



Tangents and Normals to Curves Sheet 2 Mark Scheme

Q1.

Question Number	Scheme	Marks
Q	<p>(a) $x = 2: y = 8 - 8 - 2 + 9 = 7$ (*)</p> <p>(b) $\frac{dy}{dx} = 3x^2 - 4x - 1$ $x = 2: \frac{dy}{dx} = 12 - 8 - 1 (= 3)$ $y - 7 = 3(x - 2), \quad \underline{y = 3x + 1}$</p> <p>(c) $m = -\frac{1}{3}$ (for $-\frac{1}{m}$ with their m) $3x^2 - 4x - 1 = -\frac{1}{3}, \quad 9x^2 - 12x - 2 = 0$ or $x^2 - \frac{4}{3}x - \frac{2}{9} = 0$ (o.e.) $\left(x = \frac{12 + \sqrt{144 + 72}}{18}\right) (\sqrt{216} = \sqrt{36} \sqrt{6} = 6\sqrt{6})$ or $(3x - 2)^2 = 6 \rightarrow 3x = 2 \pm \sqrt{6}$ $x = \frac{1}{3}(2 + \sqrt{6})$ (*)</p>	<p>B1 (1)</p> <p>M1 A1</p> <p>A1ft</p> <p>M1, <u>A1</u> (5)</p> <p>B1ft</p> <p>M1, A1</p> <p>M1</p> <p>A1cso (5)</p> <p style="text-align: right;">[11]</p>
(a)	<p>B1 there must be a clear attempt to substitute $x = 2$ leading to 7 e.g. $2^3 - 2 \times 2^2 - 2 + 9 = 7$</p>	
(b)	<p>1st M1 for an attempt to differentiate with at least one of the given terms fully correct. 1st A1 for a fully correct expression 2nd A1ft for sub. $x = 2$ in <u>their</u> $\frac{dy}{dx}$ ($\neq y$) accept for a correct expression e.g. $3 \times (2)^2 - 4 \times 2 - 1$</p> <p>2nd M1 for use of their "3" (provided it comes from their $\frac{dy}{dx}$ ($\neq y$) and $x=2$) to find equation of tangent. Alternative is to use (2, 7) in $y = mx + c$ to <u>find a value</u> for c. Award when $c = \dots$ is seen. No attempted use of $\frac{dy}{dx}$ in (b) scores 0/5</p>	
(c)	<p>1st M1 for forming an equation from their $\frac{dy}{dx}$ ($\neq y$) and their $-\frac{1}{m}$ (must be changed from m) 1st A1 for a correct 3TQ all terms on LHS (condone missing =0) 2nd M1 for proceeding to $x = \dots$ or $3x = \dots$ by formula or completing the square for a 3TQ. Not factorising. Condone \pm 2nd A1 for proceeding to given answer with no incorrect working seen. Can still have \pm.</p>	
ALT	<p><u>Verify (for M1AIM1A1)</u> 1st M1 for attempting to square need ≥ 3 correct values in $\frac{4+6+4\sqrt{6}}{9}$, 1st A1 for $\frac{10+4\sqrt{6}}{9}$ 2nd M1 Dependent on 1st M1 in this case for substituting in all terms of their $\frac{dy}{dx}$ 2nd A1cso for cso with a full comment e.g. "the x co-ord of Q is ..."</p>	

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Q2.

Question Number	Scheme	Marks
(a)	$\left[\frac{dy}{dx} = \right] 3kx^2 - 2x + 1$	M1 A1 (2)
(b)	Gradient of line is $\frac{7}{2}$ When $x = -\frac{1}{2}$: $3k \times \left(\frac{1}{4}\right) - 2 \times \left(-\frac{1}{2}\right) + 1 = \frac{7}{2}$	B1 M1
	$\frac{3k}{4} = \frac{3}{2} \Rightarrow k = 2$	A1 A1 (4)
(c)	$x = -\frac{1}{2} \Rightarrow y = k \times \left(-\frac{1}{8}\right) - \left(\frac{1}{4}\right) - \frac{1}{2} - 5 = -6$	M1 A1 (2)
		(8 marks)

Q3.

