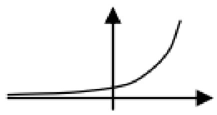
Solving Exponential Equations (From OCR 4722)

Q1, (Jun 2007, Q3)

3	$\log 3^{(2x+1)} = \log 5^{200}$	M1		Introduce logarithms throughout
	$(2x+1)\log 3 = 200 \log 5$	M1		Drop power on at least one side
		A1		Obtain correct linear equation (now containing no powers)
	$2x+1 = \frac{200\log 5}{\log 3}$	M1		Attempt solution of linear equation
	$x = 146$	A1	5	Obtain $x = 146$, or better
OR				
	$(2x + 1) = \log_3 5^{200}$	M1		Intoduce $\log_3$ on right-hand side
	$2x + 1 = 200\log_3 5$	M1		Drop power of 200
		A1		Obtain correct equation
		M1		Attempt solution of linear equation
		A1		Obtain $x = 146$, or better
			5	

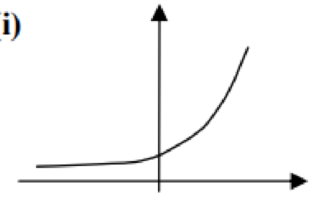
Q2, (Jun 2008, Q8)

(i)		**M1**	Attempt sketch of exponential graph (1st quad) - if seen in 2nd quad must be approx correct
		A1	Correct graph in both quadrants
		B1	State or imply (0, 2) only
		3	
(ii)	$8^x = 2 \times 3^x$		
	$\log_2 8^x = \log_2 (2 \times 3^x)$	**M1**	Form equation in x and take logs (to any consistent base, or no base) – could use $\log_8$
	$x\log_2 8 = \log_2 2 + x\log_2 3$	**M1**	Use $\log a^b = b \log a$
		M1	Use $\log ab = \log a + \log b$,or equiv with $\log {}^a/_b$
	$3x = 1 + x\log_2 3$	**M1**	Use $\log_2 8 = 3$
	$x(3 - \log_2 3) = 1$, hence $x = \dfrac{1}{3-\log_2 3}$ **A.G.**	**A1**	Show given answer correctly
OR	$8^x = 2 \times 3^x$		
	$2^{3x} = 2 \times 3^x$	**M1**	Use $8^x = 2^{3x}$
	$2^{(3x-1)} = 3^x$	**M1**	Attempt to rearrange equation to $2^k = 3^x$
	$\log_2 2^{(3x-1)} = \log_2 3^x$	**M1**	Take logs (to any base)
	$(3x - 1)\log_2 2 = x \log_2 3$	**M1**	Use $\log a^b = b \log a$
	$x(3 - \log_2 3) = 1$, hence $x = \dfrac{1}{3-\log_2 3}$ **A.G.**	**A1**	Show given answer correctly
		5	

Q3, (Jun 2009, Q3)

$\log 7^x = \log 2^{x+1}$	M1		Introduce logarithms throughout, or equiv with base 7 or 2
$x\log 7 = (x+1)\log 2$	M1		Drop power on at least one side
	A1		Obtain correct linear equation (allow with no brackets)
$x(\log 7 - \log 2) = \log 2$	M1		**Either** expand bracket and attempt to gather x terms, **or** deal correctly with algebraic fraction
$x = 0.553$	A1	5	Obtain $x = 0.55$, or rounding to this, with no errors seen
		5	

Q4, (Jan 2010, Q9)

(i) 	M1		Reasonable graph in both quadrants
	A1		Correct graph in both quadrants
	B1	**3**	State or imply (0, 6)
(ii) $9^x = 150$	M1		Introduce logarithms throughout, or equiv with $\log_9$
$x\log 9 = \log 150$	M1		Use $\log a^b = b\log a$ and attempt correct method to find x
$x = 2.28$	A1	**3**	Obtain $x = 2.28$
(iii) $6 \times 5^x = 9^x$	M1		Form eqn in x and take logs throughout (any base)
$\log_3 (6 \times 5^x) = \log_3 9^x$	M1		Use $\log a^b = b\log a$ correctly on $\log 5^x$ or $\log 9^x$ or legitimate combination of these two
$\log_3 6 + x\log_3 5 = x\log_3 9$	M1		Use $\log ab = \log a + \log b$ correctly on $\log (6 \times 5^x)$ or $\log 6$
$\log_3 3 + \log_3 2 + x\log_3 5 = 2x$	M1		Use $\log_3 9 = 2$ or equiv (need base 3 throughout that line)
$x(2 - \log_3 5) = 1 + \log_3 2$			
$x = \dfrac{1+\log_3 2}{2-\log_3 5}$ **A.G.**	A1	**5**	Obtain $x = \dfrac{1+\log_3 2}{2-\log_3 5}$ convincingly (inc base 3 throughout)
		11	

