Question 1 (Jan 2005, Q3)

Worked Solution

Part (i): $y = 5\sqrt{x}$ stretched by scale factor $\frac{1}{2}$ parallel to the x-axis.

Replace x with $2x$: $y = 5\sqrt{2x}$.

$y = 5\sqrt{2x}$

Part (ii): Transformation from $y = 5\sqrt{x}$ to $y = 5\sqrt{x} - 3$.

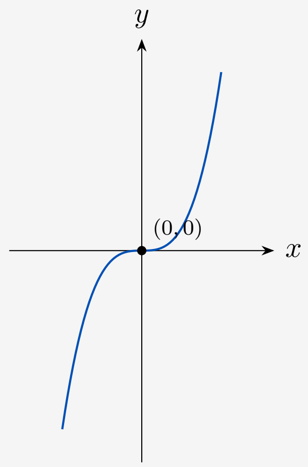
The -3 is added to the whole function, so this is a translation of $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ (3 units down).

Translation by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

Question 2 (Jun 2005, Q3)

Worked Solution

Part (i): Sketch $y = x^3$.



Part (ii): Transformation from $y = x^3$ to $y = -x^3$.

$y = -x^3 = -(x^3)$, so every y-value is negated. This is a reflection in the x-axis.

Reflection in the x-axis

Part (iii): $y = x^3$ translated p units parallel to the x-axis: replace x with $x - p$.

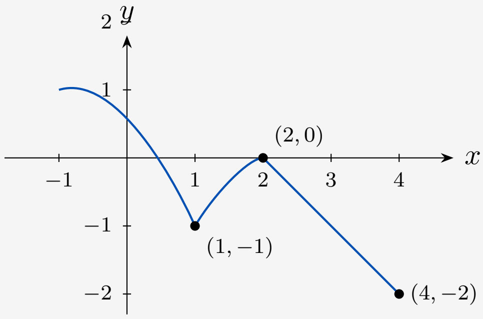
$y = (x - p)^3$

Question 3 (Jan 2007, Q5)

Worked Solution

The graph of $y = \text{f}(x)$ for $-1 \le x \le 4$ rises from $\approx (-1, -1)$ to a max near $(1, 1)$, falls to $(2, 0)$, then rises linearly to $(4, 2)$.

Part (i): Sketch $y = -\text{f}(x)$ — reflection in the x-axis. Every y-value is negated.



Part (ii): $P(1, 1)$ on $y = \text{f}(x)$ maps to Q on $y = 3\text{f}(x)$. The x-coordinate stays; y is multiplied by 3.

$Q = (1,\ 3)$

Part (iii): Transformation from $y = \text{f}(x)$ to $y = \text{f}(x + 2)$.

Replacing x with $x + 2$ shifts the graph 2 units to the left.

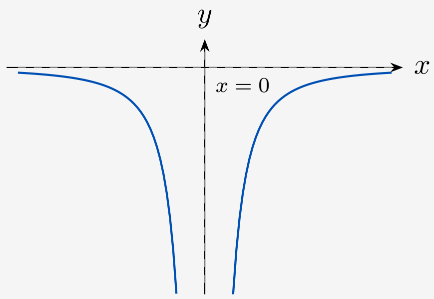
Translation of 2 units in the negative x-direction (i.e. 2 units to the left)

Question 4 (Jun 2010, Q2)

Worked Solution

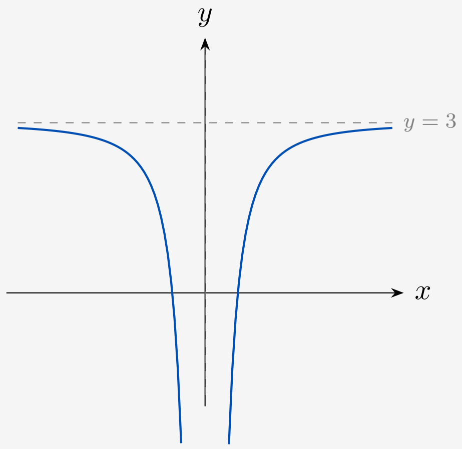
Part (i): Sketch $y = -\frac{1}{x^2}$.

This is the standard $y = \frac{1}{x^2}$ reflected in the x-axis: lies in the 3rd and 4th quadrants only, with vertical asymptote $x = 0$ and horizontal asymptote $y = 0$.



Part (ii): Sketch $y = 3 - \frac{1}{x^2}$.

This is $y = -\frac{1}{x^2}$ translated 3 units upward. Asymptote moves to $y = 3$.



Part (iii): $y = -\frac{1}{x^2}$ stretched parallel to the y-axis, scale factor 2: multiply by 2.

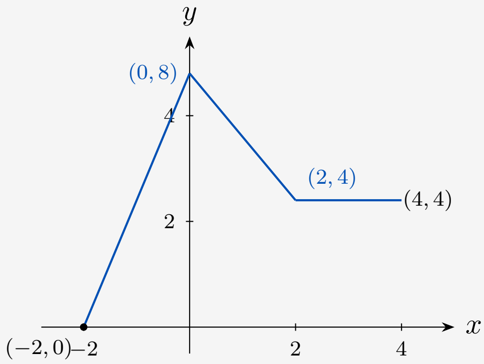
$$y = -\frac{2}{x^2}$$

Question 5 (Jan 2010, Q2)

Worked Solution

Original $f(x)$ for $-2 \leq x \leq 4$: straight line from $(-2, 0)$ to $(0, 4)$, then straight line to $(2, 2)$, then horizontal to $(4, 2)$.

Part (i): $y = 2f(x)$ — vertical stretch $\times 2$. x-coordinates unchanged; y-values doubled.