Question number	Scheme	Marks
(a)	$x = 1: y = -5 + 4 = \underline{-1}, \quad x = 2: y = -16 + 2 = \underline{-14}$ $PQ = \sqrt{(2-1)^2 + (-14 - (-1))^2} = \sqrt{170}$	(can be given in (b) or (c)) (*) 1 st B1 for - 1 2 nd B1 for - 14 M1 A1cso (4)
(b)	$y = x^3 - 6x^2 + 4x^{-1}$ $\frac{dy}{dx} = 3x^2 - 12x - 4x^{-2}$	M1 M1 A1
	$x = 1: \frac{dy}{dx} = 3 - 12 - 4 = -13$	M: Evaluate at one of the points M1
	$x = 2: \frac{dy}{dx} = 12 - 24 - 1 = -13 \quad \therefore \text{Parallel}$	A: Both correct + conclusion A1 (5)
(c)	Finding gradient of normal $\left(m = \frac{1}{13}\right)$ $y - -1 = \frac{1}{13}(x - 1)$ <u>$x - 13y - 14 = 0$</u>	M1 M1 A1ft o.e. A1cso (4)
		13



Q4.

Question Number	Scheme		Marks
(a)	$y = 2x^3 + kx^2 + 5x + 6$		
	$\left(\frac{dy}{dx}\right) = 6x^2 + 2kx + 5$	M1: $x^n \rightarrow x^{n-1}$ for one of the terms including $6 \rightarrow 0$ A1: Correct derivative	M1 A1
			[2]
(b)	Gradient of given line is $\frac{17}{2}$	Uses or states $\frac{17}{2}$ or equivalent e.g. 8.5. Must be stated or used in (b) and not just seen as part of $y = \frac{17}{2}x + \frac{1}{2}$.	B1
	$\left(\frac{dy}{dx}\right)_{x=-2} = 6(-2)^2 + 2k(-2) + 5$	Substitutes $x = -2$ into their derivative (not the curve)	M1
	$"24 - 4k + 5" = \frac{17}{2} \Rightarrow k = \frac{41}{8}$	dM1: Puts their expression = their $\frac{17}{2}$ (Allow BOD for 17 or -17 but not the normal gradient) and solves to obtain a value for k . Dependent on the previous method mark. A1: $\frac{41}{8}$ or $5\frac{1}{8}$ or 5.125	dM1 A1
	<u>Note:</u> $6x^2 + 2kx + 5 = \frac{17}{2}x + \frac{1}{2}$ scores no marks on its own but may score the first M mark if they substitute $x = -2$ into the lhs. If they rearrange this equation and then substitute $x = -2$, this scores no marks.		
(c)	$y = -16 + 4k - 10 + 6 = 4"k" - 20 = \frac{1}{2}$	M1: Substitutes $x = -2$ and their numerical k into $y = \dots$ A1: $y = \frac{1}{2}$	M1 A1
	Allow the marks for part (c) to be scored in part (b).		[2]
(d)	$y - \frac{1}{2} = \frac{17}{2}(x - 2) \Rightarrow -17x + 2y - 35 = 0$ or $y = \frac{17}{2}x + c \Rightarrow c = \dots \Rightarrow -17x + 2y - 35 = 0$ or $2y - 17x = 1 + 34 \Rightarrow -17x + 2y - 35 = 0$	M1: Correct attempt at linear equation with their 8.5 gradient (not the normal gradient) using $x = -2$ and their $\frac{1}{2}$ A1: cao (allow any integer multiple)	M1 A1
			[2]
			10 marks



Q5.