Q5, (Jun 2010, Q8a)

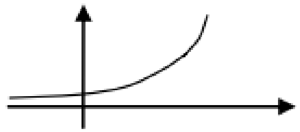
$\log 5^{3w-1} = \log 4^{250}$	**M1***		Introduce logarithms throughout
$(3w-1)\log 5 = 250\log 4$	**M1***		Use $\log a^b = b\log a$ at least once
$3w - 1 = \frac{250\log 4}{\log 5}$	**A1**		Obtain $(3w-1)\log 5 = 250\log 4$ or equiv
$w = 72.1$	**M1d***		Attempt solution of linear equation
	A1	**5**	Obtain 72.1, or better

Q6, (Jun 2014, Q5)

$(4x - 1)\log_{10}2 = (5 - 2x)\log_{10}3$	M1*	Introduce logs throughout and drop power(s)	Allow no base or base other than 10 as long as consistent, including $\log_3$ on LHS or $\log_2$ on RHS Drop single power if $\log_3$ or $\log_2$ or both powers if any other base
	A1	Obtain $(4x - 1)\log_{10}2 = (5 - 2x)\log_{10}3$	Brackets must be seen, or implied by later working Allow no base, or base other than 10 if consistent Any correct linear equation ie $4x - 1 = (5 - 2x)\log_2 3$ or $(4x - 1)\log_3 2 = 5 - 2x$
$x(4\log_{10}2 + 2\log_{10}3) = \log_{10}2 + 5\log_{10}3$	M1*	Attempt to make x the subject	Expand bracket(s) and collect like terms - as far as their $4x\log_{10}2 + 2x\log_{10}3 = \log_{10}2 + 5\log_{10}3$ Expressions could include $\log_2 3$ or $\log_3 2$ Must be working exactly, so M0 if log(s) now decimal equivs
	A1	Obtain a correct equation in which x only appears once	LHS could be $x(4\log_{10}2 + 2\log_{10}3)$, $x\log_{10}144$ or even $\log_{10}144^x$ Expressions could include $\log_2 3$ or $\log_3 2$ RHS may be two terms or single term
$x\log_{10}144 = \log_{10}486$	M1d*	Attempt correct processes to combine logs	Use $b\log a = \log a^b$, then $\log a + \log b = \log ab$ correctly on at least one side of equation (or $\log a - \log b$) Dependent on previous M1 but not the A1 so $\log_{10}486$ will get this M1 irrespective of the LHS
$x = \frac{\log_{10}486}{\log_{10}144}$	A1 **[6]**	Obtain correct final expression	Base 10 required in final answer - allow A1 if no base earlier, or if base 10 omitted at times, but A0 if different base seen previously (unless legitimate working to change base seen) Do not isw subsequent incorrect log work eg $x = {}^{\log 27}/_{\log 8}$

Q7, (Jun 2016, Q8)

(i)		2 (units) in the positive x-direction	M1 A1 **[2]**	Correct direction Fully correct description	Identify that the translation is in the x-direction (either positive or negative, so M1 for eg '2 in negative x-direction') Allow any terminology as long as intention is clear, such as in/on/along the x-axis Ignore the magnitude Must have correct magnitude and correct direction, using precise language - such as 'in the x-direction', 'parallel to the x-axis', 'horizontally' or 'to the right' A0 for in/on/along the x-axis etc Allow M1A1 for '2 in the x-direction' as positive is implied A0 for 'factor 2' 'Units' is not required, but A0 for 'places', 'spaces', 'squares' etc Allow in vector notation as well, so M1 for $\begin{pmatrix} k \\ 0 \end{pmatrix}$and M1A1 for $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$
(ii)		sf $\frac{1}{9}$ in the y-direction	M1 A1 **[2]**	Correct direction, with sf of $\frac{1}{9}$ or 9 Fully correct description	Identify that the stretch is in the y-direction, with a scale factor of either $\frac{1}{9}$ or 9 (or equiv in index notation) Allow just $\frac{1}{9}$ or 9, with no mention of 'scale factor' Allow exact decimal equiv for $\frac{1}{9}$ Allow any terminology as long as the intention is clear, such as in/on/along the y-axis Must have correct scale factor and correct direction, using precise language - such as 'in the y-direction', 'parallel to the y-axis' or 'vertically' A0 for in/on/along the y-axis etc Must now have 'scale factor' or 'factor' Allow 'positive y-direction' (not incorrect as graph is wholly above x-axis)

