Exponential Equations Exam Questions Sheet 2 MS

Q1.

Question	Scheme	Marks	AOs
	$\frac{9^{x-1}}{3^{y+2}}=81\Rightarrow\frac{3^{2x-2}}{3^{y+2}}=3^4$ or $\frac{9^{x-1}}{3^{y+2}}=81\Rightarrow\frac{9^{x-1}}{9^{\frac{1}{2}(y+2)}}=9^2$	M1	1.1b
	$\Rightarrow 2x-2-y-2=4\Rightarrow y=$ or $\Rightarrow x-1-\frac{1}{2}y-1=2\Rightarrow y=$	dM1	1.1b
	$\Rightarrow y=2x-8$	A1	1.1b
		(3)	
Alt	Eg. $\log_3\left(\frac{9^{x-1}}{3^{y+2}}\right)=\log_3 81$	M1	1.1b
	$\Rightarrow (x-1)\log_3\left(9^{x-1}\right)-(y+2)\log_3\left(3^{y+2}\right)=4$ $\Rightarrow 2(x-1)-y-2=4\Rightarrow y=$	dM1	1.1b
	$\Rightarrow y=2x-8$	A1	1.1b
			(3 marks)

Notes

M1: Attempts to set 9^{x-1} and 81 as powers of 3. Condone $9^{x-1}=3^{2x-1}$ and $9^{x-1}=3^{3x-3}$.
Alternatively attempts to write each term as a logarithm of base 3 or 9. You must see the base written to award this mark.

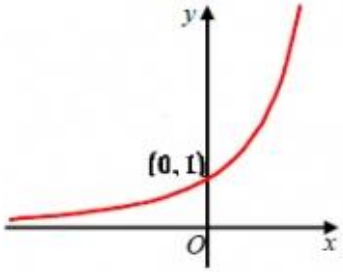
dM1: Attempts to use the addition (or subtraction) index law, or laws or logarithms, correctly and rearranges the equation to reach y in terms of x. Condone slips in their rearrangement.

A1: $y=2x-8$

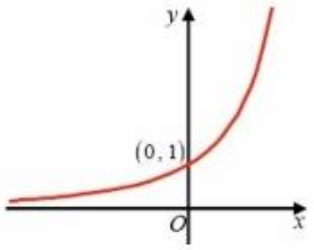
Q2.

Question Number	Scheme	Marks
(a)	$x=\frac{\log 7}{\log 5}$ or $x=\log_5 7$	M1
	1.21	A1 (2)
(b)	$\left(5^x-7\right)\left(5^x-5\right)$	M1 A1
	$\left(5^x=7 \text{ or } 5^x=5\right)$ $x=1.2$ (awrt)	A1 ft
	$x=1$	B1 (4)
		(6 marks)

Q3.

Question Number	Scheme		Marks
	Graph of $y = 3^x$ and solving $3^{2x} - 9(3^x) + 18 = 0$		
(a)		At least two of the three criteria correct. **(See notes below.)**	B1
		All three criteria correct. **(See notes below.)**	B1
		**Criteria number 1**: Correct shape of curve for $x \geq 0$ and at least touches the positive y-axis. **Criteria number 2**: Correct shape of curve for $x < 0$. Must not touch the x-axis or have any turning points. **Criteria number 3**: $(0, 1)$ stated or in a table or 1 marked on the y-axis. Allow $(1, 0)$ rather than $(0, 1)$ if marked in the "correct" place on the y-axis.	
			[2]
(b)	$(3^x)^2 - 9(3^x) + 18 = 0$ or $y = 3^x \Rightarrow y^2 - 9y + 18 = 0$	Forms a quadratic of the correct form in 3^x or in "y" where "y" $= 3^x$ or even in x where "x" $= 3^x$	M1
	$\{(y-6)(y-3) = 0$ or $(3^x - 6)(3^x - 3) = 0\}$		
	$y = 6$, $y = 3$ or $3^x = 6$, $3^x = 3$	**Both** $y = 6$ and $y = 3$.	A1
	$\{3^x = 6 \Rightarrow\}$ $x \log 3 = \log 6$ or $x = \dfrac{\log 6}{\log 3}$ or $x = \log_3 6$	A valid method for solving $3^x = k$ where $k > 0, k \neq 1, k \neq 3$ **to give either** $x \log 3 = \log k$ or $x = \dfrac{\log k}{\log 3}$ or $x = \log_3 k$	dM1
	$x = 1.63092...$	awrt 1.63	A1cso
	Provided the first M1A1 is scored, the second M1A1 can be implied by awrt 1.63		
	$x = 1$	$x = 1$ stated as a solution from *any* working.	B1
			[5]
			Total 7

Q4.

