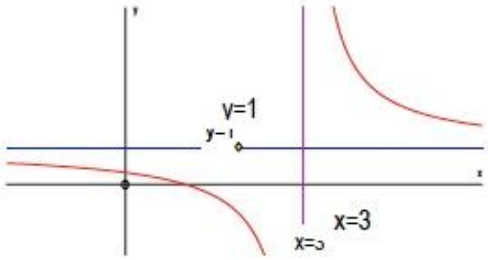
Transformation of Functions Exam Questions Sheet 2 Mark Scheme

Q1.

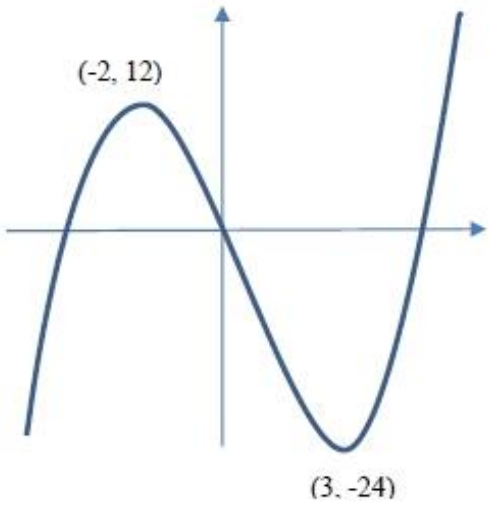
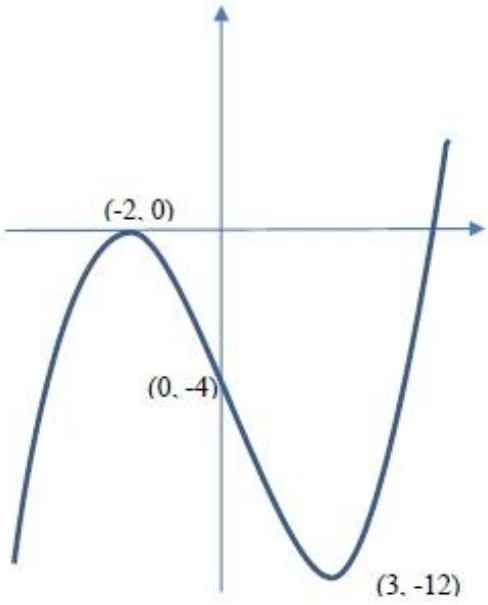
Question Number	Scheme	Marks
(a)	 Correct shape with a single crossing of each axis $y = 1$ labelled or stated $x = 3$ labelled or stated	B1 B1 B1 (3)
(b)	Horizontal translation so crosses the x-axis at (1, 0) New equation is $(y=)\dfrac{x\pm1}{(x\pm1)-2}$ When $x = 0$ $y =$ $=\dfrac{1}{3}$	B1 M1 M1 A1 (4) 7
	Notes	
(b)	B1 for point (1,0) identified - this may be marked on the sketch as 1 on x axis. Accept $x = 1$. 1st M1 for attempt at new equation and either numerator or denominator correct 2nd M1 for setting $x = 0$ in their new equation and solving as far as $y =$… A1 for $\frac{1}{3}$ or exact equivalent. Must see $y = \frac{1}{3}$ or $(0, \frac{1}{3})$ or point marked on y-axis. **Alternative** $f(-1) = \dfrac{-1}{-1-2} = \dfrac{1}{3}$ scores M1M1A0 unless $x = 0$ is seen or they write the point as $(0, \frac{1}{3})$ or give $y = 1/3$ Answers only: $x = 1$, $y = 1/3$ is full marks as is (1,0) (0, 1/3) Just 1 and 1/3 is B0 M1 M1 A0 Special case : Translates 1 unit to left (a) B0, B1, B0 (b) Mark (b) as before May score B0 M1 M1 A0 so 3/7 or may ignore sketch and start again scoring full marks for this part.	

Q2.

