The Discriminant Exam Questions Sheet 2

Q1.
The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.
(a) Show that k satisfies

$$k^2 + 2k - 3 > 0$$

(3)

(b) Find the set of possible values of k.

(4)

(Total 7 marks)

Q2.

$$f(x) = x^2 + (k+3)x + k$$

where k is a real constant.
(a) Find the discriminant of f(x) in terms of k.

(2)

(b) Show that the discriminant of f(x) can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation f(x) = 0 has real roots.

(2)

(Total 6 marks)

Q3.
The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k.

(4)

(Total 7 marks)

Q4.

f(x) = $x^2 + 4kx + (3+11k)$, where k is a constant.

(a) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.

(3)

Given that the equation f(x) = 0 has no real roots,
(b) find the set of possible values of k.

(4)

Given that $k = 1$,
(c) sketch the graph of y = f(x), showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)

(Total 10 marks)

Q5.
The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.
Find the value of p.

(4)

(Total 4 marks)

Q6.

The equation

$$(k + 3)\,x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24 < 0$$

(4)

(b) Hence find the set of possible values of k.

(3)

(Total 7 marks)

Q7.

Given the simultaneous equations

$$2x + y = 1$$
$$x^2 - 4ky + 5k = 0$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

(3)

(Total 8 marks)

Q8.

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

(Total 5 marks)