Tangents and Normals to Curves (From OCR 4721)

Q1, (Specimen Paper, Q5)

(i)	$x^2-3x+2=3x-7 \Rightarrow x^2-6x+9=0$	M1		For equating two expressions for y
		A1		For correct 3-term quadratic in x
	Hence $(x-3)^2=0$	M1		For factorising, or other solution method
	So $x=3$ and $y=2$	A1		For correct value of x
		A1	**5**	For correct value of y
(ii)	The line $y=3x-7$ is the tangent to the curve	B1		For stating tangency
	$y=x^2-3x+2$ at the point $(3, 2)$	B1	**2**	For identifying $x=3$, $y=2$ as coordinates
(iii)	Gradient of tangent is 3	B1		For stating correct gradient of given line
	Hence gradient of normal is $-\frac{1}{3}$	B1√		For stating corresponding perpendicular grad
	Equation of normal is $y-2=-\frac{1}{3}(x-3)$	M1		For appropriate use of straight line equation
	i.e. $x+3y-9=0$	A1	**4**	For correct equation in required form
			11	

Q2, (Jun 2005, Q10)

(i)	$\frac{dy}{dx}=x^2-9$			x^2-9
		B1		1 term correct
		B1	2	Both terms correct
(ii)	$x^2-9=0$	*M1		uses $\frac{dy}{dx}=0$
	$x=3, -3$	A1		$x=3, -3$
	$y=-18, 18$	A1	3	$y=-18, 18$ (1 correct pair A1 A0)
(iii)	$\frac{d^2y}{dx^2}=2x$	DM1		Looks at sign of $\frac{d^2y}{dx^2}$ or other correct method
	$x=3$ $\frac{d^2y}{dx^2}=6$	A1		$x=3$ minimum
	$x=-3$ $\frac{d^2y}{dx^2}=-6$	A1	3	$x=$-3 maximum (N.B. If no method shown but min and max correctly stated, award all 3 marks unless earlier incorrect working)

(iv)	gradient of $24x+3y+2=0$ is -8 $x^2-9=-8$ $x=\pm 1$ For line $x=1,\ y=-8\frac{2}{3}$ $x=-1,\ y=7\frac{1}{3}$ For curve $x=1,\ y=-8\frac{2}{3}$ $x=-1,\ y=8\frac{2}{3}$ $\therefore p=1, q=-8\frac{2}{3}$	B1 M1 M1 M1 A1	 5 **13**	Gradient = – 8 $x^2 - 9 = -8$ one of their x values substituted in both line <u>and</u> curve second x value substituted in both line and curve **<u>or</u>** justification that first point is the correct one $p=1, q=-8\frac{2}{3}$ seen <u>Alternative methods:</u> <u>Either:</u> Solve equations for curve and line simultaneously to get one solution (either x = 1 or x = -2) M1 Gradient of line = -8 B1 Substitution of one x value into their gradient formula and check for -8 M1 Substitution of other x value into gradient formula and check for -8 or justification as above M1 Correct q value A1 <u>Or:</u> Solve equations for curve and line simultaneously to get one solution M1 Factorise to $(x-1)^2(x+2)$ B1 State that a double root implies a tangent at x = 1 M2 Correct value for y A1

Q3, (Jan 2009, Q10)

(i)	$\frac{dy}{dx} = 2x+1$	M1	Attempt to differentiate y
	$= 5$	A1 2	cao
(ii)	Gradient of normal $= -\frac{1}{5}$	B1 ft	ft from a non-zero numerical value in (i)
	When $x = 2$, $y = 6$	B1	May be embedded in equation of line
	$y - 6 = -\frac{1}{5}(x-2)$	M1	Equation of line, any non-zero gradient, their y coordinate
	$x + 5y - 32 = 0$	A1 4	Correct equation in correct form
(iii)	$x^2 + x = kx - 4$	*M1	Equating $y_1 = y_2$
	$x^2 + (1-k)x + 4 = 0$		
	One solution => $b^2 - 4ac = 0$	DM1	Statement that discriminant = 0
	$(1-k)^2 - 4 \times 1 \times 4 = 0$	DM1	Attempt (involving k) to use a, b, c from their equation
	$(1-k)^2 = 16$	A1	Correct equation (may be unsimplified)
	$1 - k = \pm 4$	DM1	Correct method to find k, dep on 1st 3Ms
	k = -3 or 5	A1 6	Both values correct
		12	

Q4, (Jan 2010, Q3)