Question Number	Scheme	Marks
(a)	Graph of $y = 7^x$, $x \in \mathbb{R}$ and solving $7^{2x} - 4(7^x) + 3 = 0$  At least two of the three criteria correct. **(See notes below.)** All three criteria correct. **(See notes below.)**	B1 B1 (2)
(b)	$y^2 - 4y + 3 \{= 0\}$ Forming a quadratic {using "$y = 7^x$"}. $y^2 - 4y + 3 \{= 0\}$	M1 A1
	$\{(y-3)(y-1) = 0$ or $(7^x - 3)(7^x - 1) = 0\}$ $y = 3$, $y = 1$ or $7^x = 3$, $7^x = 1$ Both $y = 3$ and $y = 1$.	A1
	$\{7^x = 3 \Rightarrow\}\ x \log 7 = \log 3$ or $x = \frac{\log 3}{\log 7}$ or $x = \log_7 3$ A valid method for solving $7^x = k$ where $k > 0$, $k \neq 1$	dM1
	$x = 0.5645....$ 0.565 or awrt 0.56 $x = 0$ $x = 0$ stated as a solution.	A1 B1 (6) [8]

Notes

(a)

B1B0: Any two of the following three criteria below correct.
B1B1: All three criteria correct.
Criteria number 1: Correct shape of curve for $x \geq 0$.
Criteria number 2: Correct shape of curve for $x < 0$.
Criteria number 3: (0, 1) stated or 1 marked on the y-axis. Allow (1, 0) rather than (0, 1) if marked in the "correct" place on the y-axis.

Question Number	Scheme	Marks
(b)	1st M1 is an attempt to form a quadratic equation {using "$y = 7^x$."} 1st A1 mark is for the correct quadratic equation of $y^2 - 4y + 3 \{= 0\}$. Can use any variable here, eg: y, x or 7^x. Allow M1A1 for $x^2 - 4x + 3 \{= 0\}$. Writing $(7^x)^2 - 4(7^x) + 3 = 0$ is also sufficient for M1A1. Award M0A0 for seeing $7^{x^2} - 4(7^x) + 3 = 0$ by itself without seeing $y^2 - 4y + 3 \{= 0\}$ or $(7^x)^2 - 4(7^x) + 3 = 0$. 1st A1 mark for both $y = 3$ and $y = 1$ or both $7^x = 3$ and $7^x = 1$. Do not give this accuracy mark for both $x = 3$ and $x = 1$, unless these are recovered in later working by candidate applying logarithms on these. Award M1A1A1 for $7^x = 3$ and $7^x = 1$ written down with no earlier working. 3rd dM1 for solving $7^x = k$, $k > 0$, $k \neq 1$ to give either $x \ln 7 = \ln k$ or $x = \frac{\ln k}{\ln 7}$ or $x = \log_7 k$. dM1 is dependent upon the award of M1. 2nd A1 for 0.565 or awrt 0.56. B1 is for the solution of $x = 0$, from ***any*** working.	

Q5.

Question Number	Scheme	Marks
(a)	Attempt $f(3)$ or $f(-3)$ Use of long division is M0A0 as factor theorem was required. $f(-3)=162-63-120+21=0$ so $(x+3)$ is a factor	M1 A1 **(2)**
(b)	Either (Way 1): $f(x)=(x+3)(-6x^2+11x+7)$ $=(x+3)(-3x+7)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$	M1A1 M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain $x=-1/2$ **or** $x=7/3$ Uses trial or factor theorem to obtain both $x=-1/2$ **and** $x=7/3$ Puts three factors together (see notes below) Correct factorisation : $(x+3)(7-3x)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$ oe	M1 A1 M1 A1 **(4)**
	Or (Way 3) No working three factors $(x+3)(-3x+7)(2x+1)$ otherwise need working	M1A1M1A1 **(4)**
(c)	$2^y=\frac{7}{3}$, $\rightarrow \log\left(2^y\right)=\log\left(\frac{7}{3}\right)$ or $y=\log_2\left(\frac{7}{3}\right)$ or $\frac{\log(7/3)}{\log 2}$ $\{y=1.222392421...\} \Rightarrow y=$ awrt 1.22	B1, M1 A1 **(3)** **[9]**

