## Tangents and Normals to Curves Exam Questions Sheet 2

**Q1.**

The curve $C$ has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0$$

The point $P$ has coordinates (2, 7).

(a) Show that $P$ lies on $C$.

**(1)**

(b) Find the equation of the tangent to $C$ at $P$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants.

**(5)**

The point $Q$ also lies on $C$.

Given that the tangent to $C$ at $Q$ is perpendicular to the tangent to $C$ at $P$,

(c) show that the $x$-coordinate of $Q$ is $\frac{1}{3}(2+\sqrt{6})$.

**(5)**

**(Total 11 marks)**

**Q2.**

The curve $C$ has equation $y = kx^3 - x^2 + x - 5$, where $k$ is a constant.

(a) Find $\frac{dy}{dx}$.

**(2)**

The point $A$ with $x$-coordinate $-\frac{1}{2}$ lies on $C$. The tangent to $C$ at $A$ is parallel to the line with equation $2y - 7x + 1 = 0$.

Find

(b) the value of $k$,

**(4)**

(c) the value of the $y$-coordinate of $A$.

**(2)**

**(Total 8 marks)**

**Q3.**

The curve $C$ has equation $y = x^2(x - 6) + \frac{4}{x}$, $x > 0$.

The points $P$ and $Q$ lie on $C$ and have $x$-coordinates 1 and 2 respectively.

(a) Show that the length of $PQ$ is $\sqrt{170}$.

**(4)**

(b) Show that the tangents to $C$ at $P$ and $Q$ are parallel.

**(5)**

(c) Find an equation for the normal to $C$ at $P$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(4)**

**(Total 13 marks)**

**Q4.**

The curve $C$ has equation $y = 2x^3 + kx^2 + 5x + 6$, where $k$ is a constant.

(a) Find $\frac{dy}{dx}$

**(2)**

The point $P$, where $x = -2$, lies on $C$.

The tangent to $C$ at the point $P$ is parallel to the line with equation $2y - 17x - 1 = 0$

Find

(b) the value of $k$,

**(4)**

(c) the value of the $y$ coordinate of $P$,

**(2)**

(d) the equation of the tangent to $C$ at $P$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(2)**

**(Total for question = 10 marks)**

**Q5.**

The curve $C$ has equation

$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \quad x > 0$

(a) Find $\frac{dy}{dx}$.

**(4)**

(b) Show that the point $P(4,-8)$ lies on $C$.

**(2)**

(c) Find an equation of the normal to $C$ at the point $P$, giving your answer in the form $ax + by + c = 0$ , where $a$, $b$ and $c$ are integers.

**(6)**

**(Total 12 marks)**

**Q6.**
The curve $C$ has equation

$$y = 2x - 8\sqrt{x} + 5 \, , \, x \geq 0$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form.

**(3)**

The point $P$ on $C$ has $x$-coordinate equal to $\frac{1}{4}$

(b) Find the equation of the tangent to $C$ at the point $P$, giving your answer in the form $y = ax + b$, where $a$ and $b$ are constants.

**(4)**

The tangent to $C$ at the point $Q$ is parallel to the line with equation $2x - 3y + 18 = 0$

(c) Find the coordinates of $Q$.

**(5)**

**(Total 12 marks)**

**Q7.**
The curve $C$ has equation $y = f(x)$, $x > 0$, where

$$f'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Given that the point P(4, −8) lies on $C$,

(a) find the equation of the tangent to $C$ at $P$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants.

**(4)**

(b) Find $f(x)$, giving each term in its simplest form.

**(5)**

**(Total for question = 9 marks)**

**Q8.**
The curve $C$ has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0$$

The point $P$ on $C$ has $x$-coordinate equal to 2.

(a) Show that the equation of the tangent to $C$ at the point $P$ is $y = 1 - 2x$.

**(6)**

(b) Find an equation of the normal to $C$ at the point $P$.

**(3)**

The tangent at $P$ meets the $x$-axis at $A$ and the normal at $P$ meets the $x$-axis at $B$.

(c) Find the area of triangle $APB$.

**(4)**

**(Total 13 marks)**

**Q9.**

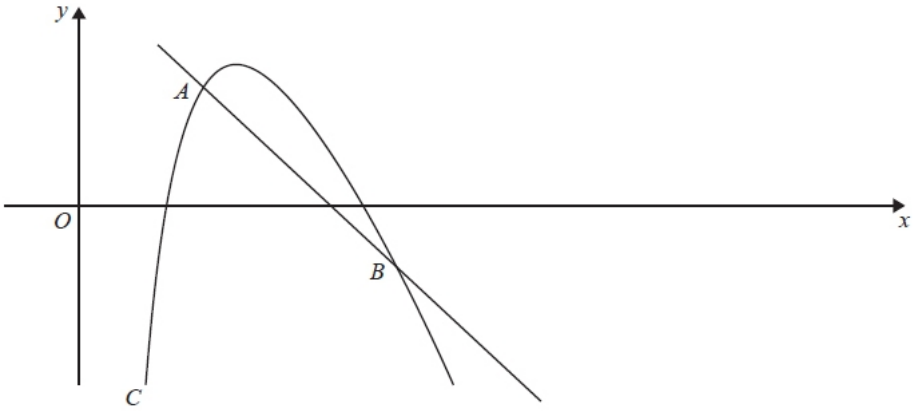


**Figure 3**

A sketch of part of the curve $C$ with equation

$$y = 20 - 4x - {}^{18}\!/_{x}, \quad x > 0$$

is shown in Figure 3.

