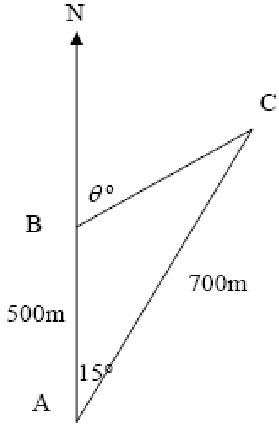
Sine and Cosine Rules and Area of a Triangle Exam Questions Sheet 2 MS

Q1.

Question Number	Scheme	Marks
		
(a)	$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700\cos 15°$ (= 63851.92…) BC = 253 awrt	M1 A1 A1 (3)
(b)	$\frac{\sin B}{700} = \frac{\sin 15}{\text{candidate's } BC}$ $\sin B = \sin 15 \times 700/253_c = 0.716$.. and giving an **obtuse** B (134.2°) dep on 1st M θ = 180° - candidate's angle B (Dep. on first M only, B can be acute) $\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9) [46 needs to be from correct working]	M1 M1 M1 A1 (4) [7]
Notes:	(a) If use cos 15° = ….., then A1 not scored until written as BC^2 = ... correctly *Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC* Finding value for BX and CX and using Pythagoras M1 $BC^2 = (500\sin 15°)^2 + (700 - 500\cos 15°)^2$ A1 BC = 253 awrt A1 (b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M)) (i) $\cos B = \frac{500^2 + \text{candidate's}BC^2 - 700^2}{2x500x\text{candidate's}BC}$ or $700^2 = 500^2 + BC_c^2 - 2x500xBC_c$ M1 Finding angle B M1 dep., then M1 as above (ii) 2 triangle approach, as defined in notes for (a) $\tan CBX = \frac{700 - valueforAX}{valueforBX}$ M1 Finding value for $\angle CBX$ ($\approx 59°$) dep M1 $\theta = [180° - (75° + candidate's\ \angle CBX)]$ M1 (iii) Using sine rule (or cos rule) to find C first: Correct use of sine or cos rule for C M1, Finding value for C M1 Either B =180° – (15° + candidate's C) or θ = (15° + candidate's C) M1 (iv) $700\cos 15° = 500 + BC\cos\theta$ M2 {first two Ms earned in this case} Solving for θ; θ = 45.8 (allow 46 or5.7, 45.8, 45.9) M1;A1 Note: S.C. In main scheme, if θ used in place of B, third M gained immediately; Other two marks likely to be earned, too, for correct value of θ stated.	

Q2.

Question number	Scheme	Marks
	(a) $4^2 = 5^2 + 6^2 - (2\times5\times6\cos\theta)$	M1
	$\cos\theta = \dfrac{5^2+6^2-4^2}{2\times5\times6}$	A1
	$\left(=\dfrac{45}{60}\right) = \dfrac{3}{4}$ (*)	A1cso (3)
	(b) $\sin^2 A + \left(\dfrac{3}{4}\right)^2 = 1$ (or equiv. Pythag. method)	M1
	$\left(\sin^2 A = \dfrac{7}{16}\right)$ $\sin A = \dfrac{1}{4}\sqrt{7}$ or equivalent exact form, e.g. $\sqrt{\dfrac{7}{16}}$, $\sqrt{0.4375}$	A1 (2)
		5
	(a) M: Is also scored for $5^2 = 4^2 + 6^2 - (2\times4\times6\cos\theta)$ or $6^2 = 5^2 + 4^2 - (2\times5\times4\cos\theta)$ or $\cos\theta = \dfrac{4^2+6^2-5^2}{2\times4\times6}$ or $\cos\theta = \dfrac{5^2+4^2-6^2}{2\times5\times4}$. 1st A: Rearranged correctly and numerically correct (possibly unsimplified), in the form $\cos\theta = ...$ or $60\cos\theta = 45$ (or equiv. in the form $p\cos\theta = q$). <u>Alternative</u> (verification): $4^2 = 5^2 + 6^2 - \left(2\times5\times6\times\dfrac{3}{4}\right)$ [M1] Evaluate correctly, at least to $16 = 25 + 36 - 45$ [A1] Conclusion (perhaps as simple as a tick). [A1cso] (Just achieving $16 = 16$ is insufficient without at least a tick). (b) M: Using a correct method to find an equation in $\sin^2 A$ or $\sin A$ which would give an exact value. <u>Correct answer without working</u> (or with unclear working or decimals): Still scores both marks.	