(iii)		 intersect at $(0, \frac{1}{9})$	B1*	Correct sketch, in both quadrants	Curve must tend towards the negative x-axis, but not touch or cross it, nor a significant flick back upwards If from plotted points then there must be enough of the graph shown to demonstrate the correct general shape, including the negative x-axis being an asymptote Ignore any numerical values given
			B1d* **[2]**	State $(0, \frac{1}{9})$	Condone $x = 0$, $y = \frac{1}{9}$ as an alternative, but $x = 0$ must be stated explicitly rather than implied Allow no brackets around the coordinates Allow exact decimal equiv for $\frac{1}{9}$ Allow just $\frac{1}{9}$ as long as marked on the y-axis Allow BOD for $(\frac{1}{9}, 0)$ on y-axis, but not if just stated Just being seen in a table of values is not sufficient Ignore any other labelled coordinates
(iv)		$\log 3^{x-2} = \log 180$ (or $x - 2 = \log_3 180$) $(x - 2)\log 3 = \log 180$ $x - 2 = 4.7268...$ $x = 6.73$	M1*	Introduce logs and drop power	Can use logs to any base, as long as consistent on both sides, and allow no explicit base as well The power must also be dropped for the M1 Brackets must be seen around the $(x - 2)$, or implied by later working If taking $\log_3$ then base must be explicit
			M1d*	Attempt to solve for x	Correct order of operations, and correct operations so M0 for $\log_3 180 - 2$ M0 if logs used incorrectly eg $x - 2 = \log(\frac{180}{3})$
			A1 **[3]**	Obtain 6.73, or better	If > 3sf, allow answer rounding to 6.727 with no errors seen 0/3 for answer only or T&I If rewriting eqn as $3^{x-2} = 3^{4.73}$ then 0/3 unless evidence of use of logs to find the index of 4.73 **SR** If using index rules first then **B1** for $3^x = 1620$ **M1** for attempting to use logs to solve $3^x = k$ **A1** for 6.73

Q8, (Jun 2011, Q8i-iv)

8(i)	translation of 3 units in negative y-direction	**B1**		State translation	Not shift, move etc.
		B1	**2**	State or imply 3 units in negative y-direction	Independent of first B1. Statement needs to clearly intend a vertical downwards move of 3, without ambiguity or contradiction, such as '3 down', '-3 in the y direction' etc or vector notation. B0 if direction unclear, such as 'in the y-axis' (could be along or towards) or 'along the y-axis' (unless direction made clear). Allow '3' or '3 units' but not '3 places', '3 spaces', '3 squares', '3 coordinates' or mention of (scale) factor of 3. If both a valid statement and an ambiguous statement are made eg '3 units down on the y-axis' then still award B1. Ignore irrelevant statements, such as where the y-intercept is, whether correct or incorrect. Give BOD on double negatives eg 'down the y-axis by – 3 units' unless clearly wrong or contradictory eg 'negative y-direction by $\begin{pmatrix}-3\\0\end{pmatrix}$'.
(ii)	$y = -2$	**B1**	**1**	State or imply $y = -2$	Just stating -2 is enough. B0 for final answer of $2^0 - 3$ or $1 - 3$. (-2, 0) is B0 unless -2 already seen or implied as y-coordinate.
(iii)	$2^x = 3$ $x = \log_2 3$	**M1**		Attempt to solve $2^x - 3 = 0$	Rearrange to $2^x = 3$, introduce logarithms (could be no base or any base as long as consistent) and then attempt expression for x. M0 for $x = \log_3 2$. M1 A0 for alternative, correct, log expressions such as ${}^{\log 3}/_{\log 2}$ or $1/\log_3 2$. Decimal equivalent of 1.58 can get M1 A0. $x = \log_2 (y + 3)$ is M0 (unless y then becomes 0).
		A1	**2**	State $\log_2 3$	Doesn't need to be $x = \ldots$ Change of base is not on the specification, but is a valid method and can gain both marks. Allow if base not initially specified, but then both logs become base 2. **NB** $x - \log_2 3 = 0$ leading to correct answer, can get full marks as there is no incorrect statement seen.

