Stationary Points Exam Questions Sheet 2 Mark Scheme

Q1.

Question Number	Scheme	Marks
	$\frac{dy}{dx} = 2 - 16x^{-3}$	M1 A1
	$2 - 16x^{-3} = 0$ so $x^{-3} =$ or $x^3 =$, or $2 - 16x^{-3} = 0$ so $x = 2$	M1
	$x = 2$ only (after correct derivative)	A1
	$y = 2 \times "2" + 3 + \frac{8}{"2^2"}$	M1
	$= 9$	A1
		(6)
		Total 6

	Notes for Question
	1st M1: At least one term **differentiated (not integrated)** correctly, so $2x \to 2$, or $\frac{8}{x^2} \to -16x^{-3}$, or $3 \to 0$ A1: This answer or equivalent e.g. $2 - \frac{16}{x^3}$ 2nd M1: Sets $\frac{dy}{dx}$ to 0, and solves to give $x^3 =$ value or $x^{-3} =$ value (or states x =2 with no working following correctly stated $2 - 16x^{-3} = 0$) A1: $x = 2$ cso (if x = -2 is included this is A0 here) 3rd M1: Attempts to substitutes **their positive** x (found from attempt to differentiate) into $y = 2x + 3 + \frac{8}{x^2}$, $x > 0$ Or may be implied by $y = 9$ or correct follow through from their positive x A1: 9 cao (Does not need to be written as coordinates) (ignore the extra (-2,1) here)

Q2.

Question Number	Scheme		Marks
	$y = 6 - 3x - \frac{4}{x^3}$		
(a)	$\frac{dy}{dx} = -3 + \frac{12}{x^4}$ or $-3 + 12x^{-4}$	M1: $x^n \to x^{n-1}$ ($x^1 \to x^0$ or $x^{-3} \to x^{-4}$ or $6 \to 0$) A1: Correct derivative	M1 A1
	$\frac{dy}{dx} = 0 \Rightarrow -3 + \frac{12}{x^4} = 0 \Rightarrow x = ...$ **or** $\frac{dy}{dx} = -3 + \frac{12}{\sqrt{2}^4}$	$y' = 0$ and attempt to solve for x May be implied by $\frac{dy}{dx} = -3 + \frac{12}{x^4} = 0 \Rightarrow \frac{12}{x^4} = 3 \Rightarrow x = ...$ **or** Substitutes $x = \sqrt{2}$ into their y'	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ **or** $\frac{dy}{dx} = -3 + \frac{12}{(\sqrt{2})^4}$ or $-3 + 12(\sqrt{2})^{-4} = 0$	Correct completion to answer with no errors by solving their $y' = 0$ or substituting $x = \sqrt{2}$ into their y'	A1
			(4)
(b)	$x = -\sqrt{2}$	Awrt -1.41	B1
			(1)
(c)	$\frac{d^2y}{dx^2} = \frac{-48}{x^5}$ or $-48x^{-5}$	Follow through their first derivative from part (a)	B1ft
			(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum		B1
	Maximum at P as $y'' < 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and there must be reference to P or to $\sqrt{2}$ and negative or < 0 and maximum. There must be no incorrect or contradictory statements (NB allow y'' = awrt-8 or -9)		
	Minimum at Q as $y'' > 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct **and part (b) must be correct** and there must be reference to P or to $-\sqrt{2}$ and positive or > 0 and minimum. There must be no incorrect or contradictory statements (NB allow y'' = awrt 8 or 9)		
			(3)
			[9]
	Other methods for identifying the nature of the turning points are acceptable. The first B1 is for finding values of y or dy/dx either side of $\sqrt{2}$ or their x at Q and the second and third B1's for fully correct solutions to identify the maximum/minimum.		

Q3.