(a)	$\left(\frac{dy}{dx}\right) = \frac{3}{2}x^2 - \frac{27}{2}x^{\frac{1}{2}} - 8x^{-2}$	M1A1A1A1 (4)
(b)	$x = 4 \Rightarrow y = \frac{1}{2} \times 64 - 9 \times 2^3 + \frac{8}{4} + 30$ $= 32 - 72 + 2 + 30 = \underline{-8} \quad *$	M1 A1cso (2)
(c)	$x = 4 \Rightarrow y' = \frac{3}{2} \times 4^2 - \frac{27}{2} \times 2 - \frac{8}{16}$ $= 24 - 27 - \frac{1}{2} = \underline{-\frac{7}{2}}$ <p>Gradient of the normal = $-1 \div \frac{7}{2}$</p> <p>Equation of normal: $y - -8 = \frac{2}{7}(x - 4)$</p> $\underline{7y - 2x + 64 = 0}$	M1 A1 M1 M1A1ft A1 (6) 12
Question Number	Scheme	Marks
<u>Notes</u>		
(a)	1 st M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$ 1 st A1 for one correct term in x 2 nd A1 for 2 terms in x correct 3 rd A1 for all correct x terms. No 30 term and no $+c$.	
(b)	M1 for substituting $x = 4$ into $y =$ and attempting $4^{\frac{3}{2}}$ A1 note this is a printed answer	
(c)	1 st M1 Substitute $x = 4$ into y' (allow slips) A1 Obtains -3.5 or equivalent 2 nd M1 for correct use of the perpendicular gradient rule using their gradient. (May be slip doing the division) Their gradient must have come from y' 3 rd M1 for an attempt at equation of tangent or normal at P 2 nd A1ft for correct use of their changed gradient to find normal at P . Depends on 1 st , 2 nd and 3 rd Ms 3 rd A1 for any equivalent form with integer coefficients	

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Q6.

Question Number	Scheme	Marks
(a)	$C: y = 2x - 8\sqrt{x} + 5, \quad x \geq 0$ So, $y = 2x - 8x^{\frac{1}{2}} + 5$ $\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}} + \{0\} \quad (x > 0)$	M1 A1 A1 [3]
(b)	(When $x = \frac{1}{4}, y = 2(\frac{1}{4}) - 8\sqrt{(\frac{1}{4})} + 5$ so) $y = \frac{3}{2}$ (gradient = $\frac{dy}{dx} \Rightarrow 2 - \frac{4}{\sqrt{(\frac{1}{4})}} \{ = -6 \}$ Either: $y - \frac{3}{2} = -6(x - \frac{1}{4})$ or: $y = -6x + c$ and $\frac{3}{2} = -6(\frac{1}{4}) + c \Rightarrow c = 3$ So $y = -6x + 3$	B1 M1 dM1 A1 [4]
(c)	Tangent at Q is parallel to $2x - 3y + 18 = 0$ ($y = \frac{2}{3}x + 6 \Rightarrow$) Gradient = $\frac{2}{3}$. so tangent gradient is $\frac{2}{3}$ So, $2 - \frac{4}{\sqrt{x}} = \frac{2}{3}$ Sets their gradient function = their numerical gradient. $\Rightarrow \frac{4}{3} = \frac{4}{\sqrt{x}} \Rightarrow x = 9$ Ignore extra answer $x = -9$ When $x = 9, y = 2(9) - 8\sqrt{9} + 5 = -1$ Substitutes their found x into equation of curve. $y = -1$.	B1 M1 A1 dM1 A1 [5]
Notes		
(a)	M1: Evidence of differentiation, so $x^n \rightarrow x^{n-1}$ at least once so $x^1 \rightarrow 1$ or x^0 or $x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}}$ not just $5 \rightarrow 0$ A1: Any two of the three terms correct – do not need to see zero – the 5 disappearing is sufficient; need not be simplified. A1: $2 - 4x^{-\frac{1}{2}}$ Both terms correct, and simplified. Do not need to include domain $x > 0$	
(b)	B1: Obtaining $y = 3/2$ or fractional or decimal equivalent (no working need be seen) M1: An attempt to substitute $x = \frac{1}{4}$ into $\frac{dy}{dx}$ to establish gradient. This may be implied by -6 or $m = -6$ but not $y = -6$. Can earn this M mark if they go on to use $m = \frac{1}{6}$ or use their numerical value of $\frac{dy}{dx}$. dM1: This depends on previous method mark. Complete method for obtaining the equation of the tangent, using their tangent gradient and their value for y_1 (obtained from $x = \frac{1}{4}$, allow slip) i.e. $y - y_1 = m_T(x - \frac{1}{4})$ with their tangent gradient and their y_1 or uses $y = mx + c$ with $(\frac{1}{4}, \text{their } y_1)$ and their tangent gradient.	
(c)	A1: $y = -6x + 3$ or $y = 3 - 6x$ or $a = -6$ and $b = 3$ B1: For the value $2/3$ not $2/3 x$ not $-3/2$ M1: Sets their gradient function $dy/dx =$ their numerical gradient A1: Obtains $x = 9$ dM1: Substitutes their x (from gradient equation) into original equation of curve C i.e. original expression $y =$ A1: $(9, -1)$ or $x = 9, y = -1$, or just $y = -1$	
Special Cases: In (b) Finds normal could get B1 M1 M0 A0 i.e. max of 2/4 In (c) Uses perpendicular instead of parallel then award B0 M1 A0 M1 A0 i.e. max 2/5 – see over		

