^\circ$ (ii) $2\tan x-3\sin x=0;\ -\pi\le x<\pi$		
(i)	$\sin(\theta+60^\circ)=\frac{4}{9}$, so $(\theta+60^\circ)=26.3877...$ $(\alpha=26.3877...)$	Sight of $\sin^{-1}\left(\frac{4}{9}\right)$ or awrt 26.4° or 0.461^c Can also be implied for $\theta=$ awrt -33.6 (i.e. $26.4-60$)	M1
	So, $\theta+60^\circ=\{153.6122...,\ 386.3877...\}$	$\theta+60^\circ=$ either "$180-$ their α" or "$360^\circ+$ their α" and not for $\theta=$ either "$180-$ their α" or "$360^\circ+$ their α". This can be implied by later working. The candidate's α could also be in radians but do not allow mixing of degrees and radians.	M1
	and $\theta=\{93.6122...,\ 326.3877...\}$	A1: At least one of awrt 93.6° or awrt 326.4° A1: Both awrt 93.6° and awrt 326.4°	A1 A1
	Both answers are cso and must come from correct work		
	Ignore extra solutions outside the range. In an otherwise fully correct solution deduct the final A1 for any extra solutions in range		
			[4]
(ii)	$2\left(\frac{\sin x}{\cos x}\right)-3\sin x=0$	Applies $\tan x=\frac{\sin x}{\cos x}$	M1
	Note: Applies $\tan x=\frac{\sin x}{\cos x}$ can be implied by $2\tan x-3\sin x=0\Rightarrow\tan x(2-3\cos x)$		
	$2\sin x-3\sin x\cos x=0$		
	$\sin x(2-3\cos x)=0$		
	$\cos x=\frac{2}{3}$	$\cos x=\frac{2}{3}$	A1
	$x=$ awrt { }	A1: One of either -48.13 or 48.13 A1ft: You can apply ft for $x=\pm\alpha$, where $\alpha=\cos^{-1}k$ and $-1\le k\le 1$	A1A1ft
	$\{\sin x=0\Rightarrow\}\ x=0$ AND -180	Both $x=0$ and $x=-180$ required from $\sin x=0$	B1
	Note solutions are $x=\{-180,\ -48.13,\ 0,\ 48.13\}$ Ignore extra solutions outside range.		

Q4.

Question Number	Scheme		Marks
(a)	Way 1 $1-\sin^2 x = 8\sin^2 x - 6\sin x$ E.g. $9\sin^2 x - 6\sin x = 1$ or $9\sin^2 x - 6\sin x - 1 = 0$ or $9\sin^2 x - 6\sin x + 1 = 2$ So $9\sin^2 x - 6\sin x + 1 = 2$ or $(3\sin x - 1)^2 - 2 = 0$ so $(3\sin x - 1)^2 = 2$ or $2 = (3\sin x - 1)^2$ *	Way 2 $2 = (3\sin x - 1)^2$ gives $9\sin^2 x - 6\sin x + 1 = 2$ so $\sin^2 x + 8\sin^2 x - 6\sin x + 1 = 2$ so $8\sin^2 x - 6\sin x = 1 - \sin^2 x$ $8\sin^2 x - 6\sin x = \cos^2 x$ *	B1 M1 A1cso* **(3)**
(b)	Way 1: $(3\sin x - 1) = (\pm)\sqrt{2}$ $\sin x = \frac{1 \pm \sqrt{2}}{3}$ or awrt 0.8047 **and** awrt -0.1381 $x = 53.58$, 126.42 (or 126.41), 352.06, 187.94	Way 2: Expands $(3\sin x - 1)^2 = 2$ and uses quadratic formula on 3TQ	M1 A1 dM1A1 A1 **(5)** **[8]**

	Notes
(a)	**Way 1** B1: Uses $\cos^2 x = 1 - \sin^2 x$ M1: Collects $\sin^2 x$ terms to form a three term quadratic or into a suitable completed square format. May be sign slips in the collection of terms. A1*: cso This needs an intermediate step from 3 term quadratic and no errors in answer and printed answer stated but allow $2 = (3\sin x - 1)^2$. If sin is used throughout instead of sin*x* it is A0. **Way 2** B1: **Needs correct expansion** and **split** M1: Collects $1 - \sin^2 x$ together A1*: Conclusion and no errors seen
(b)	M1: Square roots both sides(Way 1), or expands and uses quadratic formula (Way 2) Attempts at factorization after expanding are M0. A1: **Both** correct answers for sin*x* (need plus and minus). Need not be simplified. dM1: Uses inverse sin to give one of the given correct answers 1st A1: Need two correct angles (allow awrt) Note that the scheme allows 126.41 in place of 126.42 though 126.42 is preferred A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore) (**Premature approximation**:– in the final three marks lose first A1 then ft other angles for second A mark) Do **not** require degrees symbol for the marks **Special case: Working in radians** M1A1A0 for the *correct* 0.94, 2.21, 6.14, 3.28

Q5.