	Notes
(a)	M1 for attempting either $f(3)$ or $f(-3)$ – with **numbers substituted into expression** A1 for calculating $f(-3)$ **correctly to 0**, and they must state $(x+3)$ **is a factor** for A1 (or equivalent ie. QED, □ or a tick). A conclusion may be implied by a preamble, "if $f(-3)=0$, $(x+3)$ is a factor". $-6(-3)^3-7(-3)^2+40(-3)+21=0$ so $(x+3)$ is a factor of $f(x)$ is M1A1 providing bracketing is correct.
(b)	1st M1: attempting to divide by $(x+3)$ leading to a 3TQ beginning with the correct term, usually $-6x^2$. This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. Allow for work in part (a) if the result is used in (b). 1st A1: usually for $(-6x^2+11x+7)$ … Credit when seen and use isw if miscopied 2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together fully factorised. Accept e.g. $-3(x+3)(x-\frac{7}{3})(2x+1)$ but $(x+3)(x-\frac{7}{3})(-6x-3)$ and $(x+3)(3x-7)(-2x-1)$ are A0 as not fully factorised. Ignore subsequent work (such as a solution to a quadratic equation.) Way 2: The second M mark needs three roots together so $\pm 6(x-\alpha)(x-\beta)(x+3)$ or equivalent where they obtained α and β by trial, so if correct roots identified, then $(x+3)(3x-7)(2x+1)$ can gain M1A1M1A0. N.B. Replacing $(-6x^2+11x+7)$ (already awarded M1A1) by $(6x^2-11x-7)$ giving $(x+3)(3x-7)(2x+1)$ can have M1A0 for factorization so M1A1M1A0
(c)	B1: $2^y=\frac{7}{3}$ M1: Attempt to take logs to solve $2^y=\alpha$ or $2^y=1/\alpha$, where $\alpha>0$ and α was a root of their factorization. A1: for an answer that rounds to 1.22. If other answers are included (and not "rejected") such as $\ln(-3)$ or -1 lose final A mark **Special case:** Those who deal **throughout** with $f(x)=6x^3+7x^2-40x-21$ They may have full credit in part (a). In part (b) they can achieve a maximum of M1A0M1A0 unless they return the negative sign to give the correct answer. This is then full marks. Part (c) is fine. So they could lose 2 marks on the factorisation. (Like a misread)

Q6.

Question Number	Scheme		Marks
(a)	Either (Way 1) : Attempt f(3) or f(−3) $f(3) = 54 - 45 + 3a + 18 = 0 \Rightarrow 3a = -27 \Rightarrow a = -9$ *	Or (Way 2): Assume a = -9 and attempt f(3) or f(−3) f(3) = 0 so (x -3) is factor	M1 A1 * cso **(2)**
	Or (Way 3): $(2x^3 - 5x^2 + ax + 18) \div (x - 3) = 2x^2 + px + q$ where p is a number and q is an expression in terms of a Sets the remainder $18+3a+9 = 0$ and solves to give $a = -9$		M1 A1* cso **(2)**
(b)	Either (Way 1): $f(x) = (x - 3)(2x^2 + x - 6)$ $= (x - 3)(2x - 3)(x + 2)$		M1A1 M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain x = -2 **or** x = 3/2 Uses trial or factor theorem to obtain both x = -2 **and** x = 3/2 Puts three factors together (see notes below) Correct factorisation : $(x - 3)(2x - 3)(x + 2)$ or $(3 - x)(3 - 2x)(x + 2)$ or $2(x - 3)(x - \frac{3}{2})(x + 2)$ oe		M1 A1 M1 A1 **(4)**
	Or (Way 3) No working three factors $(x - 3)(2x - 3)(x + 2)$ otherwise need working		M1A1M1A1
(c)	$\{3^y = 3 \Rightarrow\}$ $\underline{y = 1}$ or g(1) = 0		B1
	$\{3^y = 1.5 \Rightarrow\} \log(3^y) = \log 1.5$ or $y = \log_3 1.5$		M1
	$\{y = 0.3690702...\} \Rightarrow y =$ awrt 0.37		A1 **(3)** **[9]**
	Notes for Question		
(a)	M1 for attempting either f(3) or f(−3) – with **numbers substituted into expression** A1 for applying f(3) **correctly**, setting the result **equal to 0**, and manipulating this correctly to give the result given on the paper i.e. $a = -9$. (Do not accept x = -9) Note that the answer is given in part (a). If they **assume** a = -9 and **verify** by factor theorem or division they must state $(x - 3)$ **is a factor** for A1 (or equivalent such as QED or a tick).		
(b)	1st M1: attempting to divide by $(x - 3)$ leading to a 3TQ beginning with the correct term, usually $2x^2$. (Could divide by $(3 - x)$, in which case the quadratic would begin $-2x^2$.) This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. 1st A1: usually for $2x^2 + x - 6$ … Credit when seen and use isw if miscopied 2nd M1: for a *valid** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) NB: $(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M0A0, $(x - 3)(x - \frac{3}{2})(2x + 4)$ is M1A1M1A0, but $2(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M1A1.		
(c)	B1: $\underline{y = 1}$ seen as a solution – may be spotted as answer – no working needed. Allow also for g(1) = 0. M1: Attempt to take logs to solve $3^y = \alpha$ or even $3^{ky} = \alpha$, but not $6^y = \alpha$ where $\alpha > 0$ and $\alpha \neq 3$ & was a root of f(x) = 0 (ft their factorization) A1: for an answer that rounds to 0.37. If a third answer is included (and not "rejected") such as ln(-2) lose final A mark		