Stationary Points Exam Questions (From OCR 4721)

Note: All of these questions are from the old specification and are taken from a non-calculator papers. In all of these questions, in order to prepare you for questions that require "full working" or "detailed reasoning", you should show all steps and keep calculator use to a minimum.

Q1, (Jun 2005, Q10)

(i)	$\frac{dy}{dx} = x^2 - 9$	B1 B1	 2	$x^2 - 9$ 1 term correct Both terms correct
(ii)	$x^2 - 9 = 0$ $x = 3, -3$ $y = -18, 18$	*M1 A1 A1	 3	uses $\frac{dy}{dx} = 0$ $x = 3, -3$ $y = -18, 18$ (1 correct pair A1 A0)
(iii)	$\frac{d^2y}{dx^2} = 2x$ $x = 3 \quad \frac{d^2y}{dx^2} = 6$ $x = -3 \quad \frac{d^2y}{dx^2} = -6$	DM1 A1 A1	 3	Looks at sign of $\frac{d^2y}{dx^2}$ or other correct method $x = 3$ minimum $x = -3$ maximum (N.B. If no method shown but min and max correctly stated, award all 3 marks unless earlier incorrect working)
(iv)	gradient of $24x + 3y + 2 = 0$ is -8 $x^2 - 9 = -8$ $x = \pm 1$ For line $x = 1,\ y = -8\frac{2}{3}$ $x = -1,\ y = 7\frac{1}{3}$ For curve $x = 1,\ y = -8\frac{2}{3}$ $x = -1,\ y = 8\frac{2}{3}$ $\therefore p = 1,\ q = -8\frac{2}{3}$	B1 M1 M1 M1 A1	 5	Gradient = – 8 $x^2 - 9 = -8$ one of their x values substituted in both line <u>and</u> curve second x value substituted in both line and curve **<u>or</u>** justification that first point is the correct one $p = 1, q = -8\frac{2}{3}$ seen <u>Alternative methods:</u> <u>Either:</u> Solve equations for curve and line simultaneously to get one solution (either x = 1 or x = -2) M1 Gradient of line = -8 B1 Substitution of one x value into their gradient formula and check for -8 M1 Substitution of other x value into gradient formula and check for -8 or justification as above M1 Correct q value A1 <u>Or:</u> Solve equations for curve and line simultaneously to get one solution M1 Factorise to $(x-1)^2(x+2)$ B1 State that a double root implies a tangent at x = 1 M2 Correct value for y A1

Q2, (Jun 2006, Q6)

(i)	$x^4 - 10x^2 + 25 = 0$ Let $y = x^2$ $y^2 - 10y + 25 = 0$	*M1		Use a substitution to obtain a quadratic or $(x^2 - 5)(x^2 - 5) = 0$
	$(y-5)^2 = 0$	dep*M1		Correct method to solve a quadratic
	$y = 5$ $x^2 = 5$	A1		5 (not $x = 5$ with no subsequent working)
	$x = \pm\sqrt{5}$	A1	4	$x = \pm\sqrt{5}$
(ii)	$y = \frac{2x^5}{5} - \frac{20x^3}{3} + 50x + 3$	B1		$2x^4$ or $-20x^2$ oe seen
	$\frac{dy}{dx} = 2x^4 - 20x^2 + 50$	B1	2	$2x^4 - 20x^2 + 50$ (integers required)
(iii)	$2x^4 - 20x^2 + 50 = 0$ $x^4 - 10x^2 + 25 = 0$	M1		*their* $\frac{dy}{dx} = 0$ seen (or implied by correct answer)
	which has 2 roots	A1	2	2 stationary points **www in any part**

Q3, (Jan 2007, Q8)

(i)	$\frac{dy}{dx} = 9 - 6x - 3x^2$	*M1		Attempt to differentiate y or $-y$ (at least one correct term)
		A1		3 correct terms
	At stationary points, $9 - 6x - 3x^2 = 0$	M1		Use of $\frac{dy}{dx} = 0$ (for y or $-y$)
	$3(3 + x)(1 - x) = 0$	DM1		Correct method to solve 3 term quadratic
	$x = -3$ or $x = 1$	A1		$x = -3,\ 1$
	$y = 0,\ 32$	A1ft	6	$y = 0,\ 32$ (1 correct pair www A1 A0)
(ii)	$\frac{d^2y}{dx^2} = -6x - 6$	M1		Looks at sign of $\frac{d^2y}{dx^2}$, derived correctly from $k\frac{dy}{dx}$, or other correct method
	When $x = -3, \frac{d^2y}{dx^2} > 0$	A1		$x = -3$ minimum
	When $x = 1, \frac{d^2y}{dx^2} < 0$	A1	3	$x = 1$ maximum
(iii)	$-3 < x < 1$	M1		Uses the x values of both turning points in inequality/inequalities
		A1	2	Correct inequality or inequalities. Allow $\leq$
			11	

