Surds Exam Questions Sheet 2

Mark Scheme

Q1.

Question	Scheme	Marks
(a)	$\sqrt{32} = 4\sqrt{2}$ or $\sqrt{18} = 3\sqrt{2}$	B1
	$\left(\sqrt{32}+\sqrt{18}=\right)$ $\underline{7\sqrt{2}}$	B1 (2)
(b)	$\times\dfrac{3-\sqrt{2}}{3-\sqrt{2}}$ or $\times\dfrac{-3+\sqrt{2}}{-3+\sqrt{2}}$ seen	M1
	$\left[\dfrac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}\times\dfrac{3-\sqrt{2}}{3-\sqrt{2}}=\right]\ \dfrac{a\sqrt{2}\left(3-\sqrt{2}\right)}{[9-2]} \rightarrow \dfrac{3a\sqrt{2}-2a}{[9-2]}$ (or better)	dM1
	$= \underline{3\sqrt{2}, -2}$	A1, A1 (4)
ALT	$(b\sqrt{2}+c)(3+\sqrt{2}) = 7\sqrt{2}$ leading to: $3b+c=7$, $3c+2b=0$	M1
	e.g. $3(7-3b)+2b=0$ (o.e.)	dM1
		6 marks
	Notes	
(a)	1st B1 for either surd simplified 2nd B1 for $7\sqrt{2}$ or accept $a=7$. Answer only scores B1B1 NB Common error is $\sqrt{32}+\sqrt{18}=\sqrt{50}=5\sqrt{2}$ this scores B0B0 but can use their "5" in (b) to get M1M1	
(b)	1st M1 for an attempt to multiply by $\dfrac{3-\sqrt{2}}{3-\sqrt{2}}$ (o.e.) Allow poor use of brackets 2nd dM1 for using $a\sqrt{2}$ to correctly obtain a numerator of the form $p+q\sqrt{2}$ where p and q are non-zero integers. Allow arithmetic slips e.g. $21\sqrt{2}-28$ or $3\sqrt{2}\times\sqrt{2}=3$ Follow through their $a=7$ or a new value found in (b). Ignore denominator. Allow use of letter a. Dependent on 1st M1 So $3\sqrt{32}-\sqrt{64}+3\sqrt{8}-\sqrt{36}$ is M0 until they reduce $p+q\sqrt{2}$ 1st A1 for $3\sqrt{2}$ or accept $b=3$ from correct working 2nd A1 for -2 or accept $c=-2$ from correct working	
ALT	**Simultaneous Equations** 1st M1 for $(b\sqrt{2}+c)(3+\sqrt{2})=7\sqrt{2}$ and forming 2 simultaneous equations. Ft their $a=7$ 2nd dM1 for solving their simultaneous equations: reducing to a linear equation in one variable	

Q2.

Question Number	Scheme	Marks
	$\frac{5-2\sqrt{3}}{\sqrt{3}-1}\times\frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$	M1
	$=\frac{\cdots}{2}$ denominator of 2	A1
	Numerator = $5\sqrt{3}+5-2\sqrt{3}\sqrt{3}-2\sqrt{3}$	M1
	So $\frac{5-2\sqrt{3}}{\sqrt{3}-1}=-\frac{1}{2}+\frac{3}{2}\sqrt{3}$	A1
		4
	Alternative: $(p+q\sqrt{3})(\sqrt{3}-1)=5-2\sqrt{3}$, and form simultaneous equations in p and q	M1
	$-p+3q=5$ and $p-q=-2$	A1
	Solve simultaneous equations to give $p=-\frac{1}{2}$ and $q=\frac{3}{2}$.	M1 A1
	<u>Notes</u>	
	1st M1 for multiplying numerator and denominator by same correct expression 1st A1 for a correct denominator as a single number (NB depends on M mark) 2nd M1 for an attempt to multiply the numerator by $(\sqrt{3}\pm1)$ and get 4 terms with at least 2 correct. 2nd A1 for the answer as written or $p=-\frac{1}{2}$ and $q=\frac{3}{2}$. Allow -0.5 and 1.5. (Apply isw if correct answer seen, then slip writing $p=$, $q=$)	
	Answer only (very unlikely) is full marks if correct – no part marks	

Q3.

