Question 1 (Jun 2007, Q3)

Worked Solution

Solve $3^{2x+1} = 5^{200}$, giving x correct to 3 significant figures.

Step 1: Take logarithms of both sides.

$$\log 3^{2x+1} = \log 5^{200}.$$

Step 2: Drop the powers.

$$(2x+1)\log 3 = 200\log 5.$$

Step 3: Solve for x.

$$2x + 1 = \frac{200\log 5}{\log 3} \implies 2x = \frac{200\log 5}{\log 3} - 1 \implies x = \frac{1}{2}\left(\frac{200\log 5}{\log 3} - 1\right).$$

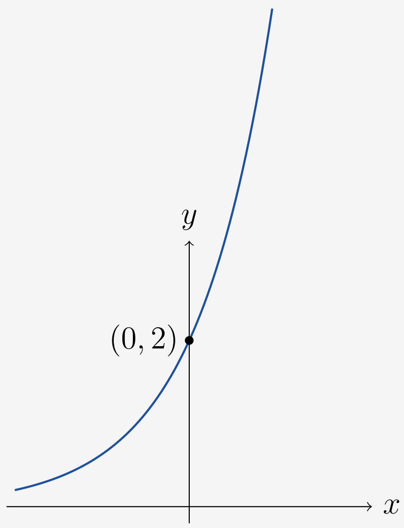
$$x = \frac{1}{2}(200 \times 1.46497\ldots - 1) = \frac{1}{2}(292.994\ldots - 1) \approx 146.$$

$$x = 146 \text{ (to 3 s.f.)}$$

Question 2 (Jun 2008, Q8)

Worked Solution

Part (i): Sketch $y = 2 \times 3^x$, stating coordinates of any axis intersections.



When $x = 0$: $y = 2 \times 1 = 2$, giving intercept $(0, 2)$. The curve does not cross the x-axis.

Part (ii): $y = 2 \times 3^x$ and $y = 8^x$ intersect at P. Show the x-coordinate of P is $\frac{1}{3 - \log_2 3}$.

Set $2 \times 3^x = 8^x$.

Step 1: Take $\log_2$ of both sides.

$$\log_2(2 \times 3^x) = \log_2 8^x.$$

Step 2: Expand using log laws.

$$\log_2 2 + x \log_2 3 = x \log_2 8.$$

$$1 + x \log_2 3 = 3x \quad (\text{since } \log_2 8 = 3).$$

Step 3: Collect x terms and solve.

$$1 = 3x - x \log_2 3 = x(3 - \log_2 3) \implies x = \frac{1}{3 - \log_2 3}. \quad \square$$

$$x = \frac{1}{3 - \log_2 3}$$

Question 3 (Jun 2009, Q3)

Worked Solution

Solve $7^x = 2^{x+1}$, giving x correct to 3 significant figures.

Step 1: Take logarithms of both sides.

$$\log 7^x = \log 2^{x+1}.$$

Step 2: Drop the powers.

$$x \log 7 = (x+1)\log 2.$$

Step 3: Expand and collect x terms.

$$x\log 7 = x\log 2 + \log 2 \implies x(\log 7 - \log 2) = \log 2 \implies x = \frac{\log 2}{\log 7 - \log 2}.$$

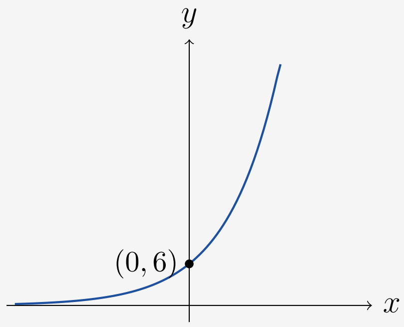
$$x = \frac{\log 2}{\log(7/2)} \approx \frac{0.30103}{0.54407} \approx 0.553.$$

$$x = 0.553 \text{ (to 3 s.f.)}$$

Question 4 (Jan 2010, Q9)

Worked Solution

Part (i): Sketch $y = 6 \times 5^x$, stating coordinates of any axis intersections.



When $x = 0$: $y = 6$, giving y-intercept $(0, 6)$. The curve never crosses the x-axis.

