Question 1 (Jun 2005, Q4)

Solve the equation $x^6 + 26x^3 - 27 = 0$.

Worked Solution

Let $k = x^3$, so the equation becomes:

$$k^2 + 26k - 27 = 0$$

$$(k + 27)(k - 1) = 0$$

$$k = -27 \quad \text{or} \quad k = 1$$

Take cube roots:

$$x^3 = -27 \Rightarrow x = -3; \qquad x^3 = 1 \Rightarrow x = 1$$

$x = -3$ and $x = 1$

Question 2 (Jan 2007, Q4)

Solve the equation $x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 10 = 0$.

Worked Solution

Let $y = x^{\frac{1}{3}}$, so the equation becomes:

$$y^2 + 3y - 10 = 0$$

$$(y - 2)(y + 5) = 0$$

$$y = 2 \quad \text{or} \quad y = -5$$

Cube both values to find x:

$$x = 2^3 = 8 \quad \text{or} \quad x = (-5)^3 = -125$$

$x = 8$ and $x = -125$

Question 3 (Jun 2007, Q6)

By using the substitution $y = (x+2)^2$, find the real roots of

$$(x+2)^4 + 5(x+2)^2 - 6 = 0.$$

Worked Solution

Let $y = (x+2)^2$. The equation becomes:

$$y^2 + 5y - 6 = 0$$

$$(y+6)(y-1) = 0$$

$$y = -6 \quad \text{or} \quad y = 1$$

Since $y = (x+2)^2 \geq 0$, discard $y = -6$.

For $y = 1$:

$$(x+2)^2 = 1 \Rightarrow x + 2 = \pm 1$$

$$x = -1 \quad \text{or} \quad x = -3$$

$x = -1$ and $x = -3$

Question 4 (Jun 2008, Q4)

Solve the equation $2x - 7x^{\frac{1}{2}} + 3 = 0$.

Worked Solution

Let $y = x^{\frac{1}{2}}$, so the equation becomes:

$$2y^2 - 7y + 3 = 0$$

$$(2y - 1)(y - 3) = 0$$

$$y = \tfrac{1}{2} \quad \text{or} \quad y = 3$$

Square both values to find x:

$$x = \left(\tfrac{1}{2}\right)^2 = \tfrac{1}{4} \quad \text{or} \quad x = 3^2 = 9$$

$$x = \frac{1}{4} \quad \text{and} \quad x = 9$$

Question 5 (Jan 2009, Q3)

Solve the equation $3x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$.

Worked Solution

Let $y = x^{\frac{1}{3}}$, so the equation becomes:

$$3y^2 + y - 2 = 0$$

$$(3y - 2)(y + 1) = 0$$

$$y = \tfrac{2}{3} \quad \text{or} \quad y = -1$$

Cube both values to find x:

$$x = \left(\tfrac{2}{3}\right)^3 = \tfrac{8}{27} \quad \text{or} \quad x = (-1)^3 = -1$$

$$x = \frac{8}{27} \quad \text{and} \quad x = -1$$

Question 6 (Jun 2012, Q7)

Solve $x - 6x^{\frac{1}{2}} + 2 = 0$, giving answers in the form $p \pm q\sqrt{r}$ where p, q, r are integers.

Worked Solution

Let $k = x^{\frac{1}{2}}$, so the equation becomes:

$$k^2 - 6k + 2 = 0$$

Complete the square:

$$(k-3)^2 - 7 = 0$$

$$k = 3 \pm \sqrt{7}$$

Square both values to find x:

$$x = (3 \pm \sqrt{7})^2 = 9 \pm 6\sqrt{7} + 7 = 16 \pm 6\sqrt{7}$$

$x = 16 + 6\sqrt{7}$ and $x = 16 - 6\sqrt{7}$

Question 7 (Jun 2014, Q3)

Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$.

Worked Solution

Let $k = x^2$, so the equation becomes:

$$4k^2 + 3k - 1 = 0$$

$$(4k - 1)(k + 1) = 0$$

$$k = \tfrac{1}{4} \quad \text{or} \quad k = -1$$

Since $k = x^2 \geq 0$, discard $k = -1$.

For $k = \frac{1}{4}$:

$$x^2 = \tfrac{1}{4} \Rightarrow x = \pm\tfrac{1}{2}$$

$$x = \frac{1}{2} \quad \text{and} \quad x = -\frac{1}{2}$$

Question 8 (Jun 2015, Q4)

Solve the equation $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$.

Worked Solution

Let $k = x^{\frac{1}{3}}$, so the equation becomes:

$$k^2 - k - 6 = 0$$

$$(k-3)(k+2) = 0$$

$$k = 3 \quad \text{or} \quad k = -2$$

Cube both values to find x:

$$x = 3^3 = 27 \quad \text{or} \quad x = (-2)^3 = -8$$

$x = 27$ and $x = -8$

End of Worked Solutions