Question Number	Scheme	Marks
(a)	$\left\{\frac{dy}{dx}=\right\}\ 2x-16x^{-\frac{1}{2}}$	M1 A1
	$2x-16x^{-\frac{1}{2}}=0 \Rightarrow\ x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$,or $2x-=16x^{-\frac{1}{2}}$ then squared then obtain $x^3=$ [or $2x-16x^{-\frac{1}{2}}=0 \Rightarrow x=4$ (no wrong work seen)]	M1
	$(x^{\frac{3}{2}}=8 \Rightarrow)x=\ 4$	A1
	$x=4,\ y=\ 4^2-32\sqrt{4}+20=-28$ (ignore $y=100$ as second answer)	M1 A1
		(6)
(b)	$\left\{\frac{d^2y}{dx^2}=\right\}\ 2+8x^{-\frac{3}{2}}$	M1 A1
	$(\frac{d^2y}{dx^2}>0 \Rightarrow)y$ is a minimum (there should be no wrong reasoning)	A1
		(3) [9]
(b)	**Alternative Method: Gradient Test:** M1 for finding the gradient either side of their x-value from part (a). A1 for both gradients calculated correctly to 1 significant figure, then using < 0 and > 0 respectively *maybe by use of sketch or table*. (See appendix for gradient values. This is **not ft their** x) A1 states minimum needs M1A1 to have been awarded.	
	Notes for Question	
(a)	1st M1: At least one term differentiated correctly, so $x^2 \to 2x$, or $32\sqrt{x} \to 16x^{-\frac{1}{2}}$, or $20 \to 0$ A1: This answer or equivalent e.g. $2x-\frac{16}{\sqrt{x}}$ 2nd M1: Sets their $\frac{dy}{dx}$ to 0, and solves to give $x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$ *or* $x^3=$ after correct squaring or spots $x=4$ (NB $\left\{\frac{d^2y}{dx^2}=0\right\}$ *so* $2+8x^{-\frac{3}{2}}=0$ **is M0**) N.B. Common error: Putting derivative = 0 and merely obtaining $x=0$ is M0A0, then M0A0 for next two marks. (The first two marks in (a) and marks for second derivative may be earned in part (b).) A1: $x=4$ cao [$x=-4$ is A0 and $x=\pm4$ is also A0] 3rd M1: Substitutes **their positive found** x **(NOT zero)** into $y=x^2-32\sqrt{x}+20,\ x>0$**.Should follow attempting to set** $\frac{dy}{dx}=0$ **and not setting** $\frac{d^2y}{dx^2}=0$ A1: -28 cao (Does not need to be written as coordinates)	
(b)	M1: Attempts differentiation of their first derivative with at least one term differentiated correctly. Should be seen or referred to (in part (b)) in determining the nature of the stationary point. A1: Answer in scheme or equivalent A1: States minimum (Second derivative should be correct- can follow incorrect positive x. Needs M1A1 to have been awarded- should not follow incorrect reasoning – (need not say $\frac{d^2y}{dx^2}>0$ but should not have said $\frac{d^2y}{dx^2}=0$ for example)	

Q4.

Question Number	Scheme	Marks
(a)	$\left[y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10\right]$	
	$[y' =]\quad 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$	M1 A1
	Puts their $\frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}} = 0$	M1
	So $x = \ ,\frac{12}{3} = 4$ (If $x = 0$ appears also as solution then lose A1)	M1, A1
	$x = 4, \Rightarrow y = 12 \times 2 - 4^{\frac{3}{2}} - 10$, so $y = 6$	dM1,A1 (7)
(b)	$y'' = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$	M1A1 (2)
(c)	[Since $x > 0$] It is a maximum	B1 (1) [10]

(a)	1^{st} M1 for an attempt to differentiate a fractional power $x^n \to x^{n-1}$ A1 a.e.f – can be unsimplified 2^{nd} M1 for forming a suitable equation using their $y' = 0$ 3^{rd} M1 for correct processing of fractional powers leading to $x = \ldots$ (Can be implied by $x = 4$) A1 is for $x = 4$ only. If $x = 0$ also seen and not discarded they lose this mark only. 4^{th} M1 for substituting their value of x back into y to find y value. Dependent on three previous M marks. Must see evidence of the substitution with attempt at fractional powers to give M1A0, but $y = 6$ can imply M1A1
(b)	M1 for differentiating their y' again A1 should be simplified
(c)	B1 . Clear conclusion needed and must follow correct y'' It is dependent on previous A mark (Do not need to have found x earlier). (Treat parts (a),(b) and (c) together for award of marks)

Q5.

Question	Scheme	Marks	AOs
(a) (i)	Uses $\frac{dy}{dx}=-3$ at $x=2\Rightarrow\ 12a+60-39=-3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a+60-39=-3\Rightarrow 12a=-24\Rightarrow a=-2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2,10)$ lies on C $\quad 10=8a+60-78+b$	M1	3.1a
	Subs $a=-2$ into $10=8a+60-78+b\Rightarrow b=44$	A1	1.1b
		(4)	
(b)	$f(x)=-2x^3+15x^2-39x+44\Rightarrow f'(x)=-6x^2+30x-39$	B1	1.1b
	Attempts to show that $-6x^2+30x-39$ has no roots Eg. calculates $b^2-4ac=30^2-4\times-6\times-39=-36$	M1	3.1a
	States that as $f'(x)\neq 0\Rightarrow$ hence $f(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3+15x^2-39x+44\equiv(x-4)(-2x^2+7x-11)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0,44)$ or $(20,0)$	B1 ft	1.1b
	Deduces both intercepts $(0,44)$ and $(20,0)$	B1 ft	2.2a
		(2)	
			(11 marks)

Q6.

Question	Scheme	Marks	AOs
(a)	Deduces $g(x)=ax^3+bx^2+ax$	B1	2.2a
	Uses $(2,9)\Rightarrow 9=8a+4b+2a$ $\Rightarrow 10a+4b=9$	M1 A1	2.1 1.1b
	Uses $g'(2)=0\Rightarrow 0=12a+4b+a$ $\Rightarrow 13a+4b=0$	M1 A1	2.1 1.1b
	Solves simultaneously $\Rightarrow a,b$	dM1	1.1b
	$g(x)=-3x^3+\frac{39}{4}x^2-3x$	A1	1.1b
		(7)	
(b)	Attempts $g''(x)=-18x+\frac{39}{2}$ and substitutes $x=2$	M1	1.1b
	$g''(2)=-\frac{33}{2}<0$ hence maximum	A1	2.4
		(2)	
			(9 marks)