Part (ii): On $y = 9^x$, find x when $y = 150$.

$$9^x = 150.$$

Take logs:

$$x \log 9 = \log 150 \implies x = \frac{\log 150}{\log 9} \approx \frac{2.17609}{0.95424} \approx 2.28.$$

$$x = 2.28 \text{ (to 3 s.f.)}$$

Part (iii): $y = 6 \times 5^x$ and $y = 9^x$ intersect at Q. Show $x = \dfrac{1 + \log_3 2}{2 - \log_3 5}$.

Set $6 \times 5^x = 9^x$. Take $\log_3$ of both sides:

$$\log_3 6 + x \log_3 5 = x \log_3 9 = 2x.$$

Now $\log_3 6 = \log_3(2 \times 3) = \log_3 2 + 1$. Substituting:

$$\log_3 2 + 1 + x \log_3 5 = 2x.$$

Collect x terms:

$$\log_3 2 + 1 = 2x - x \log_3 5 = x(2 - \log_3 5) \implies x = \frac{1 + \log_3 2}{2 - \log_3 5}. \quad \square$$

$$x = \frac{1 + \log_3 2}{2 - \log_3 5}$$

Question 5 (Jun 2010, Q8a)

Worked Solution

Solve $5^{3w-1} = 4^{250}$, giving w correct to 3 significant figures.

Step 1: Take logarithms of both sides.

$$\log 5^{3w-1} = \log 4^{250}.$$

Step 2: Drop the powers.

$$(3w-1)\log 5 = 250\log 4.$$

Step 3: Solve for w.

$$3w - 1 = \frac{250\log 4}{\log 5} \implies 3w = \frac{250\log 4}{\log 5} + 1 \implies w = \frac{1}{3}\left(\frac{250\log 4}{\log 5} + 1\right).$$

$$w = \frac{1}{3}(250 \times 0.86135\ldots + 1) = \frac{1}{3}(215.338 + 1) \approx 72.1.$$

$$w = 72.1 \text{ (to 3 s.f.)}$$

Question 6 (Jun 2014, Q5)

Worked Solution

Solve $2^{4x-1} = 3^{5-2x}$, giving the answer in the form $x = \dfrac{\log_{10} a}{\log_{10} b}$.

Step 1: Take $\log_{10}$ of both sides and drop powers.

$$(4x-1)\log_{10} 2 = (5-2x)\log_{10} 3.$$

Step 2: Expand brackets.

$$4x\log_{10} 2 - \log_{10} 2 = 5\log_{10} 3 - 2x\log_{10} 3.$$

Step 3: Collect x terms.

$$4x\log_{10} 2 + 2x\log_{10} 3 = 5\log_{10} 3 + \log_{10} 2.$$

$$x(4\log_{10} 2 + 2\log_{10} 3) = \log_{10} 2 + 5\log_{10} 3.$$

Step 4: Combine logs. Use $b\log a = \log a^b$ and $\log a + \log b = \log ab$:

$$x(\log_{10} 2^4 + \log_{10} 3^2) = \log_{10}(2 \cdot 3^5)$$

$$x\log_{10}(16 \times 9) = \log_{10}(2 \times 243)$$

$$x\log_{10} 144 = \log_{10} 486.$$

$$x = \frac{\log_{10} 486}{\log_{10} 144}$$

Question 7 (Jun 2016, Q8)

Worked Solution

Part (i): The curve $y = 3^x$ is transformed to $y = 3^{x-2}$ by a translation.

Replacing x by $x - 2$ translates the curve by $+2$ in the x-direction.

Translation of 2 units in the positive x-direction.

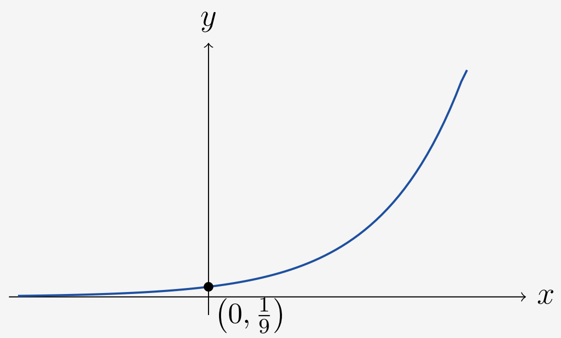
Part (ii): Describe the same transformation as a stretch.