Question Number	Scheme		Marks
	$\left\{\frac{2}{\sqrt{12}-\sqrt{8}}\right\} = \frac{2}{(\sqrt{12}-\sqrt{8})} \times \frac{(\sqrt{12}+\sqrt{8})}{(\sqrt{12}+\sqrt{8})}$	Writing this is sufficient for M1.	M1
	$= \frac{\{2(\sqrt{12}+\sqrt{8})\}}{12-8}$	For $12-8$. This mark can be implied.	A1
	$= \frac{2(2\sqrt{3}+2\sqrt{2})}{12-8}$		B1 B1
	$= \sqrt{3}+\sqrt{2}$		A1 cso
			5

Notes

M1: for a correct method to rationalise the denominator.

1st A1: $(\sqrt{12}-\sqrt{8})(\sqrt{12}+\sqrt{8}) \to 12-8$ or $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) \to 3-2$

1st B1: for $\sqrt{12} = 2\sqrt{3}$ or $\sqrt{48} = 4\sqrt{3}$ seen or implied in candidate's working.

2nd B1: for $\sqrt{8} = 2\sqrt{2}$ or $\sqrt{32} = 4\sqrt{2}$ seen or implied in candidate's working.

2nd A1: for $\sqrt{3}+\sqrt{2}$. Note: $\frac{\sqrt{3}+\sqrt{2}}{1}$ as a final answer is A0.

Note: The first accuracy mark is dependent on the first method mark being awarded.

Note: $\frac{1}{2}\sqrt{12}+\frac{1}{2}\sqrt{8} = \sqrt{3}+\sqrt{2}$ with no intermediate working implies the B1B1 marks.

Note: $\sqrt{12} = \sqrt{4}\sqrt{3}$ or $\sqrt{8} = \sqrt{4}\sqrt{2}$ are not sufficient for the B1 marks.

Note: A candidate who writes down (by misread) $\sqrt{18}$ for $\sqrt{8}$ can potentially obtain M1A0B1B1A0, where the 2nd B1 will be awarded for $\sqrt{18} = 3\sqrt{2}$ or $\sqrt{72} = 6\sqrt{2}$

Note: The final accuracy mark is for a correct solution only.

Alternative 1 solution

$\left\{\frac{2}{\sqrt{12}-\sqrt{8}}\right\} = \frac{2}{(2\sqrt{3}-2\sqrt{2})}$ B1 B1

$= \frac{1}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$ M1

$= \frac{\{(\sqrt{3}+\sqrt{2})\}}{3-2}$ A1 for $3-2$

$= \sqrt{3}+\sqrt{2}$ A1

Please record the marks in the relevant places on the mark grid.

Alternative 2 solution

$\left\{\frac{2}{\sqrt{12}-\sqrt{8}}\right\} = \frac{2}{(2\sqrt{3}-2\sqrt{2})} = \frac{1}{(\sqrt{3}-\sqrt{2})} = \sqrt{3}+\sqrt{2}$, or $\frac{2}{(2\sqrt{3}-2\sqrt{2})} = \sqrt{3}+\sqrt{2}$

with no incorrect working seen is awarded M1A1B1B1A1.

Q4.

Question Number	Scheme	Marks
	$\sqrt{7}^2+2\sqrt{7}-2\sqrt{7}-2^2$, or $7-4$ or an exact equivalent such as $\sqrt{49}-2^2$	M1
	$=3$	A1
		[2]
	M1 for an expanded expression. At worst, there can be one wrong term and one wrong sign, or two wrong signs. e.g. $7+2\sqrt{7}-2\sqrt{7}-2$ is M1 (one wrong term -2) $7+2\sqrt{7}+2\sqrt{7}+4$ is M1 (two wrong signs $+2\sqrt{7}$ and $+4$) $7+2\sqrt{7}+2\sqrt{7}+2$ is M1 (one wrong term $+2$, one wrong sign $+2\sqrt{7}$) $\sqrt{7}+2\sqrt{7}-2\sqrt{7}+4$ is M1 (one wrong term $\sqrt{7}$, one wrong sign $+4$) $\sqrt{7}+2\sqrt{7}-2\sqrt{7}-2$ is M0 (two wrong terms $\sqrt{7}$ and -2) $7+\sqrt{14}-\sqrt{14}-4$ is M0 (two wrong terms $\sqrt{14}$ and $-\sqrt{14}$) If only 2 terms are given, they must be correct, i.e. $(7-4)$ or an equivalent unsimplified version to score M1. The terms can be seen separately for the M1. Correct answer with no working scores both marks.	