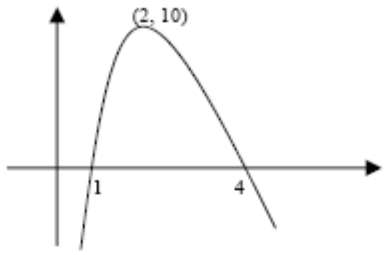
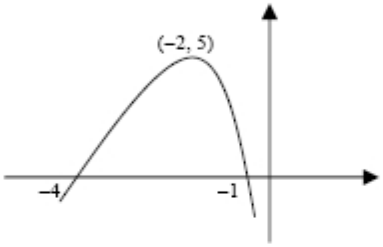
Question number	Scheme	Marks
	(a)  Translation parallel to x-axis Top branch intersects +ve y-axis Lower branch has no intersections No obvious overlap $\left(0,\frac{3}{2}\right)$ or $\frac{3}{2}$ marked on y- axis	M1 A1 B1 (3)
	(b) $x=-2,\quad y=0$	B1, B1 (2)
S.C.	[Allow ft on first B1 for $x=2$ when translated "the wrong way" but must be compatible with their sketch.]	
		5

(a)	M1 for a horizontal translation – two branches with one branch cutting $y-$ axis only. If one of the branches cuts both axes (translation up and across) this is M0. A1 for a horizontal translation to left. Ignore any figures on axes for this mark. B1 for correct intersection on positive y-axis. More than 1 intersection is B0. x=0 and $y=1.5$ in a table alone is insufficient unless intersection of their sketch is with +ve y-axis. A point marked on the graph overrides a point given elsewhere.
(b)	1st B1 for $x=-2$. NB $x\neq -2$ is B0. Can accept $x=+2$ if this is compatible with their sketch. Usually they will have M1A0 in part (a) (and usually B0 too) 2nd B1 for $y=0$.
S.C.	If $x=-2$ and $y=0$ and some other asymptotes are also given award B1B0 The asymptote equations should be clearly stated in part (b). Simply marking $x=-2$ or $y=0$ on the sketch is insufficient unless they are clearly marked "asymptote $x=-2$" etc.

Q3.

Question Number	Scheme	Notes	Marks
	Note original points are $A(-2, 4)$ and $B(3, -8)$		
(a)		Similar shape to given figure passing through the origin. A cubic shape with a maximum in the second quadrant and a minimum in the 4th quadrant. There must be evidence of a change in at least one of the y-coordinates (inconsistent changes in the y-coordinates are acceptable) but **not the x-coordinates**.	B1
		Maximum at (-2, 12) and minimum at (3, -24) with coordinates written the right way round. Condone missing brackets. The coordinates may appear on the sketch, or separately in the text (not necessarily referenced as A and B). If they are on the sketch, the x and y coordinates can be positioned correctly on the axes rather than given as coordinate pairs. In cases of ambiguity, the sketch has precedence. The origin does not need to be labelled. Nor do the x and y axes.	B1
			[2]
(b)		A positive cubic which does not pass through the origin with a maximum to the left of the y-axis and a minimum to the right of the y-axis.	M1
		Maximum at (-2, 0) and minimum at (3, -12). Condone missing brackets. For the max allow just -2 or (0, -2) if marked in the correct place. If the coordinates are in the text, they must appear as (-2, 0) and must not contradict the sketch. The curve must **touch** the x-axis at (-2, 0). For the min allow coordinates as shown or 3 and -12 to be marked in the correct places on the axes. In cases of ambiguity, the sketch has precedence.	A1
		Crosses y-axis at (0, -4). Allow just -4 (not +4) and allow (-4, 0) if marked in the correct place. If the coordinates are in the text, they must appear as (0, -4) and must not contradict the sketch. In cases of ambiguity, the sketch has precedence.	A1
			[3]
			5 marks

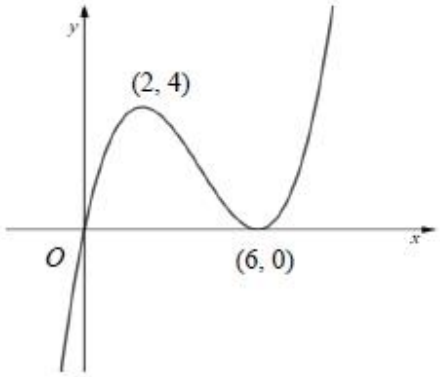
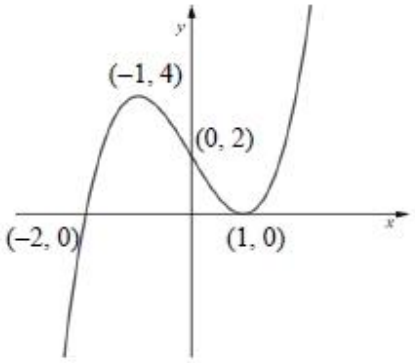
Q4.