$\frac{dy}{dx} = 3x^2 - 8x$	M1		Attempt to differentiate (one of $3x^2$, $-8x$)
	A1		Correct derivative
When $x = 2$, $\frac{dy}{dx} = -4$	M1		Substitutes $x = 2$ into their $\frac{dy}{dx}$
	A1		
$\therefore$ Gradient of normal to curve $= \frac{1}{4}$	B1 ft		Must be numerical $= -1 \div$ their m
$y + 1 = \frac{1}{4}(x-2)$	M1		Correct equation of straight line through $(2, -1)$, any non-zero numerical gradient
$x - 4y - 6 = 0$	A1	7	Correct equation in required form
		7	

Q5, (Jan 2011, Q8)

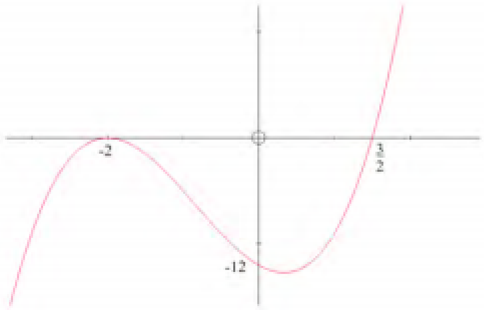
Q6, (Jun 2011, Q10)

(i)	$\frac{dy}{dx} = 6 - 2x$	M1 A1		Attempt to differentiate $\pm y$ Correct expression **cao**	One correct non-zero term
	When $x = 5$, $6 - 2x = -4$	M1		Substitute $x = 5$ into their $\frac{dy}{dx}$	
	When $x = 5$, $y = 12$	B1		Correct y coordinate	
	$y - 12 = -4(x - 5)$	M1		Correct equation of straight line through (5, their y), their non-zero, numerical gradient	Allow $\frac{y-12}{x-5}$ = their gradient If using $y = mx + c$ must attempt at evaluating c
	$4x + y - 32 = 0$	A1	6	Shows rearrangement to correct form	Allow any correct form e.g. $0 = 2y + 8x - 64$ etc.
(ii)	Q is point (8, 0)	B1ft		ft from line in (i)	.
	Midpoint of $PQ = \left(\frac{5+8}{2}, \frac{12+0}{2}\right)$	M1		Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ o.e. for their P,Q	
	$= \left(\frac{13}{2}, 6\right)$	A1	3		Do not accept $\left(\frac{13}{2}, \frac{12}{2}\right)$
(iii)	$6 - 2x = 0$	M1		Solution of their $\frac{dy}{dx} = 0$	Alternatives for Method Mark a) attempts completion of square with $\pm(x-3)^2$ b) attempts to solve quadratic (usual scheme) and to find the mid-point of the two roots c) attempts to use $x = -\frac{b}{2a}$ (allow one sign slip on substitution)
	(Line of symmetry is) $x = 3$	A1	2	Allow from $\pm[16 - (x-3)^2]$, $\pm[6 - 2x = 0]$	
(iv)	$x < 3$	M1		$x <$ their3 or $x >$ their3 **OR** attempt to solve their $\frac{dy}{dx} > 0$	May solve $\frac{dy}{dx} = 0$ then use $\frac{d^2y}{dx^2} < 0$ implies maximum point for the method mark, or sketch of curve
		A1	2 **13**	Allow from $\pm[16 - (x-3)^2]$, $\pm[6 - 2x = 0]$ in (iii)	Allow $x \le 3$

Q7, (Jun 2012, Q6)

$\frac{dy}{dx} = -12x^{-3}$	M1 A1	Attempt to differentiate (i.e. kx^{-3} seen) Correct derivative	"+ C" is A0
When $x = 2$, $\frac{dy}{dx} = -\frac{3}{2}$	A1	Correct value of $\frac{dy}{dx}$. Allow equivalent fractions.	
Gradient of normal $= \frac{2}{3}$	B1 FT	Follow through their **evaluated** $\frac{dy}{dx}$	Must be processed correctly
When $x = 2$, $y = -\frac{7}{2}$	B1	Correct y coordinate, accept equivalent forms	
$y + \frac{7}{2} = \frac{2}{3}(x-2)$	M1	Correct equation of straight line through (2, their evaluated y), any non-zero gradient	
$4x - 6y - 29 = 0$	A1 **[7]**	Correct equation in required form i.e. $k(4x - 6y - 29) = 0$ for integer k. **Must have** "=0".	