Q5.

Question Number	Scheme	Marks
	$\left(\sqrt{75}-\sqrt{27}\right)=5\sqrt{3}-3\sqrt{3}$	M1
	$=2\sqrt{3}$	A1
		2
	Notes	
	M1 for $5\sqrt{3}$ from $\sqrt{75}$ or $3\sqrt{3}$ from $\sqrt{27}$ seen anywhere A1 for $2\sqrt{3}$; allow $\sqrt{12}$ or (or $k=2, x=3$; allow $k=1, x=12$) Some Common errors $\sqrt{75}-\sqrt{27}=\sqrt{48}$ leading to $4\sqrt{3}$ is M0A0 $25\sqrt{3}-9\sqrt{3}=16\sqrt{3}$ is M0A0	

Q6.

Question number	Scheme	Marks
	(a) $(7+\sqrt{5})(3-\sqrt{5}) = 21-5+3\sqrt{5}-7\sqrt{5}$ Expand to get 3 or 4 terms	M1
	$= 16, \ -4\sqrt{5}$ (1st A for 16, 2nd A for $-4\sqrt{5}$)	A1, A1
	(i.s.w. if necessary, e.g. $16-4\sqrt{5} \rightarrow 4-\sqrt{5}$)	(3)
	(b) $\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ (This is sufficient for the M mark)	M1
	Correct denominator without surds, i.e. $9-5$ or 4	A1
	$4-\sqrt{5}$ or $4-1\sqrt{5}$	A1 (3) **[6]**
	(a) M1: Allowed for an attempt giving 3 or 4 terms, with at least 2 correct (even if unsimplified). e.g. $21-\sqrt{5}^2+\sqrt{15}$ scores M1. Answer only: $16-4\sqrt{5}$ scores full marks One term correct scores the M mark by implication, e.g. $26-4\sqrt{5}$ scores M1 A0 A1 (b) Answer only: $4-\sqrt{5}$ scores full marks One term correct scores the M mark by implication, e.g. $4+\sqrt{5}$ scores M1 A0 A0 $16-\sqrt{5}$ scores M1 A0 A0 Ignore subsequent working, e.g. $4-\sqrt{5}$ so $a=4, \ b=1$ Note that, as always, A marks are dependent upon the preceding M mark, so that, for example, $\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{\dots\dots}{4}$ is M0 A0. <u>Alternative</u> $(a+b\sqrt{5})(3+\sqrt{5}) = 7+\sqrt{5}$, then form simultaneous equations in a and b. M1 Correct equations: $3a+5b=7$ and $3b+a=1$ A1 $a=4$ and $b=-1$ A1	

Q7.

Question Number	Scheme	Marks
Q (a)	$(3\sqrt{7})^2 = 63$	B1 (1)
(b)	$(8+\sqrt{5})(2-\sqrt{5}) = 16-5+2\sqrt{5}-8\sqrt{5}$	M1
	$= 11,\ -6\sqrt{5}$	A1, A1 (3) [4]
(a)	B1 for 63 only	
(b)	M1 for an attempt to expand <u>their</u> brackets with ≥ 3 terms correct. They may collect the $\sqrt{5}$ terms to get $16-5-6\sqrt{5}$ Allow $-\sqrt{5}\times\sqrt{5}$ or $-\left(\sqrt{5}\right)^2$ or $-\sqrt{25}$ instead of the -5 These 4 values may appear in a list or table but they should have minus signs included **The next two marks should be awarded for the final answer but check that correct values follow from correct working. Do not use ISW rule** 1st A1 for 11 from $16-5$ <u>or</u> $-6\sqrt{5}$ from $-8\sqrt{5}+2\sqrt{5}$ 2nd A1 for <u>both</u> 11 and $-6\sqrt{5}$. <u>S.C - Double sign error in expansion</u> For $16-5-2\sqrt{5}+8\sqrt{5}$ leading to 11 + … allow <u>one</u> mark	

Q8.

