The Discriminant Exam Questions Sheet 2 Mark Scheme

Q1.

(a)	$b^2-4ac=(k-3)^2-4(3-2k)$ $k^2-6k+9-4(3-2k)>0$ or $(k-3)^2-12+8k>0$ or better $k^2+2k-3>0$ *	M1 M1 A1cso (3)
(b)	$(k+3)(k-1)[=0]$ Critical values are $k=1$ or -3 (choosing "outside" region) $k>1$ or $k<-3$	M1 A1 M1 A1 cao (4) **7**
	Notes	
(a)	1st M1 for attempt to find b^2-4ac with one of b or c correct 2nd M1 for a correct inequality symbol and an attempt to expand. A1cso no incorrect working seen	
(b)	1st M1 for an attempt to factorize or solve leading to $k=$ (2 values) 2nd M1 for a method that leads them to choose the "outside" region. Can follow through their critical values. 2nd A1 Allow "," instead of "or" $\geq$ loses the final A1 $1<k<-3$ scores M1A0 unless a correct version is seen before or after this one.	

Q2.

Question Number	Scheme	Marks
(a)	Discriminant: $b^2 - 4ac = (k+3)^2 - 4k$ or equivalent	M1 A1 (2)
(b)	$(k+3)^2 - 4k = k^2 + 2k + 9 = (k+1)^2 + 8$	M1 A1 (2)
(c)	For real roots, $b^2 - 4ac \geq 0$ or $b^2 - 4ac > 0$ or $(k+1)^2 + 8 > 0$ $(k+1)^2 \geq 0$ for all k, so $b^2 - 4ac > 0$, so roots are real for all k (or equiv.)	M1 A1 cso (2) **6**
	Notes (a) M1: attempt to find discriminant – substitution is required If formula $b^2 - 4ac$ is seen at least 2 of a, b and c must be correct If formula $b^2 - 4ac$ is **not** seen all 3 of a, b and c must be correct Use of $b^2 + 4ac$ is M0 A1: correct unsimplified (b) M1: Attempt at completion of square (see earlier notes) A1: both correct (no ft for this mark) (c) M1: States condition as on scheme **or** attempts to explain that their $(k+1)^2 + 8$ is greater than 0 A1: The final mark (A1cso) requires $(k+1)^2 \geq 0$ and conclusion. We will allow $(k+1)^2 > 0$ (or word positive) also allow $b^2 - 4ac \geq 0$ and conclusion.	

Q3.

Question Number	Scheme	Marks
(a)	$b^2-4ac>0 \Rightarrow \quad 16-4k(5-k)>0$ or equiv., e.g. $16>4k(5-k)$	M1A1
	So $\quad k^2-5k+4>0$ (Allow any order of terms, e.g. $4-5k+k^2>0$) (*)	A1cso (3)
(b)	Critical Values $\quad (k-4)(k-1)=0 \quad k=....$	M1
	$k = 1$ or 4	A1
	Choosing "outside" region	M1
	$\underline{k<1 \text{ or } k>4}$	A1 (4) [7]

For this question, ignore (a) and (b) labels and award marks wherever correct work is seen.

(a)

M1 for attempting to use the discriminant of the initial equation (> 0 not required, but substitution of a, b and c in the correct formula is required).
If the formula b^2-4ac is seen, at least 2 of a, b and c must be correct.
If the formula b^2-4ac is <u>not</u> seen, all 3 (a, b and c) must be correct.
This mark can still be scored if substitution in b^2-4ac is within the quadratic formula.
This mark can also be scored by comparing b^2 and $4ac$ (with substitution).
However, use of b^2+4ac is M0.

1st A1 for fully correct expression, possibly unsimplified, with > symbol. NB must appear before the last line, even if this is simply in a statement such as $b^2-4ac>0$ or 'discriminant positive'.
Condone a bracketing slip, e.g. $16-4\times k\times 5-k$ if subsequent work is correct and convincing.

2nd A1 for a fully correct derivation with no incorrect working seen.
Condone a bracketing slip if otherwise correct and convincing.

<u>Using $\sqrt{b^2-4ac}>0$</u>:
Only available mark is the first M1 (unless recovery is seen).