Question number	Scheme	Marks
	(a)  Shape: Max in 1st quadrant and 2 intersections on positive x-axis 1 and 4 labelled (in correct place) or clearly stated as coordinates (2, 10) labelled or clearly stated	B1 B1 B1 (3)
	(b)  Shape: Max in 2nd quadrant and 2 intersections on negative x-axis −1 and −4 labelled (in correct place) or clearly stated as coordinates (−2, 5) labelled or clearly stated	B1 B1 B1 (3)
	(c) $(a=)$ 2 May be implicit, i.e. $f(x+2)$ Beware: The answer to part (c) may be seen on the first page.	B1 (1) 7
	(a) and (b): 1st B: 'Shape' is generous, providing the conditions are satisfied. 2nd and 3rd B marks are dependent upon a sketch having been drawn. 2nd B marks: Allow (0, 1), etc. (coordinates the wrong way round) if the sketch is correct. Points must be labelled correctly and be in appropriate place (e.g. (−2, 5) in the first quadrant is B0). (b) Special case: If the graph is reflected in the x-axis (instead of the y-axis), B1 B0 B0 can be scored. This requires shape and coordinates to be fully correct, i.e. Shape:  Minimum in 4th quadrant and 2 intersections on positive x-axis, 1 and 4 labelled (in correct place) or clearly stated as coordinates, (2, −5) labelled or clearly stated.	

Q5.

Question Number	Scheme		Marks
(a)	(4, 7)	Accept (4, 7) or $x = 4, y = 7$ or a sketch of $y = f(x-2)$ with a maximum point marked at (4, 7). (Condone missing brackets) There should be no other coordinates.	B1
			(1)
(b)	$(x =)$ 2.5	Allow (2.5, 0) (condone missing brackets) but no other values or points. Allow a sketch of f(2x) with the only x-intercept marked at $x = 2.5$ (Allow (0, 2.5) marked in the correct place.	B1
			(1)
(c)	$y = 1$ (oe e.g. $y - 1 = 0$)	Must be an equation and not just '1' and no other asymptotes stated.	B1
			(1)
(d)	$k \leq 1$ **or** $k = 7$	Either of $k \leq 1$ or $k = 7$ Accept either of $y \leq 1$ or $y = 7$ Note that $k = 7$ may sometimes be seen embedded in e.g. $k = 0, 1, 7$ and can score B1 here.	B1
	$k \leq 1$ $k = 7$	Both correct and in terms of k with no other solutions.	B1
			(2)
			(5 marks)

Q6.

Question Number	Scheme		Marks
(a)(i)		Similar shape to the given figure passing through O (be generous if it just misses O but the intention is clear) **and** with evidence of a horizontal stretch taken from the x coordinates of the max/min point(s) but with no contradiction if both points are given. There should be no change in the y coordinates. **The origin does not need to be labelled.**	B1
		Maximum at (2, 4)	B1
		Minimum at (6, 0)	B1
	The coordinates may appear on the sketch, or separately in the text. If a point on an axis appears on the sketch it is not necessary to give both coordinates. So, for example, 6 or (0, 6) on the x - axis would get credit, but if the answer is given in the text (6, 0) is needed. If there is any ambiguity, the sketch has precedence.		
(a)(ii)		Similar shape translated horizontally. Ignore any coordinates given.	M1
		Minimum at (1, 0) and crosses or at least reaches x-axis at (−2, 0)	A1
		Maximum at (−1, 4) – must correspond to a maximum in the 2nd quadrant **and** crosses the y-axis at (0, 2)	A1
			[6]
	The coordinates may appear on the sketch, or separately in the text. If a point on an axis appears on the sketch it is not necessary to give both coordinates. So, for example, 2 or (2, 0) on the y-axis would get credit but if the answer is given in the text (0, 2) is needed. If there is any ambiguity, the sketch has precedence.		
(b)	$a = 1$ **or** $k = -4$	One correct value	B1
	$a = 1$ **and** $k = -4$	Both correct	B1
	Note that these marks may be implied by sight of e.g. "f(x) – 4" and/or "(1, 0)"		
	Note that the answer to (b) often appears at the bottom of page 1		
			[2]
			8 marks