Question Number	Scheme	Marks
(i)	$\left(5-\sqrt{8}\right)\left(1+\sqrt{2}\right)$ $=5+5\sqrt{2}-\sqrt{8}-4$ $=5+5\sqrt{2}-2\sqrt{2}-4$ — $\sqrt{8}=2\sqrt{2}$, seen or implied at any point. $=1+3\sqrt{2}$ — $1+3\sqrt{2}$ or $a=1$ and $b=3$.	M1 B1 A1 [3]
(ii)	**Method 1:** Either $\sqrt{80}+\frac{30}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$ $= 4\sqrt{5}+\ldots$ $= 4\sqrt{5}+6\sqrt{5}$ **Method 2:** Or $\left(\frac{\sqrt{400}+30}{\sqrt{5}}\right)\frac{\sqrt{5}}{\sqrt{5}}$ $=\left(\frac{20+..}{..}\right)\frac{..}{..}$ $=\left(\frac{50\sqrt{5}}{5}\right)$ $= 10\sqrt{5}$ **Method 3:** $\sqrt{80}+\frac{\sqrt{900}}{\sqrt{5}}=\sqrt{80}+\sqrt{180}$ $= 4\sqrt{5}+\ldots$ $= 4\sqrt{5}+6\sqrt{5}$	M1 B1 A1 [3]
Alternative for (i)	$\left(5-2\sqrt{2}\right)\left(1+\sqrt{2}\right)$ — This earns the B1 mark. $=5+5\sqrt{2}-2\sqrt{2}-2\sqrt{2}\sqrt{2}$ — Multiplies out correctly with $2\sqrt{2}$. This may be seen or implied and may be simplified e.g. $=5+3\sqrt{2}\ -2\sqrt{4}$ o.e. For earlier use of $2\sqrt{2}$ $=1+3\sqrt{2}$ — $1+3\sqrt{2}$ or $a=1$ and $b=3$.	M1 B1 A1 [3] **6 marks**
	Notes	
(i)	**M1:** Multiplies out brackets correctly giving four correct terms or simplifying to correct expansion. (This may be implied by correct answer) – can appear as table **B1:** $\sqrt{8}=2\sqrt{2}$, seen or implied at any point **A1:** Fully and correctly simplified to $1+3\sqrt{2}$ or $a=1$ and $b=3$.	
(ii)	**M1:** Rationalises denominator i.e. Multiplies $\left(\frac{k}{\sqrt{5}}\right)$ by $\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$ or $\left(\frac{-\sqrt{5}}{-\sqrt{5}}\right)$, seen or implied or uses Method 3 or similar e.g. $\left(\frac{30}{\sqrt{5}}\right)=\frac{6\times5}{\sqrt{5}}=6\sqrt{5}$ **B1**: (Independent mark) States $\sqrt{80}=4\sqrt{5}$ Or either $\sqrt{400}=20$ *or* $\sqrt{80}\sqrt{5}=20$ at any point if they use Method 2. **A1:** $10\sqrt{5}$ or $c=10$.	
	N.B There are other methods e.g. $\sqrt{80}=\frac{20}{\sqrt{5}}$ (B1) then add $\frac{20}{\sqrt{5}}+\frac{30}{\sqrt{5}}=\frac{50}{\sqrt{5}}$ then M1 A1as before Those who multiply initial expression by $\sqrt{5}$ to obtain $\sqrt{400}+30=20+30=50$ earn M0 B1 A0	

Q9.