Note $3^{x-2} = 3^x \cdot 3^{-2} = \frac{1}{9} \cdot 3^x$. So the curve $y = 3^x$ is stretched by scale factor $\frac{1}{9}$ in the y-direction.

Stretch, scale factor $\frac{1}{9}$, in the y-direction.

Part (iii): Sketch $y = 3^{x-2}$, stating coordinates of any axis intersections.

When $x = 0$: $y = 3^{-2} = \frac{1}{9}$. When $y = 0$: no solution (exponential never zero).



Part (iv): On $y = 3^{x-2}$ the point P has $y = 180$. Find the x-coordinate of P to 3 s.f.

$$3^{x-2} = 180.$$

Take logs:

$$(x-2)\log 3 = \log 180 \implies x - 2 = \frac{\log 180}{\log 3}.$$

$$x = 2 + \frac{\log 180}{\log 3} \approx 2 + 4.7268 \approx 6.73.$$

$x = 6.73$ (to 3 s.f.)

Question 8 (Jun 2011, Q8i–iv)

Worked Solution

The diagram shows $y = 2^x - 3$.

Part (i): Describe the transformation from $y = 2^x$ to $y = 2^x - 3$.

Subtracting 3 from $y = 2^x$ moves every point 3 units downwards.

Translation of 3 units in the negative y-direction.

Part (ii): State the y-coordinate where $y = 2^x - 3$ crosses the y-axis.

When $x = 0$: $y = 2^0 - 3 = 1 - 3 = -2$.

$y = -2$

Part (iii): Find the x-coordinate where $y = 2^x - 3$ crosses the x-axis, in the form $\log_a b$.

Set $y = 0$:

$$2^x - 3 = 0 \implies 2^x = 3 \implies x = \log_2 3.$$

$x = \log_2 3$

Part (iv): The curve passes through $(p, 62)$. Find p to 3 s.f.

$$2^p - 3 = 62 \implies 2^p = 65.$$

Take logs:

$$p \log 2 = \log 65 \implies p = \frac{\log 65}{\log 2} \approx \frac{1.8129}{0.30103} \approx 6.02.$$

$p = 6.02$ (to 3 s.f.)

Question 9 (Jun 2013, Q8)

Worked Solution

The diagram shows $y = a^x$ and $y = 4b^x$.

Part (i)(a): Coordinates of intersection of $y = a^x$ with the y-axis.

When $x = 0$: $y = a^0 = 1$.

$(0, 1)$

Part (i)(b): Coordinates of intersection of $y = 4b^x$ with the y-axis.

When $x = 0$: $y = 4b^0 = 4$.

$(0, 4)$

Part (i)(c): State a possible value for a and for b.

From the diagram, $y = a^x$ is increasing so $a > 1$, and $y = 4b^x$ is decreasing so $0 < b < 1$.

E.g. $a = 2$, $b = \frac{1}{2}$

Part (ii): Given $ab = 2$, show the x-coordinate of the intersection of $y = a^x$ and $y = 4b^x$ is $\dfrac{2}{2\log_2 a - 1}$.

Set $a^x = 4b^x$. Take $\log_2$ of both sides:

$$\log_2(a^x) = \log_2(4b^x).$$

$$x\log_2 a = \log_2 4 + x\log_2 b = 2 + x\log_2 b.$$

Since $ab = 2$, we have $b = \frac{2}{a}$, so $\log_2 b = \log_2\left(\frac{2}{a}\right) = 1 - \log_2 a$.

Substitute:

$$x\log_2 a = 2 + x(1 - \log_2 a).$$

$$x\log_2 a = 2 + x - x\log_2 a.$$

$$2x\log_2 a - x = 2 \implies x(2\log_2 a - 1) = 2 \implies x = \frac{2}{2\log_2 a - 1}. \quad \square$$

$$x = \frac{2}{2\log_2 a - 1}$$

End of Worked Solutions