Point $A$ lies on $C$ and has an $x$ coordinate equal to 2

(a) Show that the equation of the normal to $C$ at $A$ is $y = -2x + 7$

**(6)**

The normal to $C$ at $A$ meets $C$ again at the point $B$, as shown in Figure 3.

(b) Use algebra to find the coordinates of $B$.

**(5)**

**(Total 11 marks)**

**Q10.**

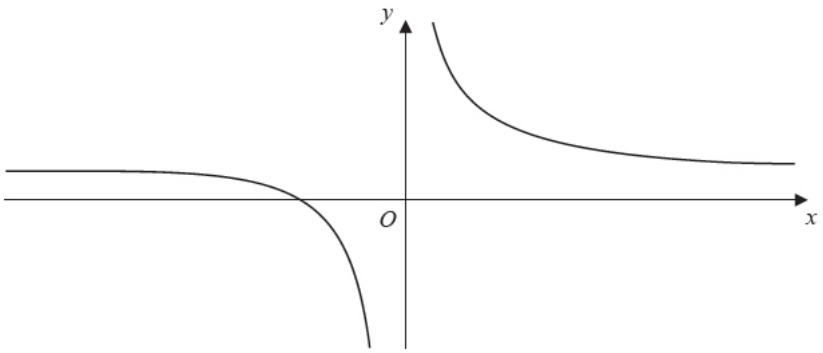


**Figure 2**

Figure 2 shows a sketch of the curve $H$ with equation $y = \frac{3}{x} + 4$, $x \neq 0$.

(a) Give the coordinates of the point where $H$ crosses the $x$-axis.

**(1)**

(b) Give the equations of the asymptotes to $H$.

**(2)**

(c) Find an equation for the normal to $H$ at the point $P(-3, 3)$.

**(5)**

This normal crosses the $x$-axis at $A$ and the $y$-axis at $B$.

(d) Find the length of the line segment $AB$. Give your answer as a surd.

**(3)**

**(Total 11 marks)**

**Q11.**

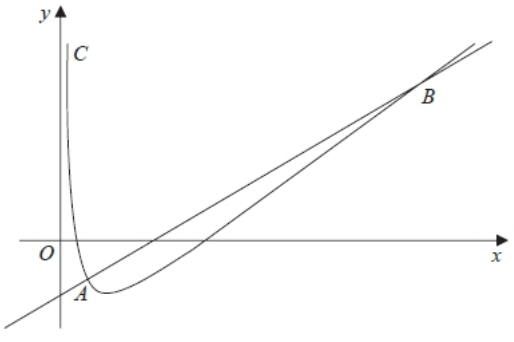


Figure 3

Figure 3 shows a sketch of part of the curve $C$ with equation

$$y = \frac{1}{2}x + \frac{27}{x} - 12, \qquad x > 0$$

The point $A$ lies on $C$ and has coordinates $\left(3, -\frac{3}{2}\right)$.

(a) Show that the equation of the normal to $C$ at $A$ can be written as $10y = 4x - 27$

**(5)**

The normal to $C$ at $A$ meets $C$ again at the point $B$, as shown in Figure 3.

(b) Use algebra to find the coordinates of $B$.

**(5)**

**(Total for question = 10 marks)**

**Q12.**

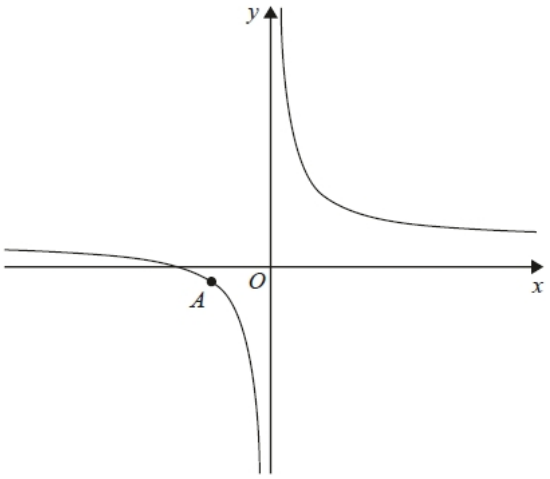


**Figure 1**

Figure 1 shows a sketch of part of the curve $H$ with equation

$$y = \frac{12}{x} + 5 \quad x \neq 0$$

(a) Find an equation for the normal to $H$ at the point $A$ $(-2, -1)$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(5)**

The points $B$ and $C$ also lie on the curve $H$.
The normal to $H$ at the point $B$ and the normal to $H$ at the point $C$ are each parallel to the straight line with equation $4y = 3x + 5$

(b) Find the coordinates of the points $B$ and $C$, given that the $x$ coordinate of $B$ is positive.

**(5)**

**(Total for question = 10 marks)**