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Q7.

Question Number	Scheme		Marks
(a)	$f'(4) = 30 + \frac{6-5 \times 4^2}{\sqrt{4}}$	Attempts to substitute $x = 4$ into $f'(x) = 30 + \frac{6-5x^2}{\sqrt{x}}$ or their algebraically manipulated $f'(x)$	M1
	$f'(4) = -7$	Gradient = -7	A1
	$y - (-8) = "-7" \times (x - 4)$ or $y = "-7" \times x + c \Rightarrow -8 = "-7" \times 4 + c$ $\Rightarrow c = \dots$	Attempts an equation of a tangent using their numeric $f'(4)$ which has come from substituting $x = 4$ into the given $f'(x)$ or their algebraically manipulated $f'(x)$ and $(4, -8)$ with the 4 and -8 correctly placed. If using $y = mx + c$, must reach as far as $c = \dots$	M1
	$y = -7x + 20$	Cao. Allow $y = 20 - 7x$ and allow the "y =" to become "detached" but it must be present at some stage. E.g. $y = \dots$ $= -7x + 20$	A1
			(4)
(b)	Allow the marks in (b) to score in (a) i.e. mark (a) and (b) together		
	$\Rightarrow f(x) = 30x + 6 \frac{x^{\frac{1}{2}}}{0.5} - 5 \frac{x^{\frac{5}{2}}}{2.5} (+c)$	M1: $30 \rightarrow 30x$ or $\frac{6}{\sqrt{x}} \rightarrow \alpha x^{\frac{1}{2}}$ or $-\frac{5x^2}{\sqrt{x}} \rightarrow \beta x^{\frac{5}{2}}$ (these cases only)	M1A1A1
		A1: Any 2 correct terms which can be simplified or un-simplified. This includes the powers - so allow $-\frac{1}{2} + 1$ for $\frac{1}{2}$ and allow $\frac{3}{2} + 1$ for $\frac{5}{2}$ (With or without $+c$)	
		A1: All 3 terms correct which can be simplified or un-simplified. (With or without $+c$)	
Ignore any spurious integral signs			
$x = 4, f(x) = -8 \Rightarrow -8 = 120 + 24 - 64 + c \Rightarrow c = \dots$	Substitutes $x = 4, f(x) = -8$ into their $f(x)$ (not $f'(x)$) i.e. a changed $f'(x)$ containing $+c$ and rearranges to obtain a value or numerical expression for c .	M1	
$\Rightarrow f(x) = 30x + 12x^{\frac{1}{2}} - 2x^{\frac{5}{2}} - 88$	Cao and cso (Allow \sqrt{x} for $x^{\frac{1}{2}}$ and e.g. $\sqrt{x^5}$ or $x^2\sqrt{x}$ for $x^{\frac{5}{2}}$). Isw here so as soon as you see the correct answer, award this mark. Note that the " $f(x) =$ " is not needed.	A1	
			(5)
			(9 marks)



Q8.

