Question 1

Worked Solution

Yacht B is 500 m due north of yacht A; yacht C is 700 m from A on bearing 015°.

The angle at A between AB (north) and AC is 15°, so $\angle BAC = 15°$.

Part (a): Find distance BC.

Using the cosine rule in triangle ABC:

$$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos(\angle BAC)$$

$$= 500^2 + 700^2 - 2(500)(700)\cos 15°$$

$$= 250000 + 490000 - 700000\cos 15°$$

$$= 740000 - 700000(0.96593\ldots)$$

$$= 740000 - 676148\ldots = 63851.9\ldots$$

$$BC = \sqrt{63851.9} = 252.7\ldots \approx 253 \text{ m}$$

$BC = 253$ m (to 3 s.f.)

Part (b): Find the bearing of C from B (i.e. find θ).

Using the sine rule in triangle ABC to find angle B:

$$\frac{\sin B}{AC} = \frac{\sin(\angle BAC)}{BC} \implies \frac{\sin B}{700} = \frac{\sin 15°}{253}$$

$$\sin B = \frac{700\sin 15°}{253} = \frac{700(0.25882\ldots)}{253} = \frac{181.17\ldots}{253} = 0.7161\ldots$$

$$B = \arcsin(0.7161) = 45.76\ldots°$$

Since C is to the east of AB and the triangle is acute (we need the obtuse possibility):

Check: $\angle BAC = 15°$, $\sin B = 0.716$. The obtuse angle would be $B' = 180° - 45.8° = 134.2°$.

With $B = 134.2°$: $\angle BAC + \angle ABC = 15° + 134.2° = 149.2° < 180°$ — consistent. With $B = 45.8°$: $\angle BAC + \angle ABC = 15° + 45.8° = 60.8° < 180°$ — also consistent.

From the diagram, C is further from B than from A ($AC = 700 > AB = 500$), and C is to the right (east) of the north direction from B. The obtuse angle at B fits the geometry of the figure.

$$\angle AB = 134.2°$$

The bearing from B looking south is 180°. The bearing of C from B:

The direction BA is due south (bearing 180°). Angle $\angle ABC = 134.2°$ is measured from BA towards BC (eastward).

$$\theta = \angle ABC - 180° + 180°$$

More carefully: bearing of C from $B = 180° - (180° - 134.2°) = 180° - 45.8° = 134.2°$.

Wait — bearing from B to A is $180°$ (due south). The angle at B between BA and BC is $134.2°$, measured on the east side. So:

$$\theta = 180° - 134.2° = 45.8°$$

$\theta = 45.8°$ (so bearing of C from B is **046**°, to nearest degree, i.e. $\theta \approx 45.8$)

Question 2

Worked Solution

Triangle ABC: $AB = 6$ cm, $BC = 4$ cm, $CA = 5$ cm.

Part (a): Show that $\cos A = \frac{3}{4}$.

Using the cosine rule with the angle at A (i.e. the angle opposite side $BC = 4$):

$$BC^2 = AB^2 + CA^2 - 2 \cdot AB \cdot CA \cdot \cos A$$

$$4^2 = 6^2 + 5^2 - 2(6)(5)\cos A$$
$$16 = 36 + 25 - 60\cos A$$
$$60\cos A = 61 - 16 = 45$$
$$\cos A = \frac{45}{60} = \frac{3}{4} \quad \checkmark$$

Part (b): Find the exact value of $\sin A$.

Using the identity $\sin^2 A + \cos^2 A = 1$:

$$\sin^2 A = 1 - \left(\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16}$$

Since A is an angle in a triangle, $\sin A > 0$:

$$\sin A = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

$\sin A = \frac{\sqrt{7}}{4}$

Question 3

Worked Solution

Triangle ABC: $AB = 16$ cm, $AC = 13$ cm, angle $ABC = 50°$, angle $BCA = x°$.

Find both possible values of x.

Using the sine rule:

$$\frac{\sin x°}{AB} = \frac{\sin(\angle ABC)}{AC} \implies \frac{\sin x}{16} = \frac{\sin 50°}{13}$$

$$\sin x = \frac{16\sin 50°}{13} = \frac{16(0.76604\ldots)}{13} = \frac{12.257\ldots}{13} = 0.9428\ldots$$

$$x = \arcsin(0.9428) = 70.5°$$

The second possible value (sine positive in first and second quadrant):

$$x = 180° - 70.5° = 109.5°$$

Check both are valid triangles:

- $x = 70.5°$: $\angle BAC = 180° - 50° - 70.5° = 59.5° > 0°$ ✓
- $x = 109.5°$: $\angle BAC = 180° - 50° - 109.5° = 20.5° > 0°$ ✓

$x = 70.5°$ and $x = 109.5°$

Question 4

Worked Solution

Triangle ABC: $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, $\angle BAC = 60°$, $\angle ACB = \theta°$.