(iv)	$2^p = 65$ $\log 2^p = \log 65$ $p\log 2 = \log 65$ $p = 6.02$	**M1***		Rearrange equation and introduce logs (or $\log_2$)	Must first rearrange to $2^p = k$, with k from attempt at 62 ± 3, before introducing logs. Can use logs to any base, as long as consistent, or equiv with $\log_2$.
		M1d*		Drop power and attempt to solve	Dependent on first M1. $p = \log_2 k$ will gain both M marks in one step. If taking logs to any other base, or no base, or $\log_2$ on both sides then need to drop power of p and attempt to solve using a sound algebraic method ie $p = {}^{\log k}/_{\log 2}$.
		A1	**3**	Obtain 6.02, or better	Decimal answer reqd, if more than 3sf it must be in range [6.022, 6.023]. Answer only, or trial and improvement, is 0/3 as no evidence of using logs as requested.

Q9, (Jun 2013, Q8)

(i)	**(a)**	(0, 1)	B1 **[1]**	State (0, 1)	Allow no brackets B1 for $x = 0$, $y = 1$ – must have $x = 0$ stated explicitly B0 for $y = a^0 = 1$ (as $x = 0$ is implicit)
	(b)	(0, 4)	B1 **[1]**	State (0, 4)	Allow no brackets B1 for $x = 0$, $y = 4$ – must have $x = 0$ stated explicitly B0 for $y = 4b^0 = 4$ (as $x = 0$ is implicit)
	(c)	State a possible value for a State a possible value for b	B1 B1 **[2]**	Must satisfy $a > 1$ Must satisfy $0 < b < 1$	Must be a single value Could be irrational eg e Must be fully correct so B0 for eg 'any positive number such as 3' Must be a single value Could be irrational eg e^{-1} Must be fully correct **SR** allow B1 if both a and b given correctly as a range of values

(ii)		$\log_2 a^x = \log_2(4b^x)$	M1	Equate a^x and $4b^x$ and introduce logarithms at some stage	Could either use the two given equations, or b could have already been eliminated so using two eqns in a only Must take logs of each side soi so M0 for $4\log_2(b^x)$ Allow just log, with no base specified, or $\log_2$ Allow logs to any base, or no base, as long as consistent
		$\log_2 a^x = \log_2 4 + \log_2 b^x$	M1	Use $\log ab = \log a + \log b$ correctly	Or correct use of $\log {}^a/_b = \log a - \log b$ Used on a correct expression eg $\log_2(4b^x)$ or $\log_2 4({}^2/_a)^x$ Equation could either have both a and b or just a Must be used on an expression associated with $a^x = 4b^x$, either before or after substitution, so M0 for $\log_2(ab) = 1$ hence $\log_2 a + \log_2 b = 1$ Could be an equiv method with indices before using logs eg $a^{2x} = 4 \times 2^x$ hence $a^{2x} = 2^{2+x}$
		$x\log_2 a = \log_2 4 + x\log_2 b$	M1	Use $\log a^b = b \log a$ correctly at least once	Allow if used on an expression that is possibly incorrect Allow M1 for $x\log_2 a = x\log_2 4b$ as one use is correct Equation could either have both a and b or just a
		$x\log_2 a = \log_2 4 + x\log_2({}^2/_a)$	B1	Use $b = {}^2/_a$ to produce a correct equation in a and x only	Can be gained at any stage, including before use of logs If logs involved then allow for no, or incorrect, base as long as equation is fully correct – ie if $\log 2^k = k$ used then base must be 2 throughout equation Could be an equiv method eg $(a \times a)^x = 4(a \times b)^x$ hence $a^{2x} = 4 \times 2^x$ Must be eliminating b, so $({}^2/_b)^x = 4b^x$ is B0 unless the equation is later changed to being in terms of a
		$x\log_2 a = 2 + x\log_2 2 - x\log_2 a$ $x(2\log_2 a - 1) = 2$ $x = \dfrac{2}{2\log_2 a - 1}$ **AG**	A1 **[5]**	Obtain given relationship with no wrong working	Proof must be fully correct with enough detail to be convincing Must use $\log_2$ throughout proof for A1 – allow 1 slip Using numerical values for a and b will gain no credit Working with equation(s) involving y is M0 unless y is subsequently eliminated