Question Number	Scheme	Marks
(a)	$\left(\frac{dy}{dx}\right) = -4 + 8x^{-2}$ (4 or $8x^{-2}$ for M1... sign can be wrong) $x = 2 \Rightarrow m = -4 + 2 = -2$ $y = 9 - 8 - \frac{8}{2} = -3$ Equation of tangent is: $y + 3 = -2(x - 2) \rightarrow y = 1 - 2x$ (*)	M1A1 M1 B1 M1 A1cso (6)
(b)	Gradient of normal = $\frac{1}{2}$ Equation is: $\frac{y+3}{x-2} = \frac{1}{2}$ or better equivalent, e.g. $y = \frac{1}{2}x - 4$	B1ft M1A1
(c)	(A:) $\frac{1}{2}$, (B:) 8 Area of triangle is: $\frac{1}{2}(x_B \pm x_A) \times y_P$ with values for all of x_B, x_A and y_P $\frac{1}{2}\left(8 - \frac{1}{2}\right) \times 3 = \frac{45}{4}$ or 11.25	(3) B1, B1 M1 A1 (4) [13]
(a)	1 st M1 for 4 or $8x^{-2}$ (ignore the signs). 1 st A1 for both terms correct (including signs). 2 nd M1 for substituting $x = 2$ into their $\frac{dy}{dx}$ (must be different from their y) B1 for $y_P = -3$, but not if clearly found from the given equation of the tangent. 3 rd M1 for attempt to find the equation of tangent at P , follow through their m and y_P . Apply general principles for straight line equations (see end of scheme). NO DIFFERENTIATION ATTEMPTED: Just assuming $m = -2$ at this stage is M0 2 nd A1cso for correct work leading to printed answer (allow equivalents with $2x, y$, and 1 terms... such as $2x + y - 1 = 0$).	
(b)	B1ft for correct use of the perpendicular gradient rule. Follow through their m , but if $m \neq -2$ there must be clear evidence that the m is thought to be the gradient of the tangent. M1 for an attempt to find normal at P using their changed gradient and their y_P . Apply general principles for straight line equations (see end of scheme).	
(c)	A1 for any correct form as specified above (correct answer only). 1 st B1 for $\frac{1}{2}$ and 2 nd B1 for 8. M1 for a full method for the area of triangle ABP . Follow through their x_A, x_B and their y_P , but the mark is to be awarded 'generously', condoning sign errors.. The final answer must be positive for A1, with negatives in the working condoned. Determinant: Area = $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 2 & -3 & 1 \\ 0.5 & 0 & 1 \\ 8 & 0 & 1 \end{vmatrix} = \dots$ (Attempt to multiply out required for M1) Alternative: $AP = \sqrt{(2 - 0.5)^2 + (-3)^2}$, $BP = \sqrt{(2 - 8)^2 + (-3)^2}$, Area = $\frac{1}{2} AP \times BP = \dots$ M1 Intersections with y -axis instead of x -axis: Only the M mark is available B0 B0 M1 A0.	

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Q9.