Q8, (Jun 2014, Q10)

i)			**B1** **B1** **B1** **[3]**	Positive cubic with max and min Correct y intercept – graph must be drawn Double root shown at $x = -2$ and single root at $x = \frac{3}{2}$ with no extras – graph must be drawn	For first mark must clearly be a cubic – must not stop at either axis, do not allow straight line sections/tending to extra turning points etc.
ii)		$x^2 + 4x + 4$ or $2x^2 + x - 6$ $2x^3 + 5x^2 - 4x - 12$ $\frac{dy}{dx} = 6x^2 + 10x - 4$ When $x = -1$, gradient $= -8$ When $x = -1$, $y = -5$ $y + 5 = -8(x + 1)$ $8x + y + 13 = 0$	**B1** **M1** **A1** **M1*** **M1dep*** **A1ft** **B1** **M1** **A1**	Obtain one quadratic factor Multiply their three term quadratic by linear factor to obtain at least 5 term cubic If simplified, must be correct Attempt to differentiate (power of at least one term involving x reduced by one) Substitutes to find gradient at $x = -1$ Correct gradient found **ft** their derivative, differentiation of their expression must be fully correct to earn this mark Correct y value Correct equation of straight line through (-1, their y), their gradient from differentiation	**Check for working for this in 10 (i)** Alternative using product rule: Clear attempt at product rule **M1*** Differentiates $(x + 2)^2$ correctly **A1** Both expressions fully correct **A2 (1 each)**, then as main scheme y must have been found, do not allow use of gradient of normal instead of tangent i.e. $k(8x + y + 13) = 0$. Must have "=0".
			[9]	Correct answer in correct form	Note If $x = 1$ used instead of $x = -1$, then max possible from last 5 marks is **M1 M1** only

Q9, (Jun 2015, Q7)

(a)		$(x^2+3)(5-x)=5x^2-x^3+15-3x$	M1	Attempt to multiply out brackets, Must have four terms, at least three correct	Alternative using product rule: Clear attempt at correct rule **M1*** Both expressions fully correct **A1** Expand brackets of both parts **M1*dep** Fully correct expression **A1**
			A1	Fully correct expression. Do not ISW if signs then changed. Max 2/4.	
		$\frac{dy}{dx}=10x-3x^2-3$	M1	Attempt to differentiate their expression, (power of at least one term involving x reduced by one)	
			A1 **[4]**		
(b)		$\frac{dy}{dx}=-\frac{1}{3}x^{-\frac{4}{3}}$	M1	Attempt to differentiate i.e. $-\frac{1}{3}x^{-\frac{k}{3}}$ soi for positive integer k	$x^{-\frac{1}{3}}$ misread as $x^{\frac{1}{3}}$ earns max 2/4: $\frac{dy}{dx}=\frac{1}{3}x^{-\frac{2}{3}}$ **M1 A0 MR**
			A1	Fully correct	
		When $x=-8$ $\frac{dy}{dx}=-\frac{1}{3}\times(-8)^{-\frac{4}{3}}$	B1	$(-8)^{-\frac{4}{3}}=\frac{1}{16}$ **www** Must use -8	$(-8)^{-\frac{2}{3}}=\frac{1}{4}$ **B1**
		$\frac{dy}{dx}=-\frac{1}{3}\times\frac{1}{16}=-\frac{1}{48}$	A1 **[4]**	Final answer	Final answer $\frac{1}{12}$ **A0 MR**

Q10, (Jun 2017, Q11)

(i)		Gradient of given line = 6	B1	soi as gradient of the line	Can be implied by use of $-\frac{1}{6}$
		Perpendicular gradient = $-\frac{1}{6}$	M1	Uses product of perpendicular gradients is – 1 at some point; may be implied by later working.	
		$\frac{dy}{dx} = -2kx^{-3}$	M1 A1	Attempt to differentiate (ax^{-3} seen) Fully correct	
		$-\frac{1}{6} = -2k(-3)^{-3}$	M1	Equates their derivative at $x = -3$ with their perpendicular gradient	e.g. $-\frac{27}{2k} = 6$ (implies first M1)
		$k = -\frac{9}{4}$	A1 **[6]**	Correct value of k. Allow $-\frac{27}{12}$ etc.	
(ii)		When $x = -3$, $y = -\frac{9}{4(-3)^2} = -\frac{1}{4}$	B1	Correct value of y www	
		$y + \frac{1}{4} = 6(x+3)$	M1	Attempts equation of straight line through $(-3, y)$, any non-zero gradient. y must be from their k but allow slips for M mark.	
			A1ft	Correct equation in any form – gradient 6 but ft their value of $\frac{k}{9}$. Allow $6(x - -3)$	For the first A mark, allow follow through their value of k – straight line through (–3, their $\frac{k}{9}$) with correct gradient of 6 e.g. $k = 81$ leads to $y - 9 = 6(x+3)$
		$24x - 4y + 71 = 0$	A1 **[4]**	Correct equation in required form i.e. $a(24x - 4y + 71) = 0$ for integer a, terms in any order. cao	