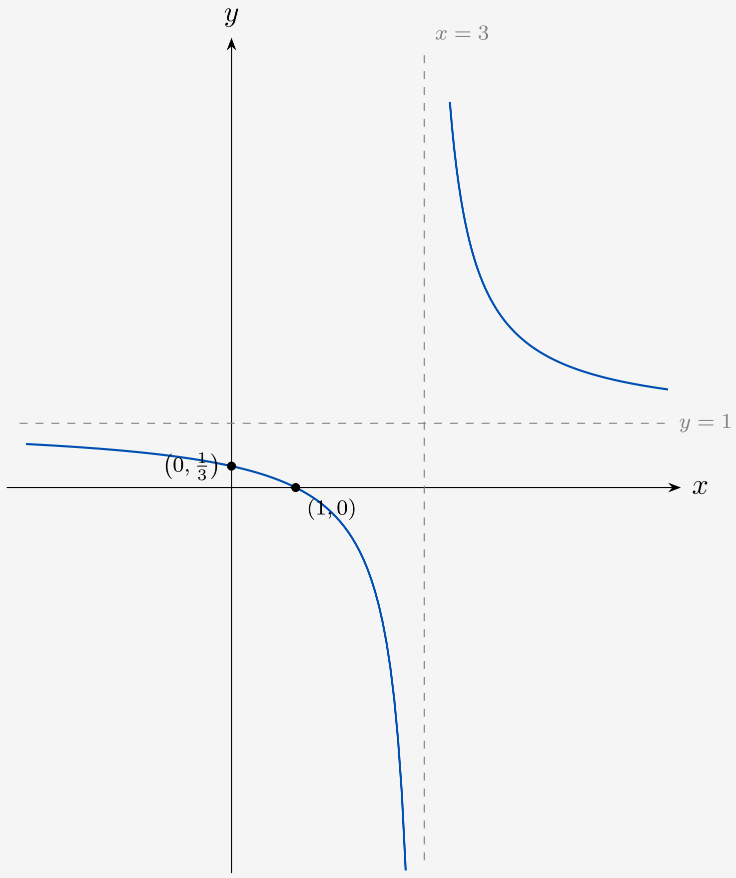
Question 1

Worked Solution

$f(x) = \dfrac{x}{x-2}$. Original curve: asymptotes $x = 2$, $y = 1$; passes through origin.

$y = f(x-1)$ is a translation of $+1$ in the x-direction, so asymptotes shift to $x = 3$ and $y = 1$.

Part (a): Sketch of $y = f(x-1)$:



Asymptotes: $x = 3$ and $y = 1$

Part (b): Axis crossings of $y = f(x-1) = \dfrac{x-1}{x-3}$.

x-intercept: set $y = 0$: $x - 1 = 0 \Rightarrow x = 1$. Point: $(1, 0)$.

y-intercept: set $x = 0$: $y = \dfrac{-1}{-3} = \dfrac{1}{3}$. Point: $\left(0, \dfrac{1}{3}\right)$.

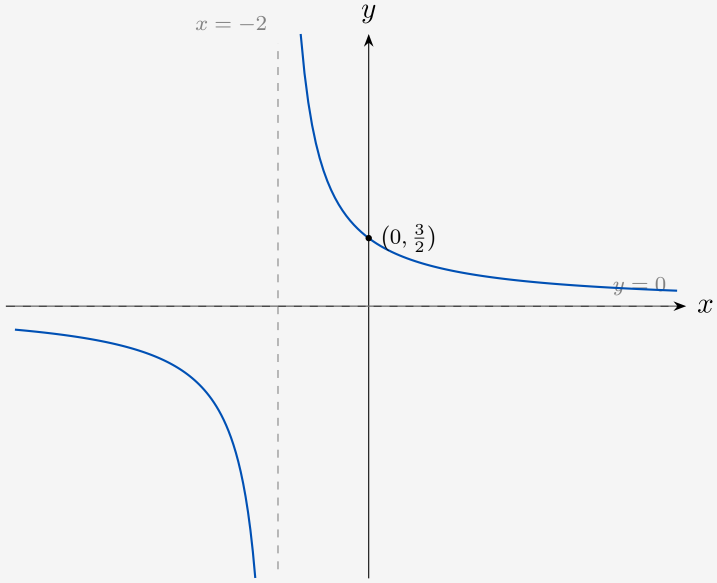
$(1, 0)$ and $\left(0, \dfrac{1}{3}\right)$

Question 2

Worked Solution

Original curve: $y = \frac{3}{x}$. The curve $y = \frac{3}{x+2}$ is a translation of -2 in the x-direction.