(b)

1st M1 for attempt to solve an appropriate 3TQ
1st A1 for both $k=1$ and 4 (only the critical values are required, so accept, e.g. $k>1$ and $k>4$). **
2nd M1 for choosing the "outside" region. A diagram or table alone is not sufficient.
Follow through their values of k.
The set of values must be 'narrowed down' to score this M mark… listing everything $k<1$, $1<k<4$, $k>4$ is M0.
2nd A1 for correct answer only, condone "$k<1$, $k>4$" and even "$k<1$ and $k>4$", but "$1>k>4$" is A0.

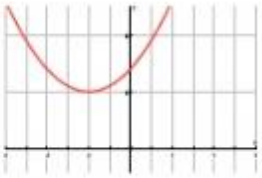
** Often the statement $k>1$ and $k>4$ is followed by the correct final answer. Allow full marks.

<u>Seeing 1 and 4 used as critical values</u> gives the first M1 A1 by implication.

In part (b), condone working with x's except for the final mark, where the set of values must be a set of values of k (i.e. 3 marks out of 4).

Use of ≤ (or ≥) in the final answer loses the final mark.

Q4.

Question number	Scheme	Marks
	(a) $(x+2k)^2$ or $\left(x+\frac{4k}{2}\right)^2$	M1
	$(x\pm F)^2 \pm G \pm 3 \pm 11k$ (where F and G are <u>any</u> functions of k, not involving x)	M1
	$(x+2k)^2-4k^2+(3+11k)$ Accept unsimplified equivalents such as $\left(x+\frac{4k}{2}\right)^2-\left(\frac{4k}{2}\right)^2+3+11k$, <u>and i.s.w. if necessary</u>.	A1 (3)
	(b) Accept part (b) solutions seen in part (a).	
	"$4k^2-11k-3"=0$ $(4k+1)(k-3)=0$ $k=\ldots,$	M1
	[Or, 'starting again', $b^2-4ac=(4k)^2-4(3+11k)$ and proceed to $k=\ldots$]	
	$-\frac{1}{4}$ and 3 (Ignore any inequalities for the first 2 marks in (b)).	A1
	Using $b^2-4ac<0$ for no real roots, i.e. "$4k^2-11k-3"<0$, to establish inequalities involving their <u>two</u> critical values m and n (<u>even if the inequalities are wrong</u>, e.g. $k<m$, $k<n$).	M1
	$-\frac{1}{4}<k<3$ (See conditions below) Follow through their critical values.	A1ft (4)
	The final A1ft is still scored if the answer $m<k<n$ follows $k<m$, $k<n$. <u>Using x instead of k in the final answer</u> loses only the 2nd A mark, (condone use of x in earlier working).	
	(c)  Shape $\vee$ (seen in (c))	B1
	Minimum in correct quadrant, <u>not</u> touching the x-axis, <u>not</u> on the y-axis, and there must be no other minimum or maximum.	B1
	(0, 14) or 14 on y-axis. Allow (14, 0) marked on y-axis.	B1 (3)
	n.b. Minimum is at $(-2,10)$, (but there is no mark for this).	[10]
	(b) 1st M: Forming and solving a 3-term quadratic in k (usual rules.. see general principles at end of scheme). The quadratic must come from "b^2-4ac", or from the "q" in part (a).	
	Using <u>wrong discriminant</u>, e.g. "b^2+4ac" will score <u>no marks</u> in part (b).	
	2nd M: As defined in main scheme above.	
	2nd A1ft: $m<k<n$, where $m<n$, for their critical values m and n.	
	Other possible forms of the answer (in each case $m<n$):	
	(i) $n>k>m$	
	(ii) $k>m$ <u>and</u> $k<n$	
	In this case the word "and" must be seen (implying intersection).	
	(iii) $k\in(m,n)$ (iv) $\{k:k>m\}\cap\{k:k<n\}$	
	<u>Not</u> just a number line.	
	<u>Not</u> just $k>m$, $k<n$ (without the word "and").	
	(c) Final B1 is dependent upon a sketch having been attempted in part (c).	

Q5.

