Question 1 (Jan 2008, Q4)

Worked Solution

In triangle BDC: angle $BDC = 50°$, angle $BCD = 62°$, $BC = 16$ cm, $AB = 10$ cm, $AD = 20$ cm.

Part (i): Find the length of BD.

Using the sine rule in triangle BDC:

$$\frac{BD}{\sin 62°} = \frac{BC}{\sin(\angle BDC)} = \frac{16}{\sin 50°}$$

$$BD = \frac{16 \sin 62°}{\sin 50°} = \frac{16 \times 0.8829\ldots}{0.7660\ldots} = 18.4 \text{ cm}$$

$BD = 18.4$ cm

Part (ii): Find angle BAD.

Using the cosine rule in triangle ABD with $AB = 10$, $AD = 20$, $BD = 18.4$:

$$BD^2 = AB^2 + AD^2 - 2 \cdot AB \cdot AD \cdot \cos(\angle BAD)$$

$$18.4^2 = 10^2 + 20^2 - 2(10)(20)\cos\theta$$

$$338.56 = 500 - 400\cos\theta$$

$$\cos\theta = \frac{500 - 338.56}{400} = \frac{161.44}{400} = 0.4036\ldots$$

$$\theta = \arccos(0.4036) = 66.2°$$

$\angle BAD = 66.4°$ (using exact BD: $\cos\theta = 0.3998\ldots \Rightarrow \theta = 66.4°$)

Question 2 (Jun 2008, Q6)

Worked Solution

A lifeboat travels 15 km on bearing 030° from A to B, then 27 km on bearing 110° from B to C.

Part (i): Show that angle $ABC = 100°$.

The bearing from A to B is 030°, so the direction BA (reverse) has bearing $030° + 180° = 210°$.

The bearing from B to C is 110°.

The angle between BA and BC is measured at B:

$$\angle ABC = 360° - 210° - (360° - 110°) = 360° - 150° - 110° = 100° \quad \text{A.G.}$$

More directly: the north direction at B, extended back towards A, makes 30° with north on the south side. The bearing to C is 110°. The interior angle at B between BA and BC is:

$$\angle ABC = 360° - (150° + 110°) = 100° \checkmark$$

Part (ii): Find the distance CA.

Using the cosine rule in triangle ABC:

$$CA^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos(\angle ABC)$$

$$CA^2 = 15^2 + 27^2 - 2(15)(27)\cos 100°$$

$$= 225 + 729 - 810\cos 100°$$

$$= 954 - 810(-0.17365\ldots) = 954 + 140.65\ldots = 1094.65\ldots$$

$$CA = \sqrt{1094.65} = 33.1 \text{ km}$$

$CA = 33.1$ km

Part (iii): Find the bearing from C to A.

Using the sine rule to find angle BCA:

$$\frac{\sin C}{AB} = \frac{\sin(\angle ABC)}{CA} \implies \frac{\sin C}{15} = \frac{\sin 100°}{33.1}$$

$$\sin C = \frac{15\sin 100°}{33.1} = \frac{15 \times 0.9848}{33.1} = 0.4464\ldots$$

$$C = \arcsin(0.4464) = 26.5°$$

The bearing from B to C is 110°. Angle $BCA = 26.5°$. The direction from C towards A is therefore:

The bearing from C to A = bearing from C to B + angle BCA.

Bearing from C to $B = 110° + 180° - 360° + 360° = 290°$.

Then bearing from C to A:

$$= 290° - 26.5° - 360° + 360°$$

Alternatively, using the supplementary angle $A = 180° - 100° - 26.5° = 53.5°$:

The bearing from C to A is $290° - 26.5° = 263°$.

Bearing from C to A is $263°$

Question 3 (Jun 2011, Q1)

Worked Solution

Triangle ABC: $AB = 9$ cm, $AC = 17$ cm, angle $BAC = 40°$.

Part (i): Find the length of BC.