Key points: $(-2, 0)$, $(0, 8)$, $(2, 4)$, $(4, 4)$.

$y = 2f(x)$: points $(-2, 0)$, $(0, 8)$, $(2, 4)$, $(4, 4)$

Part (ii): Transformation from $f(x)$ to $f(x - 1)$: replace x with $x - 1$, so the graph shifts 1 unit to the right.

Translation of 1 unit in the positive x-direction (1 unit to the right)

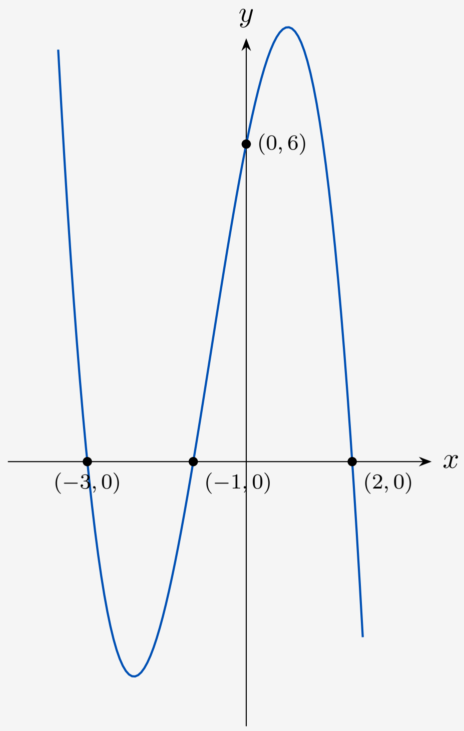
Question 6 (Jan 2013, Q3)

Worked Solution

Part (i): Sketch $y = (1 + x)(2 - x)(3 + x)$.

Roots: $x = -1$, $x = 2$, $x = -3$. Leading term: $x \cdot (-x) \cdot x = -x^3$ — so it is a negative cubic.

y-intercept: $x = 0$: $y = (1)(2)(3) = 6$.



x-intercepts: $(-3, 0)$, $(-1, 0)$, $(2, 0)$; y-intercept: $(0, 6)$

Part (ii): $(1 + x)(2 - x)(3 + x) \to (1 - x)(2 + x)(3 - x)$.

Replace x with $-x$: $(1 - x)(2 + x)(3 - x) = (1 + (-x))(2 - (-x))(3 + (-x))$. This is a reflection in the y-axis.

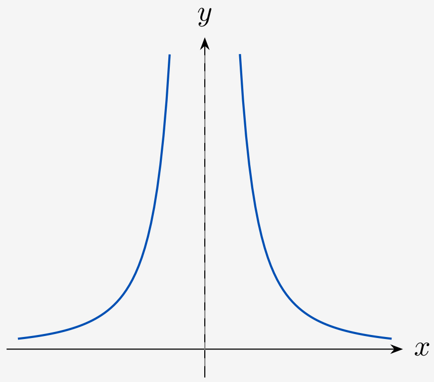
Reflection in the y-axis

Question 7 (Jun 2013, Q5)

Worked Solution

Part (i): Sketch $y = \dfrac{2}{x^2}$.

Two branches, both above the x-axis (1st and 2nd quadrants), with asymptotes $x = 0$ and $y = 0$.



Part (ii): $y = \dfrac{2}{x^2}$ translated 5 units in the negative x-direction: replace x with $x + 5$.

$$y = \frac{2}{(x+5)^2}$$

Part (iii): Transformation from $y = \dfrac{2}{x^2}$ to $y = \dfrac{1}{x^2}$.

The y-values are halved: this is a stretch parallel to the y-axis with scale factor $\dfrac{1}{2}$.

Stretch, scale factor $\frac{1}{2}$, parallel to the y-axis

Question 8 (OCR 4722, Jun 2016, Q8 – Modified)

Worked Solution

Part (i): $y = 3^x$ to $y = 3^{x-2}$ by translation.

$3^{x-2} = 3^x$ with x replaced by $x - 2$: the graph shifts 2 units to the right.

Translation of 2 units in the positive x-direction

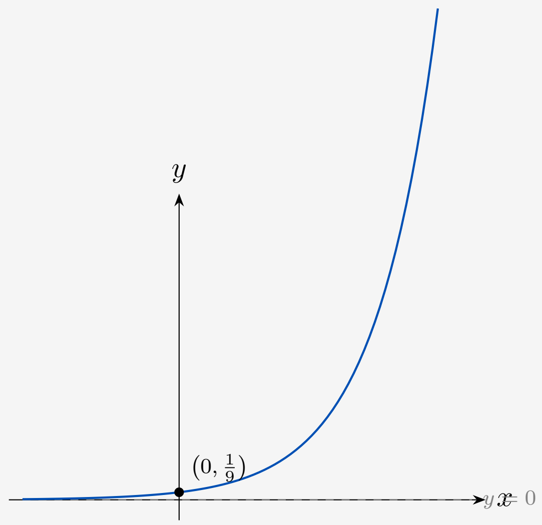
Part (ii): $y = 3^x$ to $y = 3^{x-2}$ by stretch.

$3^{x-2} = 3^x \cdot 3^{-2} = \frac{1}{9} \cdot 3^x$. So y-values are multiplied by $\frac{1}{9}$: stretch parallel to the y-axis, scale factor $\frac{1}{9}$.

Stretch, scale factor $\frac{1}{9}$, parallel to the y-axis

Part (iii): Sketch $y = 3^{x-2}$.

Same shape as $y = 3^x$, approaching $y = 0$ for $x \to -\infty$. When $x = 0$: $y = 3^{-2} = \frac{1}{9}$.



y-intercept $\left(0, \dfrac{1}{9}\right)$; asymptote $y = 0$