Question Number	Scheme	Marks
(a)	$\sin(2\theta-30)=-0.6$ or $2\theta-30=-36.9$ or implied by 216.9	B1
	$2\theta-30=216.87$ $=(180+36.9)$	M1
	$\theta=\dfrac{216.87+30}{2}=123.4$ or 123.5	A1
	$2\theta-30=360-36.9$ or 323.1	M1
	$\theta=\dfrac{323.1+30}{2}=176.6$	A1 (5)
(b)	$9\cos^2 x-11\cos x+3(1-\cos^2 x)=0$ or $6\cos^2 x-11\cos x+3(\sin^2 x+\cos^2 x)=0$	M1
	$6\cos^2 x-11\cos x+3=0$ {as $(\sin^2 x+\cos^2 x)=1$}	A1
	$(3\cos x-1)(2\cos x-3)=0$ implies $\cos x=$	M1
	$\cos x=\dfrac{1}{3}, \left(\dfrac{3}{2}\right)$	A1
	$x=70.5$	B1
	$x=360-$"70.5"	M1
	$x=289.5$	A1cao (7)
		Total 12

Notes for Question

(a)

B1: This statement seen and must contain no errors or may implied by – 36.9

M1: Uses 180 – arcsin (-0.6) i.e. 180 + 36.9 (or $\pi+\arcsin(0.6)$ in radians) (in 3rd quadrant)

A1: allow answers which round to 123.4 or 123.5 must be in degrees

M1: Uses 360 + arcsin (-0.6) i.e. 360 – 36.9 (or $2\pi+\arcsin(-0.6)$ in radians) (in 4th quadrant)

A1: allow answers which round to 176.6 must be in degrees (A1 implies M1)

Ignore extra answers outside range but lose final A1 for extra answers in the range if both B and A marks have been earned)

Working in radians may earn B1M1A0M1A0

(b)

M1: Use of $\sin^2 x=(1-\cos^2 x)$ or $(\sin^2 x+\cos^2 x)=1$ in the given equation

A1: Correct three term quadratic in any equivalent form

M1: Uses standard method to solve quadratic and obtains cos$x$ =

A1: A1 for $\dfrac{1}{3}$ with $\dfrac{3}{2}$ ignored but A0 if $\dfrac{3}{2}$ is incorrect

B1: 70.5 or answers which round to this value

M1: 360 –arcos(*their*1/3) (or $2\pi-\arccos(\textit{their}1/3)$ in radians)

A1: Second answer

Working in radians in (b) may earn M1A1M1A1B0M1A0

Extra values in the range coming from arcos (1/3) – deduct final A mark - so A0

Q6.

Question	Scheme	Marks	AOs
(a)	$\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{3+2\cos\theta}$	M1	1.1b
	$\equiv\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$	A1	1.1b
	$\equiv\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	1.1b
	$\equiv 4-5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4+3\sin x=4-5\cos x\Rightarrow\tan x=-\dfrac{5}{3}$	M1	2.1
	$x=$ awrt $121°, 301°$	A1 A1	1.1b 1.1b
		(3)	
			(7 marks)

Notes

(a)

M1: Uses the identity $\sin^2\theta=1-\cos^2\theta$ within the fraction

A1: Correct (simplified) expression in just $\cos\theta$ $\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$ or exact equivalent such as $\dfrac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$ Allow for $\dfrac{12-7u-10u^2}{3+2u}$ where they introduce $u=\cos\theta$

We would condone mixed variables here.

M1: A correct attempt to factorise the numerator, usual rules. Allow candidates to use $u=\cos\theta$ oe

A1*: A fully correct proof with correct notation and no errors.

Only withhold the last mark for (1) Mixed variable e.g. θ and x's (2) Poor notation $\cos\theta^2\leftrightarrow\cos^2\theta$ or $\sin^2=1-\cos^2$ within the solution.

Don't penalise incomplete lines if it is obvious that it is just part of their working

E.g. $\dfrac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\dfrac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{=}\dfrac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$

(b)

M1: Attempts to use part (a) and proceeds to an equation of the form $\tan x=k,\quad k\neq 0$

Condone $\theta\leftrightarrow x$ Do not condone $a\tan x=0\Rightarrow\tan x=b\Rightarrow x=\ldots$

Alternatively squares $3\sin x=-5\cos x$ and uses $\sin^2 x=1-\cos^2 x$ oe to reach $\sin x=A,-1<A<1$ or $\cos x=B,-1<B<1$

A1: Either $x=$ awrt $121°$ **or** $301°$. Condone awrt 2.11 or 5.25 which are the radian solutions

A1: Both $x=$ awrt $121°$ **and** $301°$ and no other solutions.

Answers without working, or with no incorrect working in (b).

Question states hence or otherwise so allow

For 3 marks both $x=$ awrt $121°$ **and** $301°$ and no other solutions.

For 1 marks scored SC 100 for either $x=$ awrt $121°$ **or** $301°$

Q7.