Question Number	Scheme	Marks
Q	b^2-4ac attempted, in terms of p. $(3p)^2-4p=0$ o.e. Attempt to solve for p e.g. $p(9p-4)=0$ Must potentially lead to $p=k,\ k\neq 0$ $p=\frac{4}{9}$ (Ignore $p=0$, if seen)	M1 A1 M1 A1cso [4]
	1st M1 for an attempt to substitute into b^2-4ac or $b^2=4ac$ with b or c correct Condone x's in one term only. This can be inside a square root as part of the quadratic formula for example. **Use of inequalities can score the M marks only** 1st A1 for any correct equation: $(3p)^2-4\times1\times p=0$ or better 2nd M1 for an attempt to factorize or solve their quadratic expression in p. Method must be sufficient to lead to their $p=\frac{4}{9}$. Accept factors or use of quadratic formula or $\left(p\pm\frac{2}{9}\right)^2=k^2$ (o.e. eg) $\left(3p\pm\frac{2}{3}\right)^2=k^2$ or equivalent work on their eqn. $9p^2=4p\Rightarrow\frac{9p^{\cancel{2}}}{\cancel{p}}=4$ which would lead to $9p=4$ is OK for this 2nd M1 ALT Comparing coefficients M1 for $(x+\alpha)^2=x^2+\alpha^2+2\alpha x$ and A1 for a correct equation eg $3p=2\sqrt{p}$ M1 for forming solving leading to $\sqrt{p}=\frac{2}{3}$ or better Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.	

Q6.

Question Number	Scheme	Marks
(a)	**Method 1:** Attempts $b^2 - 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$	M1
	$b^2 - 4ac = 6^2 - 4(k+3)(k-5)$	A1
	$(b^2 - 4ac =) \quad -4k^2 + 8k + 96$ or $-(b^2 - 4ac =) \quad 4k^2 - 8k - 96$ (with no prior algebraic errors)	B1
	As $b^2 - 4ac > 0$, then $-4k^2 + 8k + 96 > 0$ and so, $k^2 - 2k - 24 < 0$	A1 *
	Method 2: Considers $b^2 > 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$	M1
	$6^2 > 4(k+3)(k-5)$	A1
	$4k^2 - 8k - 96 < 0$ or $-4k^2 + 8k + 96 > 0$ or $9 > (k+3)(k-5)$ (with no prior algebraic errors)	B1
	and so, $k^2 - 2k - 24 < 0$ following correct work	A1 *
		[4]
(b)	Attempts to solve $k^2 - 2k - 24 = 0$ to give $k =$ ($\Rightarrow$ Critical values, $k = 6, -4$.)	M1
	$k^2 - 2k - 24 < 0$ gives $-4 < k < 6$	M1 A1
		[3]
		7 marks
	Notes	
(a)	**Method 1: M1:** Attempts $b^2 - 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ or uses quadratic formula and has this expression under square root. (ignore > 0, < 0 or $= 0$ for first 3 marks)	
	A1: Correct expression for $b^2 - 4ac$ - need not be simplified (may be under root sign)	
	B1: Uses algebra to manipulate result **without error** into **one of these** three term quadratics. Again may be under root sign in quadratic formula. If inequality is used early in "proof" may see $4k^2 - 8k - 96 < 0$ and B1 would be given for $4k^2 - 8k - 96$ correctly stated.	
	A1: Applies $b^2 - 4ac > 0$ correctly (**or writes** $b^2 - 4ac > 0$) to achieve the **result given** in the question. No errors should be seen. Any incorrect line of argument should be penalised here. There are several ways of reaching the answer; either multiplication of both sides of inequality by –1, or taking every term to other side of inequality. **Need conclusion i.e. printed answer.**	
	Method 2: M1: Allow $b^2 > 4ac$ $b^2 < 4ac$ or $b^2 = 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$	
	A1: Correct expressions on either side (ignore $>$, $<$ or $=$).	
	B1: Uses algebra to manipulate result into one of the two three term quadratics or divides both sides by 4 **again without error**	
	A1: Produces result with no errors seen from initial consideration of $b^2 > 4ac$.	
(b)	**M1:** Uses factorisation, formula, completion of square method to find two values for k , or finds two **correct** answers with no obvious method	
	M1: Their Lower Limit $< k <$ Their Upper Limit Allow the M mark mark for $\leq$. (Allow $k <$ upper and $k >$ lower)	
	A1: $-4 < k < 6$ Lose this mark for $\leq$ Allow (-4, 6) [not square brackets] or $k > -4$ **and** $k < 6$ (must be **and** not **or**) Can also use intersection symbol $\cap$ **NOT** $k > -4$, $k < 6$ (M1A0)	
	Special case : **In part (a)** uses $c = k$ instead of k - 5 - scores 0 . Allow $k + 5$ for method marks	
	Special Case: In part (b) Obtaining $-6 < k < 4$ This is a common wrong answer. Give M1 M1 A0 special case.	
	Special Case: In part (b) Use of x instead of k – M1M1A0	
	Special Case: $-4 < k < 6$ and $k < -4$, $k > 6$ both given is M0A0 for last two marks. Do not treat as isw.	