Q3.

Question Number	Scheme	Marks
	$\frac{\sin x}{16} = \frac{\sin 50^0}{13}$	M1
	$(\sin x) = \frac{16\times\sin 50}{13}$ $(= 0.943 \text{ but accept } 0.94)$	A1
	$x =$ awrt 70.5(3) **and** 109.5 **or** 70.6 and 109.4	dM1 A1 **(4)** **[4]**
	Notes M1: use sine formula correctly **in any form.** Allow awrt 0.77 for $\sin 50°$ A1: give the correct value or correct expression for $\sin x$ (this implies the M1 mark). If it is given as expression they do not need degrees symbol. $\frac{\sin 50\times 16}{13}$ is fine, If this is given as a decimal allow answers which round to 0.94. Allow awrt −0.323 (radians) here but no further marks are available. If they give this as x (not sin$x$) and do not recover this is A0 dM1: Correct work leading to x=… (via inverse sin) expression or value for sin$x$ If the previous A mark has been awarded for a correct expression then this is for getting to awrt 70.5 or 109.5 (allow for 70.6 or 109.4). If the previous A mark was not gained, e.g. rounding errors were made in rearranging the correct sine formula then award dM1 for evidence of use of **inverse sin in degrees** on their value for sin$x$ (may need to check on calculator). **NB** 70.5 following a correct sine formula will gain M1A1M1. A1: deduce and state **both of the answers** $x = 70.5$ and 109.5 (do not need degrees) Accept awrt these. Also accept 70.6 and 109.4. (Second answer is sometimes obtained by a long indirect route but still scores A1) If working in radians throughout, answers are 1.23 and 1.91 and this can be awarded M1 A1 M1 A0 (Working with 50 radians gives probable answers of -0.3288 and 3.47 giving M1A1M0A0) **Special case:** Wrong labelling of triangle. This simplifies the problem as there is only one solution for angle x. So it is not treated as a misread. If they find the missing side as awrt 12.6 then proceed to find an angle or its sine or cosine then give M1A0M0A0	
	Alternative Method using cosine rule Let $BC = a$. M1: uses the cosine rule to form to form a three term quadratic equation in a (e.g. $a^2 - 32a\cos 50^\circ + 87 = 0$ or $a^2 - \text{awrt}\,20.6a + 87 = 0$ though allow slips in signs rearranging) A1: Solves and obtains a correct value for a of awrt 14.6 **or** awrt 5.95. dM1: A **correct full** method to find (at least) one of the two angles. May use cosine rule again, or find angle BAC and then use sine rule. As in the main scheme, if the previous A mark has been awarded then they should obtain one of the correct angles for this mark. A1: deduces both correct answer as in main scheme. **NB** obtaining only one correct angle will usually score M1A1M1A0 in any method.	

Q4.

Question	Scheme	Marks	AOs
(a)(i)	$(3x+10)^2=(x+2)^2+(7x)^2-2(x+2)(7x)\cos 60°$ oe	M1	3.1a
	Uses cos 60° = ½, expands the brackets and proceeds to a 3 term quadratic equation	dM1	1.1b
	$17x^2-35x-48=0$ *	A1*	2.1
		(3)	
(ii)	$x=3$	B1	3.2a
		(1)	
(b)	$\dfrac{5}{\sin ACB}=\dfrac{19}{\sin 60°}\Rightarrow \sin ACB=...\left(\dfrac{5\sqrt{3}}{38}\right)$ or e.g. $5^2=21^2+19^2-2\times 19\times 21\cos ACB\Rightarrow \cos ACB=...\left(\dfrac{37}{38}\right)$	M1	1.1b
	$\theta=$ awrt 13.2	A1	1.1b
		(2)	
		(6 marks)	