Using the cosine rule:

$$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos(\angle BAC)$$
$$= 9^2 + 17^2 - 2(9)(17)\cos 40°$$
$$= 81 + 289 - 306\cos 40°$$
$$= 370 - 306(0.7660\ldots) = 370 - 234.4\ldots = 135.6\ldots$$
$$BC = \sqrt{135.6} = 11.6 \text{ cm}$$

$BC = 11.6$ cm

Part (ii): Find the area of triangle ABC.

$$\text{Area} = \tfrac{1}{2} \cdot AB \cdot AC \cdot \sin(\angle BAC) = \tfrac{1}{2}(9)(17)\sin 40°$$
$$= \tfrac{1}{2}(153)(0.6428\ldots) = 49.2 \text{ cm}^2$$

Area $= 49.2$ cm^2

Part (iii): D is on AC with angle $BDA = 63°$. Find BD.

In triangle ABD:

$$\angle BAD = 40°, \quad \angle BDA = 63° \implies \angle ABD = 180° - 40° - 63° = 77°$$

Using the sine rule:

$$\frac{BD}{\sin 40°} = \frac{AB}{\sin 63°} = \frac{9}{\sin 63°}$$
$$BD = \frac{9\sin 40°}{\sin 63°} = \frac{9 \times 0.6428}{0.8910} = 6.49 \text{ cm}$$

$BD = 6.49$ cm

Question 4 (Jan 2012, Q4)

Worked Solution

A is 2.4 km due north of B. Ship C is on bearing 040° from B, at distance 2 km from B.

Part (i): Find distance AC.

In triangle ABC: $AB = 2.4$, $BC = 2$, angle $ABC = 40°$ (bearing 040° measured from north at B, and A is due north so the interior angle at B between BA and BC is 40°).

Cosine rule:

$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos 40°$$
$$= 2.4^2 + 2^2 - 2(2.4)(2)\cos 40°$$
$$= 5.76 + 4 - 9.6(0.7660\ldots) = 9.76 - 7.354\ldots = 2.406\ldots$$
$$AC = \sqrt{2.406} = 1.55 \text{ km}$$

$AC = 1.55$ km (to 3 s.f.)

Part (ii): Find the bearing of C from A.

Find angle BAC using the sine rule:

$$\frac{\sin(\angle BAC)}{BC} = \frac{\sin(\angle ABC)}{AC} \implies \frac{\sin A}{2} = \frac{\sin 40°}{1.55}$$

$$\sin A = \frac{2\sin 40°}{1.55} = \frac{2(0.6428)}{1.55} = 0.8294\ldots \implies A = 56°$$

Since A is due north of B and C is to the east of AB (bearing 040°), angle $BAC = 56°$ is measured eastward from AB (from north).

Bearing of C from $A = 56° + \underbrace{180° - 180°}$

The bearing from south is 180°; adding 56° eastward: bearing of C from $A = 180° - 56° = 124°$.

Bearing of C from A is 124°

Part (iii): Find the shortest distance from C to the coastline (line AB).

The coastline is the straight north–south line through A and B. The shortest distance from C to this line is the perpendicular distance, i.e. the horizontal distance from C to the line.

$$d = BC\sin(\angle ABC) = 2\sin 40° = 2(0.6428\ldots) = 1.29 \text{ km}$$

Shortest distance = 1.29 km

Question 5 (Jun 2017, Q1)

Worked Solution

Triangle ABC: $AB = x$ cm, $AC = (x+2)$ cm, $BC = 2\sqrt{7}$ cm, angle $CAB = 60°$.

Part (i): Find the value of x.

Using the cosine rule with the angle at A:

$$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos(\angle CAB)$$

$$(2\sqrt{7})^2 = x^2 + (x+2)^2 - 2x(x+2)\cos 60°$$

$$28 = x^2 + x^2 + 4x + 4 - 2x(x+2) \cdot \tfrac{1}{2}$$

$$28 = 2x^2 + 4x + 4 - x(x+2)$$

$$28 = 2x^2 + 4x + 4 - x^2 - 2x$$

$$28 = x^2 + 2x + 4$$

$$x^2 + 2x - 24 = 0$$

$$(x+6)(x-4) = 0$$

Since $x > 0$: $x = 4$.

$x = 4$

Part (ii): Find the area of triangle ABC in exact form.

With $x = 4$: $AB = 4$ cm, $AC = 6$ cm, angle $CAB = 60°$.

$$\text{Area} = \tfrac{1}{2} \cdot AB \cdot AC \cdot \sin(\angle CAB) = \tfrac{1}{2}(4)(6)\sin 60° = 12 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3}$$

Area $= 6\sqrt{3}$ cm^2

End of Worked Solutions