Part (a): Sketch of $y = \frac{3}{x+2}$:



The upper branch crosses the y-axis at $\left(0, \frac{3}{2}\right)$; the lower branch does not cross either axis.

y-axis crossing: $\left(0, \dfrac{3}{2}\right)$

Part (b): Asymptotes.

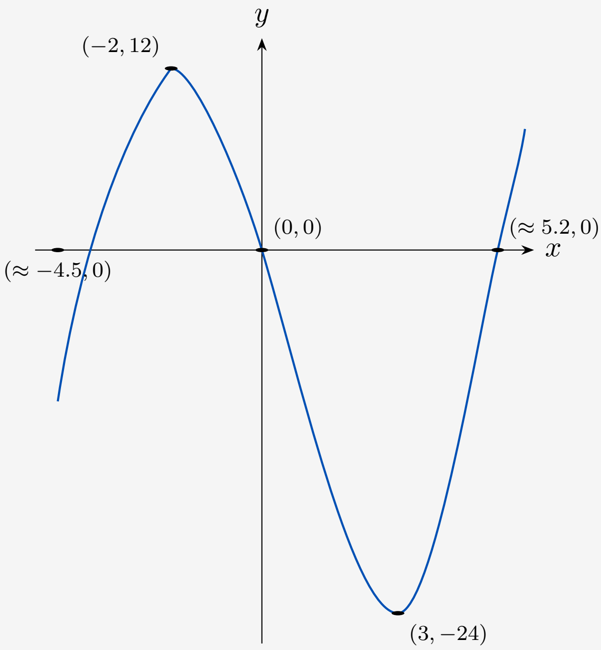
$x = -2$ and $y = 0$

Question 3

Worked Solution

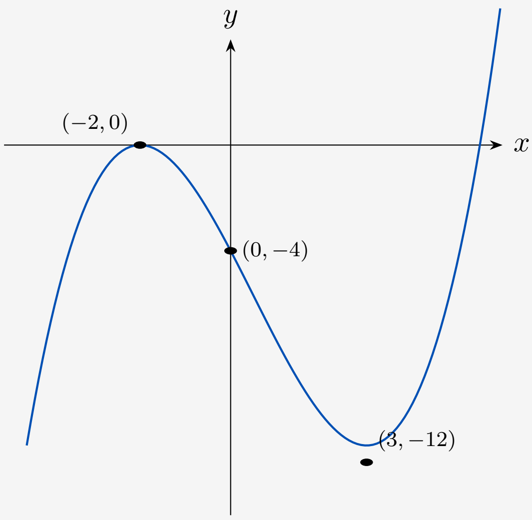
Original: max $A(-2,4)$, min $B(3,-8)$, passes through origin.

Part (a): $y = 3\mathrm{f}(x)$ — vertical stretch $\times 3$. x-coordinates unchanged; y-coordinates multiplied by 3.



Max at $(-2,\ 12)$; Min at $(3,\ -24)$; passes through $(0,0)$

Part (b): $y = \mathrm{f}(x) - 4$ — translation -4 in y-direction. x-coordinates unchanged; y-coordinates decrease by 4.



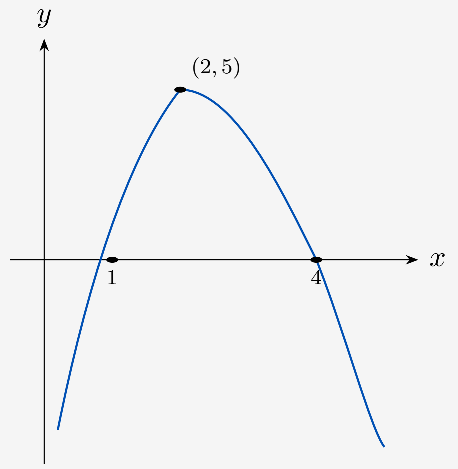
Max at $(-2,\ 0)$; Min at $(3,\ -12)$; y-intercept $(0,-4)$

Question 4

Worked Solution

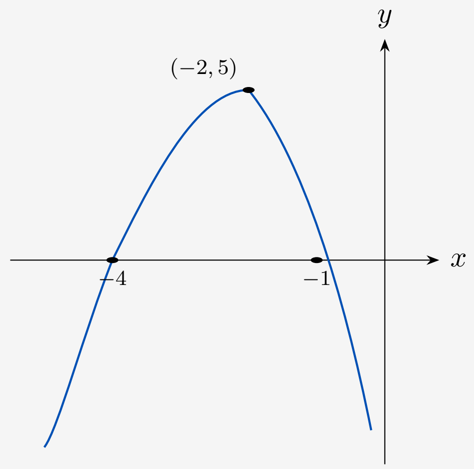
Original: x-intercepts $(1, 0)$ and $(4, 0)$; max $(2, 5)$.