Q7.

Question Number	Scheme	Marks	
(a)	$x^2-4k(1-2x)+5k(=0)$	Makes y the subject from the first equation and substitutes into the second equation (= 0 not needed here) or eliminates y by a correct method.	M1
	So $x^2+8kx+k=0$ *	Correct completion to printed answer. There must be no incorrect statements.	A1cso
			(2)
(b)	$(8k)^2-4k$	M1: Use of b^2-4ac (Could be in the quadratic formula or an inequality, = 0 not needed yet). There must be some correct substitution but there must be no x's. No formula quoted followed by e.g. $8k^2-4k=0$ is M0.	M1 A1
		A1: Correct expression. Do not condone missing brackets unless they are implied by later work but can be implied by $(8k)^2>4k$ etc.	
	$k=\frac{1}{16}$ (oe)	Cso (Ignore any reference to $k=0$) but there must be no contradictory earlier statements. A fully correct solution with no errors.	A1
			(3)
(b) Way 2 Equal roots	$\Rightarrow x^2+8kx+k=(x+\sqrt{k})^2$ $\Rightarrow 8k=2\sqrt{k}$	M1: Correct strategy for equal roots	M1A1
		A1: Correct equation	
	$k=\frac{1}{16}$ (oe)	Cso (Ignore any reference to $k=0$)	A1
(b) Way 3	Completes the Square $x^2+8kx+k=(x+4k)^2-16k^2+k$ $\Rightarrow 16k^2-k=0$	M1: $(x\pm 4k)^2\pm p\pm k,\ p\neq 0$	M1A1
		A1: Correct equation	
	$k=\frac{1}{16}$ (oe)	Cso (Ignore any reference to $k=0$)	A1
			(3)
(c)	$x^2+\frac{1}{2}x+\frac{1}{16}=0$ so $(x+\frac{1}{4})^2=0\Rightarrow x=$	Substitutes their value of k into the given quadratic and attempt to solve their 2 or 3 term quadratic as far as $x=$ (may be implied by substitution into the quadratic formula) or starts again and substitutes their value of k into the second equation and solves simultaneously to obtain a value for x.	M1
	$x=-\frac{1}{4}, y=1\frac{1}{2}$	First A1 one answer correct, second A1 both answers correct.	A1A1
	Special Case: $x^2+\frac{1}{2}x+\frac{1}{16}=0\Rightarrow x=-\frac{1}{4},\frac{1}{4}\Rightarrow y=1\frac{1}{2},\frac{1}{2}$ allow M1A1A0		
			(3)
			[8]

Q8.

Question Number	Scheme	Marks
(a)	[No real roots implies $b^2-4ac<0$.] $b^2-4ac=q^2-4\times 2q\times(-1)$	M1
	So $q^2-4\times 2q\times(-1)<0$ i.e. $q^2+8q<0$ (*)	A1 cso (2)
(b)	$q(q+8)=0$ or $(q\pm 4)^2\pm 16=0$	M1
	(q) = 0 or −8 (2 cvs)	A1
	$-8<q<0$ or $q\in(-8,0)$ or $q<0$ and $q>-8$	A1 ft (3)
		(5 marks)