Question 1

Worked Solution

Part (a) — Simplify $\sqrt{32}+\sqrt{18}$ in the form $a\sqrt{2}$:

$$\sqrt{32}=\sqrt{16\times 2}=4\sqrt{2} \qquad \sqrt{18}=\sqrt{9\times 2}=3\sqrt{2}$$
$$\sqrt{32}+\sqrt{18}=4\sqrt{2}+3\sqrt{2}=7\sqrt{2}$$

$$\boxed{\sqrt{32}+\sqrt{18}=7\sqrt{2} \quad (a=7)}$$

Part (b) — Simplify $\dfrac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}$ in the form $b\sqrt{2}+c$:

Using part (a), the numerator is $7\sqrt{2}$. Multiply top and bottom by the conjugate $(3-\sqrt{2})$:

$$\frac{7\sqrt{2}}{3+\sqrt{2}}\times\frac{3-\sqrt{2}}{3-\sqrt{2}}=\frac{7\sqrt{2}(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$$

Denominator:

$$(3+\sqrt{2})(3-\sqrt{2})=9-2=7$$

Numerator:

$$7\sqrt{2}(3-\sqrt{2})=21\sqrt{2}-7\times 2=21\sqrt{2}-14$$

So:

$$\frac{21\sqrt{2}-14}{7}=3\sqrt{2}-2$$

$$\boxed{\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}=3\sqrt{2}-2 \quad (b=3,\ c=-2)}$$

Question 2

Worked Solution

Simplify $\dfrac{5-2\sqrt{3}}{\sqrt{3}-1}$ in the form $p+q\sqrt{3}$.

Multiply top and bottom by the conjugate $(\sqrt{3}+1)$:

$$\frac{5-2\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{(5-2\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

<u>Denominator:</u>

$$(\sqrt{3}-1)(\sqrt{3}+1)=3-1=2$$

<u>Numerator:</u>

$$(5-2\sqrt{3})(\sqrt{3}+1)=5\sqrt{3}+5-2\sqrt{3}\cdot\sqrt{3}-2\sqrt{3}$$
$$=5\sqrt{3}+5-6-2\sqrt{3}=3\sqrt{3}-1$$

<u>So:</u>

$$\frac{3\sqrt{3}-1}{2}=-\frac{1}{2}+\frac{3}{2}\sqrt{3}$$

$$\frac{5-2\sqrt{3}}{\sqrt{3}-1}=-\frac{1}{2}+\frac{3}{2}\sqrt{3}\quad\left(p=-\tfrac{1}{2},\ q=\tfrac{3}{2}\right)$$

Question 3

Worked Solution

Show that $\dfrac{2}{\sqrt{12}-\sqrt{8}}$ can be written in the form $\sqrt{a}+\sqrt{b}$.

Multiply top and bottom by the conjugate $(\sqrt{12}+\sqrt{8})$:

$$\frac{2}{\sqrt{12}-\sqrt{8}} \times \frac{\sqrt{12}+\sqrt{8}}{\sqrt{12}+\sqrt{8}} = \frac{2(\sqrt{12}+\sqrt{8})}{(\sqrt{12})^2-(\sqrt{8})^2}$$

Denominator:

$$12-8=4$$

Numerator — simplify the surds:

$$\sqrt{12}=\sqrt{4\times 3}=2\sqrt{3} \qquad \sqrt{8}=\sqrt{4\times 2}=2\sqrt{2}$$

$$2(\sqrt{12}+\sqrt{8})=2(2\sqrt{3}+2\sqrt{2})=4\sqrt{3}+4\sqrt{2}$$

So:

$$\frac{4\sqrt{3}+4\sqrt{2}}{4}=\sqrt{3}+\sqrt{2}$$

$$\frac{2}{\sqrt{12}-\sqrt{8}}=\sqrt{3}+\sqrt{2} \quad (a=3,\ b=2)\square$$

Question 4

Worked Solution

Expand and simplify $(\sqrt{7}+2)(\sqrt{7}-2)$.

This is a difference of two squares:

$$(\sqrt{7}+2)(\sqrt{7}-2) = (\sqrt{7})^2 - 2^2 = 7 - 4 = 3$$

$$(\sqrt{7}+2)(\sqrt{7}-2) = 3$$

Question 5

Worked Solution

Write $\sqrt{75} - \sqrt{27}$ in the form $k\sqrt{x}$.