Part (a): $y = 2\mathrm{f}(x)$ — vertical stretch $\times 2$. x-intercepts stay; y-values doubled.



x-intercepts $(1, 0)$ and $(4, 0)$; Max at $(2,\ 10)$

Part (b): $y = \mathrm{f}(-x)$ — reflection in the y-axis. $x \to -x$, so intercepts at $(-1, 0)$ and $(-4, 0)$; max moves to $(-2, 5)$.



x-intercepts $(-1, 0)$ and $(-4, 0)$; Max at $(-2,\ 5)$

Part (c): The max of $\mathrm{f}(x + a)$ is on the y-axis. The original max is at $x = 2$, so $2 + a = 0 \Rightarrow a = 2$.

$a = 2$

Question 5

Worked Solution

Original: crosses y-axis at $(0, 4)$, x-axis at $(5, 0)$; max $(2, 7)$; asymptote $y = 1$.

Part (a): $y = \mathrm{f}(x - 2)$ shifts right by 2. Max moves from $(2, 7)$ to $(4, 7)$.

Turning point at (4, 7)

Part (b): $\mathrm{f}(2x) = 0$ when $2x = 5 \Rightarrow x = 2.5$.

$x = 2.5$

Part (c): $y = \mathrm{f}(-x)$ is a reflection in the y-axis. Asymptote $y = 1$ is horizontal — unchanged.

$y = 1$

Part (d): A horizontal line $y = k$ meets the curve at only one point. From the sketch, the curve rises from the asymptote $y = 1$ to the max at $(2, 7)$, then falls back toward $y = 1$. A line $y = k$ meets the curve once if it is at or above the maximum, or at or below the asymptote level. Since the asymptote is $y = 1$ (approached from above for large $|x|$), the line meets the curve once when $k = 7$ (tangent at max) or $k \leq 1$ (below or at the asymptote, touching the tail).

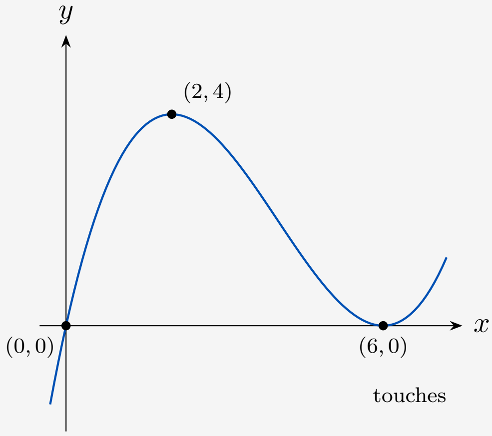
$k \leq 1$ or $k = 7$

Question 6

Worked Solution

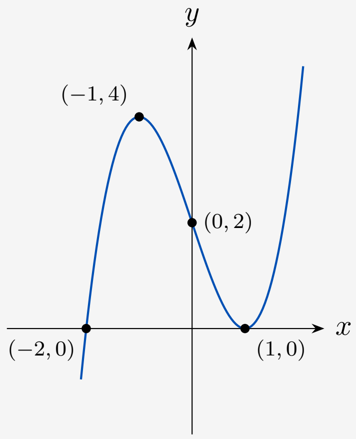
$f(x) = x(3-x)^2$. Passes through origin; touches x-axis at $(3,0)$; max $(1,4)$; min $(3,0)$.

Part (a)(i): $y = f\left(\frac{1}{2}x\right)$ — horizontal stretch $\times 2$. All x-coordinates doubled; y-coordinates unchanged.



Passes through $(0,0)$; touches x-axis at $(6,0)$; Max at $(2, 4)$

Part (a)(ii): $y = f(x+2)$ — translation -2 in x-direction. All x-coordinates decrease by 2.



Passes through $(-2,0)$; touches x-axis at $(1,0)$; Max $(-1, 4)$; y-intercept $(0,2)$

Part (b): $y = f(x)+k$ has max at $(a,0)$. Original max is $(1,4)$, so the max of $f(x)+k$ is $(1, 4+k)$. For this to be $(a,0)$: $a = 1$ and $4+k = 0 \Rightarrow k = -4$.

$a = 1, \quad k = -4$

End of Worked Solutions