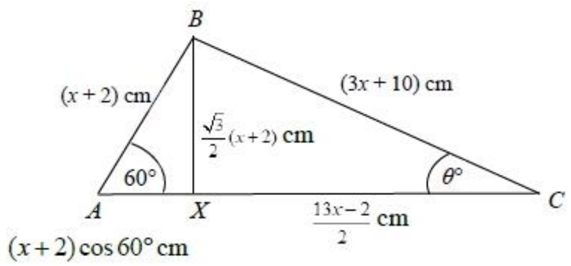
Notes

(a)(i) Mark (a) and (b) together

M1: Recognises the need to apply the cosine rule and attempts to use it with the sides in the correct positions and the formula applied correctly. Condone invisible brackets and slips on $3x+10$ as $3x-10$.
Alternatively, uses trigonometry to find *AX* and then equates two expressions for the length *BX*. You may see variations of this if they use Pythagoras or trigonometry to find *BX* and then apply Pythagoras to the triangle *BXC*. See the diagram below to help you.
The angles and lengths must be in the correct positions. Cos 60 may be $\frac{1}{2}$ from the start

dM1: Uses cos 60° = ½, expands the brackets and proceeds to a 3TQ. You may see the use of cos 60° = ½ in earlier work, but they must proceed to a 3TQ as well to score this mark. It is dependent on the first method mark.

A1*: Obtains the correct quadratic equation with the = 0 with no errors seen in the main body of their solution. Condone the recovery of invisible brackets as long as the intention is clear. You do not need to explicitly see cos 60 to score full marks.



(a)(ii)

B1: Selects the appropriate value i.e. $x=3$ only. The other root must **either** be rejected if found **or** $x=3$ must be the only root used in part (b). Can be implied by awrt 13.2 in (b)

(b)

M1: Using their value for x this mark is for either:

- applying the sine rule correctly (or considers 2 right angled triangles) and proceeding to obtain a value for $\sin ACB$ or
- applying the cosine rule correctly and proceeding to obtain a value for $\cos ACB$.

Condone slips calculating the lengths AB, BC and AC. At least one of them should be found correctly for their value for x

(Also allow if the sine rule or cosine rule is applied correctly to find a value for $\sin ABC$ $\left(=\frac{21\sqrt{3}}{38}\right)$ or $\cos ACB\left(=-\frac{11}{38}\right)$)

A1: awrt 13.2 (answers with little working eg just lengths on the diagram can score M1A1)

Q5.

Question Number	Scheme	Marks
(a)	$N(2, -1)$	B1, B1 (2)
(b)	$r=\sqrt{\frac{169}{4}}=\frac{13}{2}=6.5$	B1 (1)
(c)	Complete Method to find x coordinates, $x_2-x_1=12$ and $\frac{x_1+x_2}{2}=2$ then solve	M1
	To obtain $x_1=-4$, $x_2=8$	A1ft A1ft
	Complete Method to find y coordinates, using equation of circle or Pythagoras i.e. let d be the distance below N of A *then* $d^2=6.5^2-6^2 \Rightarrow d=2.5 \Rightarrow y=..$	M1
	So $y_2=y_1=-3.5$	A1 (5)
(d)	Let $A\hat{N}B=2\theta \Rightarrow \sin\theta=\frac{6}{"6.5"} \Rightarrow \theta=(67.38)...$	M1
	So angle ANB is 134.8 *	A1 (2)
(e)	AP is perpendicular to AN so using triangle ANP $\tan\theta=\frac{AP}{"6.5"}$	M1
	Therefore $AP=15.6$	A1cao (2)
		[12]
(a)	B1 for 2 (α), B1 for -1	
(b)	B1 for 6.5 o.e.	
(c)	1st M1 for finding x coordinates – may be awarded if either x co-ord is correct A1ft,A1ft are for $\alpha-6$ and $\alpha+6$ if x coordinate of N is α 2nd M1 for a method to find y coordinates – may be given if y co-ordinate is correct A marks is for -3.5 only.	
(d)	M1 for a full method to find θ or angle ANB (eg sine rule or cosine rule directly or finding another angle and using angles of triangle.) **ft their 6.5 from radius or wrong *y*.** (cos $ANB=\frac{"6.5"^2+"6.5"^2-12^2}{2\times"6.5"\times"6.5"}$=-0.704) A1 is a printed answer and must be 134.8 – do not accept 134.76.	
(e)	M1 for a full method to find AP <u>Alternative Methods</u> N.B. May use triangle AXP where X is the mid point of AB. Or may use triangle ABP. From circle theorems may use angle $BAP=67.38$ or some variation. Eg $\frac{AP}{\sin 67.4}=\frac{12}{\sin 45.2}$, $AP=\frac{6}{\sin 22.6}$ or $AP=\frac{6}{\cos 67.4}$ are each worth M1	