Question Number	Scheme	Marks		
(a)	<p>Substitutes $x = 2$ into $y = 20 - 4 \times 2 - \frac{18}{2}$ and gets 3</p> $\frac{dy}{dx} = -4 + \frac{18}{x^2}$ <p>Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)</p> <table border="0" style="width: 100%;"> <tr> <td style="width: 50%; vertical-align: top;"> <p>Method 1</p> <p>States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3)</p> <p>to deduce that $y = -2x + 7$ *</p> </td> <td style="width: 50%; vertical-align: top;"> <p>Method 2</p> <p>Or: Check that (2, 3) lies on the line $y = -2x + 7$</p> <p>Deduce equation of normal as it has the same gradient and passes through a common point</p> </td> </tr> </table>	<p>Method 1</p> <p>States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3)</p> <p>to deduce that $y = -2x + 7$ *</p>	<p>Method 2</p> <p>Or: Check that (2, 3) lies on the line $y = -2x + 7$</p> <p>Deduce equation of normal as it has the same gradient and passes through a common point</p>	<p>B1</p> <p>M1 A1</p> <p>dM1</p> <p>dM1</p> <p>A1*</p>
<p>Method 1</p> <p>States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3)</p> <p>to deduce that $y = -2x + 7$ *</p>	<p>Method 2</p> <p>Or: Check that (2, 3) lies on the line $y = -2x + 7$</p> <p>Deduce equation of normal as it has the same gradient and passes through a common point</p>			
(b)	<p>Put $20 - 4x - \frac{18}{x} = -2x + 7$ and simplify to give $2x^2 - 13x + 18 = 0$</p> <p>Or put $y = 20 - 4\left(\frac{7-y}{2}\right) - \frac{18}{\left(\frac{7-y}{2}\right)}$ to give $y^2 - y - 6 = 0$</p> <p>$(2x - 9)(x - 2) = 0$ so $x =$ or $(y - 3)(y + 2) = 0$ so $y =$</p> $x = \frac{9}{2}, y = -2$	<p>M1 A1</p> <p>dM1</p> <p>A1, A1</p>		

(6)

(5)

(11 marks)

PTO for notes on this question.



Q10.

Question Number	Scheme		Marks
(a)	$\left(-\frac{3}{4}, 0\right)$. Accept $x = -\frac{3}{4}$		B1
			(1)
(b)	$y = 4$	B1: One correct asymptote	B1B1
	$x = 0$ or 'y-axis'	B1: Both correct asymptotes and no extra ones.	
	Special case $x \neq 0$ and $y \neq 4$ scores B1B0		
			(2)
(c)	$\frac{dy}{dx} = -3x^{-2}$	$\frac{dy}{dx} = kx^{-2}$ (Allow $\frac{dy}{dx} = kx^{-2} + 4$)	M1
	At $x = -3$, gradient of curve = $-\frac{1}{3}$	Cao (may be un-simplified but must be a fraction with no powers) e.g. $-3(-3)^{-2}$ scores A0 unless evaluated as e.g. $\frac{-3}{9}$ or is implied by their normal gradient.	A1
	Gradient of normal = $-1/m$	Correct perpendicular gradient rule applied to a numerical gradient that must have come from substituting $x = -3$ into their derivative. Dependent on the previous M1.	dM1
	Normal at P is $(y-3) = 3(x+3)$	M1: Correct straight line method using $(-3, 3)$ and a "changed" gradient. A wrong equation with no formula quoted is M0. Also dependent on the first M1.	dM1A1
		A1: Any correct equation	
			(5)
(d)	$(-4, 0)$ and $(0, 12)$.	Both correct (May be seen on a sketch)	B1
	So AB has length $\sqrt{160}$ or AB^2 has length 160	M1: Correct use of Pythagoras for their A and B one of which lies on the x -axis and the other on the y -axis, obtained from their equation in (c). A correct method for AB^2 or AB .	M1 A1eso
		A1: $\sqrt{160}$ or better e.g. $4\sqrt{10}$ with no errors seen	
			(3)
			[11]

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Q11.