Part (a)(i): Show that $17x^2 - 35x - 48 = 0$.

Using the cosine rule with the angle at A opposite side BC:

$$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos(\angle BAC)$$

$$(3x + 10)^2 = (x + 2)^2 + (7x)^2 - 2(x + 2)(7x)\cos 60°$$

Using $\cos 60° = \frac{1}{2}$:

$$(3x + 10)^2 = (x + 2)^2 + 49x^2 - (x + 2)(7x)$$

Expand each term:

$$(3x + 10)^2 = 9x^2 + 60x + 100$$
$$(x + 2)^2 = x^2 + 4x + 4$$
$$(x + 2)(7x) = 7x^2 + 14x$$

Substituting:

$$9x^2 + 60x + 100 = x^2 + 4x + 4 + 49x^2 - 7x^2 - 14x$$

$$9x^2 + 60x + 100 = 43x^2 - 10x + 4$$

$$0 = 34x^2 - 70x - 96$$

Dividing by 2:

$$0 = 17x^2 - 35x - 48 \quad \checkmark$$

Part (a)(ii): Find the value of x.

Factorising $17x^2 - 35x - 48 = 0$:

$$(17x + 16)(x - 3) = 0$$

$x = 3$ (taking the positive root since lengths must be positive).

$x = 3$

Part (b): Find θ to 1 d.p.

With $x = 3$: $AB = 5$ cm, $BC = 19$ cm, $AC = 21$ cm.

Using the sine rule:

$$\frac{\sin\theta}{AB} = \frac{\sin(\angle BAC)}{BC} \implies \frac{\sin\theta}{5} = \frac{\sin 60°}{19}$$

$$\sin\theta = \frac{5\sin 60^\circ}{19} = \frac{5 \times \frac{\sqrt{3}}{2}}{19} = \frac{5\sqrt{3}}{38}$$

$$\sin\theta = \frac{5(1.7321\ldots)}{38} = \frac{8.660\ldots}{38} = 0.22791\ldots$$

$$\theta = \arcsin(0.22791) = 13.2^\circ$$

$\theta = 13.2^\circ$

Question 5

Worked Solution

Circle C has centre N and equation $(x-2)^2 + (y+1)^2 = \dfrac{169}{4}$.

Part (a): Coordinates of N.

Comparing with $(x-a)^2 + (y-b)^2 = r^2$:

$N = (2, -1)$

Part (b): Radius of C.

$$r^2 = \frac{169}{4} \implies r = \frac{13}{2} = 6.5$$

$r = 6.5$

Part (c): Find coordinates of A and B.

Chord AB is parallel to the x-axis, below the x-axis, of length 12.

Since $AB \parallel x$-axis, A and B have the same y-coordinate, say y_1.

The midpoint of AB is directly below N (since N is the centre and $AB \parallel x$-axis), so the midpoint has x-coordinate 2.

With $AB = 12$, we have $x_1 = 2 - 6 = -4$ and $x_2 = 2 + 6 = 8$.

To find y_1, use the circle equation with $x = -4$ (or $x = 8$):

$$(-4-2)^2 + (y_1+1)^2 = \frac{169}{4}$$

$$36 + (y_1+1)^2 = \frac{169}{4}$$

$$(y_1+1)^2 = \frac{169}{4} - 36 = \frac{169-144}{4} = \frac{25}{4}$$

$$y_1 + 1 = \pm\frac{5}{2}$$

Since AB is below the x-axis: $y_1 + 1 = -\frac{5}{2}$, so $y_1 = -\frac{5}{2} - 1 = -3.5$.

$A = (-4,\ -3.5)$ and $B = (8,\ -3.5)$

Part (d): Show that angle $ANB = 134.8°$.

Let $\angle ANB = 2\theta$. The perpendicular from N to AB bisects AB, so half-chord $= 6$.

$$\sin\theta = \frac{6}{r} = \frac{6}{6.5} \implies \theta = \arcsin\left(\frac{6}{6.5}\right) = 67.38\ldots°$$

$$\angle ANB = 2\theta = 134.76\ldots° = 134.8° \quad \text{(to nearest 0.1°)}✓$$

Part (e): Find the length AP.

The tangent at A is perpendicular to NA. Tangents from P to A and B are equal; P lies on the perpendicular bisector of AB, so triangle ANP has a right angle at A.

In triangle ANP, angle $ANP = \frac{1}{2}\angle ANB = 67.38\ldots°$.

$$\tan(\angle ANP) = \frac{AP}{AN} \implies AP = AN\tan(\angle ANP) = 6.5\tan(67.38°)$$
$$AP = 6.5 \times 2.399\ldots = 15.6 \text{ (to 3 s.f.)}$$

$AP = 15.6$ (to 3 s.f.)