Q4, (Jan 2008, Q8)

(i)	$\frac{dy}{dx} = 3x^2 + 2x - 1$	*M1 A1	Attempt to differentiate (at least one correct term) 3 correct terms
	At stationary points, $3x^2 + 2x - 1 = 0$	M1	Use of $\frac{dy}{dx} = 0$
	$(3x-1)(x+1) = 0$	DM1	Correct method to solve 3 term quadratic
	$x = \frac{1}{3},\ x = -1$	A1	$x = \frac{1}{3},\ x = -1$
	$y = \frac{76}{27},\ y = 4$	A1 6	$y = \frac{76}{27},\ 4$
			SR one correct (x, y) pair **www** **B1**
(ii)	$\frac{d^2y}{dx^2} = 6x + 2$	M1	Looks at sign of $\frac{d^2y}{dx^2}$ for at least one of their x-values or other correct method
	$x = \frac{1}{3},\ \frac{d^2y}{dx^2} > 0$	A1	$x = \frac{1}{3}$, minimum point CWO
	$x = -1,\ \frac{d^2y}{dx^2} < 0$	A1 3	$x = -1$, maximum point CWO
(iii)	$-1 < x < \frac{1}{3}$	M1	Any inequality (or inequalities) involving both their x values from part (i)
		A1 2	Correct inequality (allow $<$ or $\le$)
		11	

Q5, (Jun 2008, Q8)

(i)	$\frac{dy}{dx} = 3x^2 - 2kx + 1$	**B1**	One term correct
		B1	Fully correct
		2	
(ii)	$3x^2 - 2kx + 1 = 0$ when $x = 1$	**M1**	their $\frac{dy}{dx} = 0$ soi
	$3 - 2k + 1 = 0$	**M1**	$x = 1$ substituted into their $\frac{dy}{dx} = 0$
	$k = 2$	**A1√**	
		3	
(iii)	$\frac{d^2y}{dx^2} = 6x - 4$	**M1**	Substitutes $x = 1$ into their $\frac{d^2y}{dx^2}$ and looks at sign
	When $x = 1$, $\frac{d^2y}{dx^2} > 0 \therefore$ min pt	**A1**	States minimum **CWO**
		2	
(iv)	$3x^2 - 4x + 1 = 0$	**M1**	their $\frac{dy}{dx} = 0$
	$(3x-1)(x-1) = 0$	**M1**	correct method to solve 3-term quadratic
	$x = \frac{1}{3}, x = 1$		
	$x = \frac{1}{3}$	**A1**	**WWW** at any stage
		3	

Q6, (Jan 2009, Q9)

$\frac{dy}{dx} = 3x^2 + 2px$	M1 A1		Attempt to differentiate Correct expression cao
When $x = 4$, $\frac{dy}{dx} = 0$	M1		Setting their $\frac{dy}{dx} = 0$
$\therefore 3 \times 4^2 + 8p = 0$	M1		Substitution of $x = 4$ into their $\frac{dy}{dx} = 0$ to evaluate p
$8p = -48$			
$p = -6$	A1		
$\frac{d^2y}{dx^2} = 6x - 12$ When $x = 4$, $6x - 12 > 0$	M1		Looks at sign of $\frac{d^2y}{dx^2}$, derived correctly from their $\frac{dy}{dx}$, or other correct method
Minimum point	A1	7	Minimum point CWO
		7	

Q7, (Jun 2011, Q8)

(i)	$\frac{dy}{dx} = 6x + 6x^{-2}$	**M1** **A1**		Attempt to differentiate (one non-zero term correct) Completely correct	**NB** – $x = -1$ (and therefore possibly $y = 7$) can be found from equating the incorrect differential
	$6x + \frac{6}{x^2} = 0$	**M1**		Sets their $\frac{dy}{dx} = 0$	$\frac{dy}{dx} = 6x + 6$ to 0. This could score **M1A0 M1A0A1 ft**
	$x = -1$	**A1**		Correct value for x - **www**	
	$y = 7$	**A1 ft**	5	Correct value of y for *their* value of x	If more than one value of x found, allow **A1 ft** for one correct value of y
(ii)	$\frac{d^2y}{dx^2} = 6 - 12x^{-3}$	**M1**		Correct method e.g. substitutes their x from (i) into their $\frac{d^2y}{dx^2}$ (must involve x) and considers sign.	Allow comparing signs of their $\frac{dy}{dx}$ either side of their "– 1", comparing values of y to their "7"
	When $x = -1$, $\frac{d^2y}{dx^2} > 0$ so minimum pt	**A1 ft**	2 **7**	**ft** from their $\frac{dy}{dx}$ differentiated correctly and correct substitution of *their* value of x and consistent final conclusion **NB** If second derivate evaluated, it must be correct (18 for $x = -1$). If more than one value of x used, max **M1 A0**	**SC** $\frac{d^2y}{dx^2}$ = a constant correctly obtained from their $\frac{dy}{dx}$ and correct conclusion (ft) **B1**

