Question 1

Worked Solution

Express $\dfrac{5}{2-\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are integers.

Multiply numerator and denominator by the conjugate $(2+\sqrt{3})$:

$$\frac{5}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{5(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$$

Denominator (difference of two squares):

$$(2-\sqrt{3})(2+\sqrt{3}) = 4-3 = 1$$

So:

$$\frac{5(2+\sqrt{3})}{1} = 10+5\sqrt{3}$$

$$\frac{5}{2-\sqrt{3}} = 10+5\sqrt{3} \quad (a=10,\ b=5)$$

Question 2

Worked Solution

Express $\sqrt{45} + \dfrac{20}{\sqrt{5}}$ in the form $k\sqrt{5}$, where k is an integer.

Simplify $\sqrt{45}$:

$$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

Rationalise $\dfrac{20}{\sqrt{5}}$:

$$\frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$$

Add:

$$3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$$

$$\sqrt{45} + \frac{20}{\sqrt{5}} = 7\sqrt{5} \quad (k = 7)$$

Question 3

Worked Solution

Express $\dfrac{8+\sqrt{7}}{2+\sqrt{7}}$ in the form $a+b\sqrt{7}$, where a and b are integers.

Multiply numerator and denominator by the conjugate $(2-\sqrt{7})$:

$$\frac{8+\sqrt{7}}{2+\sqrt{7}} \times \frac{2-\sqrt{7}}{2-\sqrt{7}} = \frac{(8+\sqrt{7})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})}$$

Denominator:

$$(2+\sqrt{7})(2-\sqrt{7}) = 4-7 = -3$$

Numerator:

$$(8+\sqrt{7})(2-\sqrt{7}) = 16-8\sqrt{7}+2\sqrt{7}-7 = 9-6\sqrt{7}$$

So:

$$\frac{9-6\sqrt{7}}{-3} = -3+2\sqrt{7}$$

$$\frac{8+\sqrt{7}}{2+\sqrt{7}} = -3+2\sqrt{7} \quad (a=-3,\ b=2)$$

Question 4

Worked Solution

Part (i) — Express $\dfrac{12}{3+\sqrt{5}}$ in the form $a - b\sqrt{5}$:

Multiply by the conjugate $(3-\sqrt{5})$:

$$\frac{12}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{12(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

Denominator:

$$(3+\sqrt{5})(3-\sqrt{5}) = 9 - 5 = 4$$

So:

$$\frac{12(3-\sqrt{5})}{4} = 3(3-\sqrt{5}) = 9 - 3\sqrt{5}$$

$$\frac{12}{3+\sqrt{5}} = 9 - 3\sqrt{5} \quad (a = 9,\ b = 3)$$

Part (ii) — Express $\sqrt{18} - \sqrt{2}$ in simplified surd form:

$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$$
$$\sqrt{18} - \sqrt{2} = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$$

$$\sqrt{18} - \sqrt{2} = 2\sqrt{2}$$

Question 5

Worked Solution

Part (i) — Express $\sqrt{300} - \sqrt{48}$ in the form $k\sqrt{3}$:

Simplify each surd:

$$\sqrt{300} = \sqrt{100 \times 3} = 10\sqrt{3}$$
$$\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$$

Subtract:

$$10\sqrt{3} - 4\sqrt{3} = 6\sqrt{3}$$

$$\sqrt{300} - \sqrt{48} = 6\sqrt{3} \quad (k = 6)$$

Part (ii) — Express $\dfrac{15 + \sqrt{40}}{\sqrt{5}}$ in the form $a\sqrt{5} + b\sqrt{2}$:

Multiply numerator and denominator by $\sqrt{5}$:

$$\frac{15 + \sqrt{40}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}(15 + \sqrt{40})}{5}$$

Note $\sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$, so expand:

$$\frac{15\sqrt{5} + \sqrt{5} \cdot \sqrt{40}}{5} = \frac{15\sqrt{5} + \sqrt{200}}{5}$$

Now $\sqrt{200} = \sqrt{100 \times 2} = 10\sqrt{2}$:

$$\frac{15\sqrt{5} + 10\sqrt{2}}{5} = 3\sqrt{5} + 2\sqrt{2}$$

$$\frac{15 + \sqrt{40}}{\sqrt{5}} = 3\sqrt{5} + 2\sqrt{2} \quad (a = 3,\ b = 2)$$

Question 6

Worked Solution

Express $\dfrac{15+\sqrt{3}}{3-\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b are integers.

Multiply by the conjugate $(3+\sqrt{3})$:

$$\frac{15+\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{(15+\sqrt{3})(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})}$$

Denominator:

$$(3-\sqrt{3})(3+\sqrt{3}) = 9-3 = 6$$

Numerator:

$$(15+\sqrt{3})(3+\sqrt{3}) = 45+15\sqrt{3}+3\sqrt{3}+3 = 48+18\sqrt{3}$$

So:

$$\frac{48+18\sqrt{3}}{6} = 8+3\sqrt{3}$$

$$\frac{15+\sqrt{3}}{3-\sqrt{3}} = 8+3\sqrt{3} \quad (a=8,\ b=3)$$

Question 7

Worked Solution

Express $\dfrac{3+\sqrt{20}}{3+\sqrt{5}}$ in the form $a+b\sqrt{5}$.

First simplify $\sqrt{20}$:

$$\sqrt{20} = \sqrt{4\times 5} = 2\sqrt{5}$$

So the expression becomes:

$$\frac{3+2\sqrt{5}}{3+\sqrt{5}}$$

Multiply by the conjugate $(3-\sqrt{5})$:

$$\frac{3+2\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{(3+2\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

Denominator:

$$(3+\sqrt{5})(3-\sqrt{5}) = 9-5 = 4$$

Numerator:

$$(3+2\sqrt{5})(3-\sqrt{5}) = 9-3\sqrt{5}+6\sqrt{5}-2(5) = 9+3\sqrt{5}-10 = -1+3\sqrt{5}$$

So:

$$\frac{-1+3\sqrt{5}}{4} = -\frac{1}{4}+\frac{3}{4}\sqrt{5}$$

$$\frac{3+\sqrt{20}}{3+\sqrt{5}} = -\frac{1}{4}+\frac{3}{4}\sqrt{5} \quad \left(a=-\tfrac{1}{4},\ b=\tfrac{3}{4}\right)$$