Question 1 (Specimen Q3)

$x^2 + kx + k = 0$ has no real roots. (i) Write down the discriminant. (ii) Find the set of values k can take.

Worked Solution

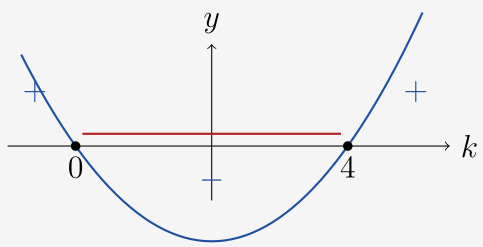
Part (i):

$$\Delta = k^2 - 4k$$

Part (ii):

For no real roots, $\Delta < 0$: $k^2 - 4k < 0$, i.e. $k(k-4) < 0$.

Critical values: $k = 0$ or $k = 4$.



$0 < k < 4$

Question 2 (Jun 2007, Q4)

(i) Find the discriminant of $kx^2 - 4x + k$ in terms of k. (ii) $kx^2 - 4x + k = 0$ has equal roots: find the possible values of k.

Worked Solution

Part (i):

$$\Delta = (-4)^2 - 4 \cdot k \cdot k = 16 - 4k^2$$

Part (ii):

Equal roots $\Rightarrow \Delta = 0$:

$$16 - 4k^2 = 0 \implies k^2 = 4 \implies k = \pm 2$$

$k = 2$ or $k = -2$

Question 3 (Jan 2010, Q10)

$kx^2 - 30x + 25k = 0$ has equal roots. Find the possible values of k.

Worked Solution

Equal roots $\Rightarrow \Delta = 0$. Here $a = k$, $b = -30$, $c = 25k$:

$$(-30)^2 - 4(k)(25k) = 0$$

$$900 - 100k^2 = 0$$

$$k^2 = 9$$

$$k = \pm 3$$

$k = 3 \quad \text{or} \quad k = -3$

Question 4 (Jan 2013, Q8)

$kx^2 + (3k-1)x - 4 = 0$ has no real roots. Find the set of possible values of k.

Worked Solution

For no real roots, $\Delta < 0$. Here $a = k$, $b = 3k-1$, $c = -4$:

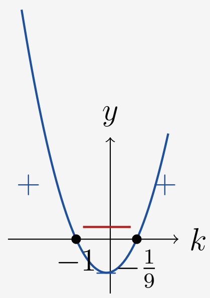
$$(3k-1)^2 - 4(k)(-4) < 0$$

$$9k^2 - 6k + 1 + 16k < 0$$

$$9k^2 + 10k + 1 < 0$$

$$(9k+1)(k+1) < 0$$

Critical values: $k = -\frac{1}{9}$ or $k = -1$.



$$-1 < k < -\frac{1}{9}$$

Question 5 (Jun 2015, Q8)

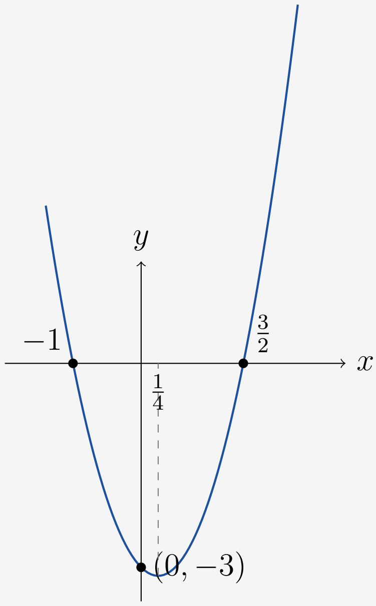
(i) Sketch $y = 2x^2 - x - 3$. (ii) Solve $2x^2 - x - 3 > 0$. (iii) $2x^2 - x - 3 = k$ has no real roots: find values of k.

Worked Solution

Part (i):

Factorise: $2x^2 - x - 3 = (2x - 3)(x + 1) = 0 \Rightarrow x = \frac{3}{2}$ or $x = -1$.

y-intercept: $(0, -3)$. Minimum at $x = \frac{1}{4}$, $y = 2(\frac{1}{16}) - \frac{1}{4} - 3 = -\frac{25}{8}$.



Part (ii):

From the sketch, $2x^2 - x - 3 > 0$ in the regions outside the roots:

$$x < -1 \quad \text{or} \quad x > \frac{3}{2}$$

Part (iii):

$2x^2 - x - 3 = k$ rearranges to $2x^2 - x - (3 + k) = 0$.

For no real roots, $\Delta < 0$:

$$1 - 4(2)(-(3 + k)) < 0 \implies 1 + 8(3 + k) < 0 \implies 25 + 8k < 0 \implies k < -\frac{25}{8}$$

This corresponds to k being below the minimum value of the curve.

$$k < -\frac{25}{8}$$

Question 6 (Jun 2016, Q9)

Find the set of values of k for which $x^2 + 2x + 11 = k(2x - 1)$ has two distinct real roots.

Worked Solution

Rearrange to standard form:

$$x^2 + 2x + 11 - k(2x - 1) = 0$$

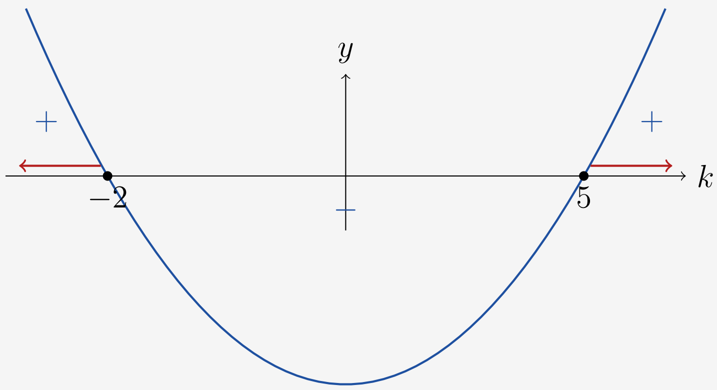
$$x^2 + (2 - 2k)x + (11 + k) = 0$$

For two distinct real roots, $\Delta > 0$:

$$(2 - 2k)^2 - 4(11 + k) > 0$$

$$4 - 8k + 4k^2 - 44 - 4k > 0$$
$$4k^2 - 12k - 40 > 0$$
$$k^2 - 3k - 10 > 0$$
$$(k - 5)(k + 2) > 0$$

Critical values: $k = 5$ or $k = -2$.



$k < -2$ or $k > 5$

End of Worked Solutions