Q8, (Jun 2013, Q10)

(i)	$y = -x^3 - 3x^2 + 4x - kx + k$ $\frac{dy}{dx} = -3x^2 - 6x + 4 - k$ When $x = -3$, $\frac{dy}{dx} = 0$ $-27 + 18 + 4 - k = 0$ $k = -5$	M1 A1 M1 A1 M1* DM1* A1 [7]	Attempt to multiply out brackets Can be unsimplified Attempt to differentiate **their** expansion (**M0** if signs have changed throughout) Sets $\frac{dy}{dx} = 0$ Substitutes $x = -3$ into their $\frac{dy}{dx} = 0$ **www**	Must have $\pm x^3$ and 5 or 6 terms **<u>If using product rule:</u>** Clear attempt at correct rule **M1*** Differentiates both parts correctly **A1** Expand brackets of both parts ***DM1** Then as main scheme
(ii)	$\frac{d^2y}{dx^2} = -6x - 6$ When $x = -3$, $\frac{d^2y}{dx^2}$ is positive so min point	M1 A1 [2]	Evaluates second derivative at $x = -3$ or other fully correct method No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 12. (Ignore errors in k value)	**Alternate valid methods include:** 1) Evaluating gradient at either side of −3 2) Evaluating y at either side of −3 3) Finding other turning point and stating "negative cubic so min before max"
(iii)	$-3x^2 - 6x + 9 = 9$ $3x(x + 2) = 0$ $x = 0$ or $x = -2$ When $x = 0$, $y = -9$ for line $y = -5$ for curve When $x = -2$, $y = -27$ for line $y = -27$ for curve $x = -2$, $y = -27$	M1 A1 M1 M1 A1 [5]	Sets their gradient function from (i) (or from a restart) to 9 Correct x-values One of their x-values substituted into both curve and line/substituted into one and verified to be on the other Conclusion that $x = -2$ is the correct value **<u>or</u>** Second x-value substituted into both curve and line/verified as above $x = -2$, $y = -27$ **www *(Check k correct)***	Allow first **M** even if k not found but look out for correct answer from wrong working. **<u>SEE NEXT PAGE FOR ALTERNATIVE METHODS</u>** **Note:** Putting a value into $x^3 + 3x^2 - 4 = 0$ (where the line and curve meet) is equivalent <u>If curve equated to line before differentiating:</u> **M0 A0**, can get **M1M1** but **A0 ww** Maximum mark **2/5**

Q9, (Jun 2014, Q8)

i)	$\frac{dy}{dx} = 9x^2 - 7 - 2x^{-2}$ When $x = 1$, $\frac{dy}{dx} = 9 - 7 - 2 = 0$ Therefore a stationary point	**M1*** **A1** **A1** **M1dep** **A1** **[5]**	Attempt to differentiate, any term correct Two correct terms Fully correct Substitute $x = 1$ into their derivative Correctly obtain zero **www** and state conclusion **AG**	Alternative for the last two marks: Sets derivative to zero and makes valid attempt to solve resulting quartic **M1dep** Correctly establishes $x = 1$ as solution and draws clear conclusion **A1www**
ii)	$\frac{d^2y}{dx^2} = 18x + 4x^{-3}$ When $x = 1$, $\frac{d^2y}{dx^2} > 0$ so minimum	**M1** **A1** **[2]**	Correct method to find nature of stationary point e.g. substituting $x = 1$ into second derivative (at least one term correct from their first derivative in (i)) No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 22.	**Alternate valid methods include:** 1) Evaluating gradient at either side of 1($x > 0$) 2) Evaluating y at 1 and either side of 1 ($x > 0$) If using alternatives, working must be fully correct to obtain the **A** mark
iii)	When $x=1$, $y = -2$ (0, –2)	**B1** **B1** **[2]**	Finding $y = -2$ at $x = 1$ Correct coordinate **www**	

Q10, (Jun 2015, Q9)