Simplify each surd:

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$
$$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$$

Subtract:

$$5\sqrt{3} - 3\sqrt{3} = 2\sqrt{3}$$

$$\sqrt{75} - \sqrt{27} = 2\sqrt{3} \quad (k = 2,\ x = 3)$$

Question 6

Worked Solution

Part (a) — Expand and simplify $(7+\sqrt{5})(3-\sqrt{5})$:

$$(7+\sqrt{5})(3-\sqrt{5}) = 21 - 7\sqrt{5} + 3\sqrt{5} - (\sqrt{5})^2$$
$$= 21 - 4\sqrt{5} - 5 = 16 - 4\sqrt{5}$$

$$(7+\sqrt{5})(3-\sqrt{5}) = 16 - 4\sqrt{5}$$

Part (b) — Express $\dfrac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$:

Multiply by the conjugate $(3-\sqrt{5})$:

$$\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{(7+\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

Denominator:

$$(3+\sqrt{5})(3-\sqrt{5}) = 9 - 5 = 4$$

Numerator (from part (a)):

$$(7+\sqrt{5})(3-\sqrt{5}) = 16 - 4\sqrt{5}$$

So:

$$\frac{16-4\sqrt{5}}{4} = 4 - \sqrt{5}$$

$$\frac{7+\sqrt{5}}{3+\sqrt{5}} = 4 - \sqrt{5} \quad (a = 4,\ b = -1)$$

Question 7

Worked Solution

Part (a) — Simplify $(3\sqrt{7})^2$:

$$(3\sqrt{7})^2 = 3^2 \times (\sqrt{7})^2 = 9 \times 7 = 63$$

$$(3\sqrt{7})^2 = 63$$

Part (b) — Simplify $(8+\sqrt{5})(2-\sqrt{5})$:

$$(8+\sqrt{5})(2-\sqrt{5}) = 16 - 8\sqrt{5} + 2\sqrt{5} - (\sqrt{5})^2$$
$$= 16 - 6\sqrt{5} - 5 = 11 - 6\sqrt{5}$$

$$(8+\sqrt{5})(2-\sqrt{5}) = 11 - 6\sqrt{5}$$

Question 8

Worked Solution

Part (i) — Express $(5-\sqrt{8})(1+\sqrt{2})$ in the form $a+b\sqrt{2}$:

First note $\sqrt{8}=2\sqrt{2}$, so:

$$(5-2\sqrt{2})(1+\sqrt{2}) = 5+5\sqrt{2}-2\sqrt{2}-2(\sqrt{2})^2$$

$$= 5+3\sqrt{2}-4 = 1+3\sqrt{2}$$

$$(5-\sqrt{8})(1+\sqrt{2}) = 1+3\sqrt{2} \quad (a=1,\ b=3)$$

Part (ii) — Express $\sqrt{80}+\dfrac{30}{\sqrt{5}}$ in the form $c\sqrt{5}$:

Simplify $\sqrt{80}$:

$$\sqrt{80}=\sqrt{16\times 5}=4\sqrt{5}$$

Rationalise $\dfrac{30}{\sqrt{5}}$:

$$\frac{30}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{30\sqrt{5}}{5}=6\sqrt{5}$$

Add:

$$4\sqrt{5}+6\sqrt{5}=10\sqrt{5}$$

$$\sqrt{80}+\frac{30}{\sqrt{5}}=10\sqrt{5} \quad (c=10)$$

Question 9

Worked Solution

Simplify $\dfrac{7+\sqrt{5}}{\sqrt{5}-1}$ in the form $a+b\sqrt{5}$.

Multiply by the conjugate $(\sqrt{5}+1)$:

$$\frac{7+\sqrt{5}}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{(7+\sqrt{5})(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)}$$

Denominator:

$$(\sqrt{5}-1)(\sqrt{5}+1) = 5-1 = 4$$

Numerator:

$$(7+\sqrt{5})(\sqrt{5}+1) = 7\sqrt{5}+7+5+\sqrt{5} = 8\sqrt{5}+12$$

So:

$$\frac{12+8\sqrt{5}}{4} = 3+2\sqrt{5}$$

$$\frac{7+\sqrt{5}}{\sqrt{5}-1} = 3+2\sqrt{5} \quad (a=3,\ b=2)$$

Question 10

Worked Solution

Solve $10 + x\sqrt{8} = \dfrac{6x}{\sqrt{2}}$, giving the answer in the form $a\sqrt{b}$.

Simplify both sides:

$$\sqrt{8} = 2\sqrt{2} \qquad \frac{6x}{\sqrt{2}} = \frac{6x}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6x\sqrt{2}}{2} = 3x\sqrt{2}$$

The equation becomes:

$$10 + 2x\sqrt{2} = 3x\sqrt{2}$$

Collect x terms:

$$10 = 3x\sqrt{2} - 2x\sqrt{2} = x\sqrt{2}$$

Solve for x:

$$x = \frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

$$x = 5\sqrt{2} \quad (a = 5,\ b = 2)$$

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