Question Number	Scheme		Marks
(a)	$\frac{dy}{dx} = \frac{1}{2} - \frac{27}{x^3}$	M1: $\frac{1}{2}$ or $-\frac{27}{x^2}$	M1A1
		A1: $\frac{dy}{dx} = \frac{1}{2} - \frac{27}{x^2}$ oe e.g. $\frac{1}{2}x^0 - 27x^{-2}$	
	$x = 3 \Rightarrow \frac{dy}{dx} = \frac{1}{2} - \frac{27}{9} = \left(-\frac{5}{2}\right)$	Substitutes $x = 3$ into their $\frac{dy}{dx}$ to obtain a numerical gradient	M1
	$m_T = -\frac{5}{2} \Rightarrow m_N = -1 \div -\frac{5}{2}$ $\Rightarrow y - \left(-\frac{3}{2}\right) = \frac{2}{5}(x - 3)$	The correct method to find the equation of a normal. Uses $-\frac{1}{m_T}$ with $\left(3, -\frac{3}{2}\right)$ where m_T has come from calculus. If using $y = mx + c$ must reach as far as $c = \dots$	M1
	$10y = 4x - 27^*$	Cso (correct equation must be seen in (a))	A1*
			(5)
(b)	$\frac{1}{2}x + \frac{27}{x} - 12 = \frac{4x - 27}{10}$ or $y = \frac{10y + 27}{8} + \frac{108}{10y + 27} - 12$	Equate equations to produce an equation just in x or just in y . Do not allow e.g. $\frac{1}{2}x^2 + 27 - 12x = \frac{4x - 27}{10}$ Unless $\frac{1}{2}x + \frac{27}{x} - 12 = \frac{4x - 27}{10}$ was seen previously. Allow sign slips only.	M1
	$x^2 - 93x + 270 = 0$ or $20y^2 - 636y - 999 = 0$	Correct 3 term quadratic equation (or any multiple of). Allow terms on both sides e.g. $x^2 - 93x = -270$ (The “= 0” may be implied by their attempt to solve)	A1
	$(x - 90)(x - 3) = 0 \Rightarrow x = \dots$ or $x = \frac{93 \pm \sqrt{93^2 - 4 \times 270}}{2}$ or $(10y - 333)(2y + 3) = 0 \Rightarrow y = \dots$ or $y = \frac{636 \pm \sqrt{636^2 - 4 \times 20 \times (-999)}}{2 \times 20}$	Attempt to solve a 3TQ (see general guidance) leading to at least one for x or y . Dependent on the first method mark.	dM1
	$x = 90$ or $y = 33.3$ oe	Cso. The x must be 90 and the y an equivalent number such as e.g. $\frac{333}{10}$	A1
	$x = 90$ and $y = 33.3$ oe	Cso. The x must be 90 and the y an equivalent number such as e.g. $\frac{333}{10}$	A1
			(5)
			(10 marks)

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Q12.

Question Number	Scheme	Notes	Marks
(a)	$y = \frac{12}{x} + 5 \Rightarrow \frac{dy}{dx} = -\frac{12}{x^2}$	$\frac{12}{x} \rightarrow \frac{k}{x^2}$ (or kx^{-2})	M1
	At $x = -2$ $\frac{dy}{dx} = -\frac{12}{4}$ or -3	Correct value (may be implied by later work)	A1
	Gradient of normal is $-1 \div -\frac{12}{4} \left(= \frac{1}{3} \right)$	Correct application of the perpendicular gradient rule. May be implied by use of $-\frac{1}{12}$ as the normal gradient for those candidates who think the gradient is 12.	M1
	$y + 1 = \frac{1}{3}(x + 2)$ or $y = \frac{1}{3}x + c$ and $-1 = \frac{1}{3}(-2) + c \Rightarrow c = \dots$	A correct straight line method using their changed gradient and the point $(-2, -1)$. This must follow use of calculus to find the gradient.	M1
	$3y - x + 1 = 0$	Correct equation in the required form. (Allow any integer multiple)	A1
			[5]
(b)	Gradient of given line is $\frac{3}{4}$	May be implied by use of $-\frac{4}{3}$	B1
	$\frac{x^2}{12} = \frac{3}{4} \Rightarrow x = \dots$	Sets up a correct equation using what they think is the gradient of the given line and attempts to solve.	M1
	$x = \pm 3$	Both correct values required	A1
	$x = \dots \Rightarrow \frac{12}{x} + 5 = \dots$	Uses at least one x to find a value for y using $y = \frac{12}{x} + 5$. Dependent on the first method mark.	dM1
	$(3, 9)$ and $(-3, 1)$ or e.g. $x = 3, y = 9$ $x = -3, y = 1$	Correct coordinates correctly paired	A1
			10 marks