(i)		$\frac{dy}{dx} = 6x^2 - 2ax + 8$	M1 A1	Attempt to differentiate, at least two non-zero terms correct Fully correct	
		When $x = 4$, $\frac{dy}{dx} = 104 - 8a$	M1	Substitutes $x = 4$ into their $\frac{dy}{dx}$	These Ms may be awarded in either order
		$\frac{dy}{dx} =$ 0 gives $a = 13$	M1 A1 **[5]**	Sets their $\frac{dy}{dx}$ to 0. Must be seen	
(ii)		$\frac{d^2y}{dx^2} = 12x - 26$ When $x = 4$, $\frac{d^2y}{dx^2} > 0$ so minimum	M1 A1 **[2]**	Correct method to find nature of stationary point e.g. substituting $x = 4$ into second derivative (at least one term correct from their first derivative in (i)) and consider the sign **www**	**Alternate valid methods include:** 1) Evaluating gradient at either side of 4($x > \frac{1}{3}$) e.g. at 3, –16 at 5, 28 2) Evaluating $y = -46$ at 4 and either side of 4 ($x > \frac{1}{3}$) e.g. (3, – 37), (5, – 33) If using alternatives, working must be fully correct to obtain the **A** mark
(iii)		$6x^2 - 26x + 8 = 0$	M1	Sets their derivative to zero	
		$(3x - 1)(x - 4) = 0$	M1	Correct method to solve quadratic **(appx 1)**	Could be $(6x - 2)(x - 4) = 0$ or $(3x - 1)(2x - 8) = 0$
		$x = \frac{1}{3}$	A1 **[3]**	oe	

Q11, (Jun 2016, Q11)

$y = 4x^2 + ax^{-1} + 5$	**B1**	ax^{-1} **soi**	
$\frac{dy}{dx} = 8x - ax^{-2}$	**M1**	Attempt to differentiate – at least one non-zero term correct	
	A1	Fully correct	
At stationary point, $8x - ax^{-2} = 0$	**M1**	Sets their derivative to 0	
$a = 8x^3$ **oe**	**A1**	Obtains expression for a in terms of x, or x in terms of a **www**	$x = \frac{\sqrt[3]{a}}{2}$ **oe**, $a = 18x$ **oe** also fine
When $a = 8x^3$, $y = 32$ $32 = 4x^2 + 8x^2 + 5$	**M1**	Substitutes their expression and 32 into equation of the curve to form single variable equation	
$x = \frac{3}{2}$ **oe**	**A1**	Obtains correct value for x. Allow $x = \sqrt{\frac{27}{12}}$. Ignore $-\frac{3}{2}$ given as well.	or expression for a e.g. $a^{\frac{2}{3}} = 9$
$a = 27$	**A1** **[8]**	Obtains correct value for a. Ignore –27 given as well.	
OR			
$y = 4x^2 + ax^{-1} + 5$	**B1**	ax^{-1} **soi**	
$\frac{dy}{dx} = 8x - ax^{-2}$	**M1**	Attempt to differentiate – at least one non-zero term correct	
	A1	Fully correct	
$32 = 4x^2 + ax^{-1} + 5$	**M1**	Substitutes 32 into equation of the curve to find expression for a	
$a = 27x - 4x^3$	**A1**	Obtains expression for a in terms of x **www**	
At stationary point, $8x - ax^{-2} = 0$ $8x - (27x - 4x^3)x^{-2} = 0$	**M1**	Sets derivate to zero **and** forms single variable equation	
$x = \frac{3}{2}$ **oe**	**A1**	Obtains correct value for x. Allow $x = \sqrt{\frac{27}{12}}$. Ignore $-\frac{3}{2}$ given as well.	
$a = 27$	**A1**	Obtains correct value for a. Ignore –27 given as well.	

Q12, (Jun 2018, Q10)

$(y=)\ (x+1)^2(x-3)$	B1	$(x-3)$ seen	<u>Alt – using simultaneous equations</u>
	B1	$(x+1)$ or $(x+1)^2$ seen	$-1+p-q+r=0$ **B1**
$(y=)(x^2+2x+1)(x-3)$	M1	Multiply repeated root by linear factor to obtain at least five terms	$27+9p+3q+r=0$ **B1**
$(y=)\ x^3-x^2-5x-3$	A1	Correct values of p, q, r obtained	$\frac{dy}{dx}=3x^2+2px+q$ **B1**
$\frac{dy}{dx}=3x^2-2x-5$	B1 ft	Correct differentiation of their cubic	Sets gradient to zero at $x=-1$ **M1**
$3x^2-2x-5=0$	M1	Sets derivative equal to zero	$3-2p+q=0$
$(3x-5)(x+1)=0$	M1	Correct method to find roots. **See appendix1**	Correct method to solve three simultaneous equations in three variables; do not allow incorrectly obtained equations e.g. from $\frac{dy}{dx}=0$ at $x=3$. Must get as far as finding at least one of p, q, r **M1**
$x=\frac{5}{3}$	A1	Correct value of x obtained	Correct values of p, q, r obtained **A1**
$y=-\frac{256}{27}$	A1	Correct value of y obtained	Correct method to find x **M1**
			Correct x **A1**
			Correct y **A1**