Question Number	Scheme		Marks
	$\frac{7+\sqrt{5}}{\sqrt{5}-1}\times\frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$	Multiplies top and bottom by a correct expression. This statement is sufficient.	M1
	(Allow to multiply top and bottom by $k(\sqrt{5}+1)$)		
	$=\frac{\dots}{4}$	Obtains a denominator of 4 or sight of $(\sqrt{5}-1)(\sqrt{5}+1)=4$	A1cso
	Note that M0A1 is not possible. The 4 must come from a correct method.		
	$(7+\sqrt{5})(\sqrt{5}+1)=7\sqrt{5}+5+7+\sqrt{5}$	An attempt to multiply the numerator by $(\pm\sqrt{5}\pm1)$ and get 4 terms with at least 2 correct for their $(\pm\sqrt{5}\pm1)$. (May be implied)	M1
	$3+2\sqrt{5}$	Answer as written or $a = 3$ and $b=2$. (Allow $2\sqrt{5}+3$)	A1cso
	Correct answer with no working scores full marks		
			[4]
Way 2	$\frac{7+\sqrt{5}}{\sqrt{5}-1}\times\frac{(-\sqrt{5}-1)}{(-\sqrt{5}-1)}$	Multiplies top and bottom by a correct expression. This statement is sufficient.	M1
	(Allow to multiply top and bottom by $k(-\sqrt{5}-1)$)		
	$=\frac{\dots}{-4}$	Obtains a denominator of -4	A1cso
	$(7+\sqrt{5})(-\sqrt{5}-1)=-7\sqrt{5}-5-7-\sqrt{5}$	An attempt to multiply the numerator by $(\pm\sqrt{5}\pm1)$ and get 4 terms with at least 2 correct for their $(\pm\sqrt{5}\pm1)$. (May be implied)	M1
	$3+2\sqrt{5}$	Answer as written or $a = 3$ and $b=2$	A1cso
	Correct answer with no working scores full marks		
			[4]
	Alternative using Simultaneous Equations: $\frac{(7+\sqrt{5})}{\sqrt{5}-1}=a+b\sqrt{5}\Rightarrow 7+\sqrt{5}=(a-b)\sqrt{5}+5b-a$ M1 Multiplies and collects rational and irrational parts $a-b=1,\ \ 5b-a=7$ A1 Correct equations $a=3,\ \ b=2$ M1 for attempt to solve simultaneous equations A1 both answers correct		

Q10.

Question Number	Scheme	Marks
Method 1	$x\sqrt{8}+10=\dfrac{6x}{\sqrt{2}}$ $\times\sqrt{2}\Rightarrow x\sqrt{16}+10\sqrt{2}=6x$ $4x+10\sqrt{2}=6x\Rightarrow 2x=10\sqrt{2}$ $x=5\sqrt{2}$ or $a=5$ and $b=2$	 M1,A1 M1A1 (4)
Method 2	$x\sqrt{8}+10=\dfrac{6x}{\sqrt{2}}$ $2\sqrt{2}x+10=3\sqrt{2}x$ $\sqrt{2}x=10\Rightarrow x=\dfrac{10}{\sqrt{2}}=\dfrac{10\sqrt{2}}{2},=5\sqrt{2}$ oe	 M1A1 M1,A1 (4)

Method 1

M1 For multiplying both sides by √2 – allow a slip e.g. $\sqrt{2}x\sqrt{8}+10=\dfrac{6x}{\sqrt{2}}\times\sqrt{2}$ or

$\sqrt{2}\times10+x\sqrt{8}=\dfrac{6x}{\sqrt{2}}\times\sqrt{2}$, where one term has an error or the correct $\sqrt{2}(10+x\sqrt{8})=\dfrac{6x}{\sqrt{2}}\times\sqrt{2}$

NB $x\sqrt{8}+10=6x\sqrt{2}$ is M0

A1 A correct equation in x with no fractional terms. Eg $x\sqrt{16}+10\sqrt{2}=6x$ oe.

M1 An attempt to solve their linear equation in x to produce an answer of the form $a\sqrt{2}$ or $a\sqrt{50}$

A1 $5\sqrt{2}$ oe (accept $1\sqrt{50}$)

Method 2

M1 For writing √8 as 2√2 or $\dfrac{6}{\sqrt{2}}$ as 3√2

A1 A correct equation in x with no fractional terms. Eg $2\sqrt{2}x+10=3\sqrt{2}x$ *or* $x\sqrt{8}+10=3\sqrt{2}x$ oe.

M1 An attempt to solve their linear equation in x to produce an answer of the form $a\sqrt{2}$ or $a\sqrt{50}$

$\sqrt{2}x=10\Rightarrow x=\dfrac{10}{\sqrt{2}}=\dfrac{10\sqrt{2}}{2},=5\sqrt{2}$

or $\sqrt{2}x=10\Rightarrow 2x^2=100\Rightarrow x^2=50\Rightarrow x=\sqrt{50}$ or $5\sqrt{2}$

A1 $5\sqrt{2}$ oe Accept $1\sqrt{50}$