Question Number	Scheme	Marks
(i)	$(\lvert\alpha\rvert = 56.3099...)$	
	$x = \{\alpha + 40 = 96.309993...\}$ = **awrt 96.3**	B1
	$x - 40^{\circ} = -180 + "56.3099"...$ or $x - 40^{\circ} = -\pi + "0.983"...$	M1
	$x = \{-180 + 56.3099... + 40 = -83.6901...\}$ = **awrt -83.7**	A1
		(3)
(ii)(a)	$\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) = 3\cos\theta + 2$	M1
	$\left(\frac{1-\cos^2\theta}{\cos\theta}\right) = 3\cos\theta + 2$	dM1
	$1-\cos^2\theta = 3\cos^2\theta + 2\cos\theta \Rightarrow 0 = 4\cos^2\theta + 2\cos\theta - 1$ *	A1 cso *
		(3)
(b)	$\cos\theta = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	M1
	or $4(\cos\theta \pm \frac{1}{4})^2 \pm q \pm 1 = 0$, or $(2\cos\theta \pm \frac{1}{2})^2 \pm q \pm 1 = 0$, $q \neq 0$ so $\cos\theta = ...$	
	One solution is 72° or 144°, Two solutions are 72° and 144°	A1, A1
	$\theta = \{72, 144, 216, 288\}$	M1 A1
		(5)
		[11]

	Notes for Question
(i)	B1: 96.3 by any or no method M1: Takes 180 degrees from arctan (1.5) or from their "96.3" May be implied by A1. (Could be obtained by adding 180, then subtracting 360). A1: awrt –83.7 Extra answers: ignore extra answers outside range. Any extra answers in range lose final A mark (if earned) Working in radians – could earn M1 for $x - 40^{\circ} = -\pi + "0.983"...$ so B0M1A0
(ii) (a)	M1: uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ or equivalent in equation (not just $\tan = \frac{\sin}{\cos}$, with no argument) dM1: uses $\sin^2\theta = 1 - \cos^2\theta$ (quoted correctly) in equation A1: completes proof correctly, with no errors to give printed answer*. Need at least three steps in proof and need to achieve the correct quadratic with all terms on one side and "=0"
(b)	M1: Attempts to solve quadratic by correct quadratic formula, or completion of the square . Factorisation attempts score M0. 1st A1: Either 72 **or** 144, 2nd A1: both 72 **and** 144 (allow 72.0 etc.) M1: 360 – "a previous solution" (provided that cos was being used) (not dependent on previous M) A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore) (**Premature approximation**: e.g. 71.9, 144.1, 288.1 and 215.9 – lose first A1 then ft other angles) Do **not** require degrees symbol for the marks **Special case: Working in radians** M1: as before, A1 for either $\theta = \frac{2}{5}\pi$ or $\theta = \frac{4}{5}\pi$ or decimal equivalents, and 2nd A1: both M1: $2\pi - \alpha_1$ or $2\pi - \alpha_2$ then A0 so 4/5

Q8.

Question number	Scheme	Marks
(a)	States or uses $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$ $\dfrac{\sin 2x}{\cos 2x} = 5\sin 2x \Rightarrow \sin 2x - 5\sin 2x\cos 2x = 0 \Rightarrow \sin 2x(1-5\cos 2x) = 0$ *	M1 A1 (2)
(b)	$\sin 2x = 0$ gives $2x = 0, 180, 360$ so $x = 0, 90, 180$ — B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1 $\cos 2x = \frac{1}{5}$ gives $2x = 78.46$ (or 78.5 or 78.4) **or** $2x = 281.54$ (or 281.6) $x = 39.2$ (or 39.3), 140.8 (or 141)	B1, B1 M1 A1, A1 (5) **7 marks**
Notes	(a) **M1**: Statement that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or Replacement of tan (wherever it appears). Must be a correct statement but may involve θ instead of $2x$. **A1**: the answer is given so all steps should be given. N.B. $\sin 2x - 5\sin 2x\cos 2x = 0$ or $-5\sin 2x\cos 2x + \sin 2x = 0$ or $\sin 2x(\dfrac{1}{\cos 2x} - 5) = 0$ o.e. **must be seen** and be followed by printed answer for A1 mark $\sin 2x = 5\sin 2x\cos 2x$ is not sufficient. (b) Statement of 0 and 180 with no working gets B1 B0 (bod) as it is two solutions **M1**: This mark for one of the two statements given (must relate to $2x$ not just to x) **A1, A1**: first A1 for 39.2, second for 140.8 *Special case* solving $\cos 2x = -1/5$ giving $2x = 101.5$ or 258.5 is awarded M1A0A0 140.8 omitted would give M1A1A0 Allow answers which round to 39.2 or 39.3 and which round to 140.8 and allow 141 Answers in radians lose last A1 awarded (These are 0, 0.68, 1.57, 2.46 and 3.14) Excess answers **in range** lose last A1 Ignore excess answers outside range. All 5 correct answers with no extras and no working gets **full marks** in part (b). The answers imply the method here	