## Question 1 (Jun 2010, Q7)

Solve the simultaneous equations

$$x + 2y - 6 = 0, \qquad 2x^2 + y^2 = 57$$

**Worked Solution**

From the linear equation: $x = 6 - 2y$.

Substitute into $2x^2 + y^2 = 57$:

$$2(6 - 2y)^2 + y^2 = 57$$

$$2(36 - 24y + 4y^2) + y^2 = 57$$
$$72 - 48y + 8y^2 + y^2 = 57$$
$$9y^2 - 48y + 15 = 0$$
$$3y^2 - 16y + 5 = 0$$
$$(3y - 1)(y - 5) = 0$$
$$y = \tfrac{1}{3} \quad \text{or} \quad y = 5$$

Back-substitute $x = 6 - 2y$:

$$y = \tfrac{1}{3} \Rightarrow x = \tfrac{16}{3}; \qquad y = 5 \Rightarrow x = -4$$

$$x = \frac{16}{3},\ y = \frac{1}{3} \quad \text{and} \quad x = -4,\ y = 5$$

## Question 2 (Jun 2011, Q4)

Solve the simultaneous equations

$$y = 2(x-2)^2, \qquad 3x + y = 26$$

**Worked Solution**

From the linear equation: $y = 26 - 3x$.

Substitute into $y = 2(x-2)^2$:

$$26 - 3x = 2(x-2)^2$$

$$26 - 3x = 2(x^2 - 4x + 4)$$
$$26 - 3x = 2x^2 - 8x + 8$$
$$2x^2 - 5x - 18 = 0$$
$$(2x - 9)(x + 2) = 0$$
$$x = \tfrac{9}{2} \quad \text{or} \quad x = -2$$

Back-substitute $y = 26 - 3x$:

$$x = \tfrac{9}{2} \Rightarrow y = 26 - \tfrac{27}{2} = \tfrac{25}{2}; \qquad x = -2 \Rightarrow y = 32$$

$$x = \frac{9}{2},\ y = \frac{25}{2} \quad \text{and} \quad x = -2,\ y = 32$$

## Question 3 (Jan 2013, Q4)

**(i)** Solve the simultaneous equations

$$y = 2x^2 - 3x - 5, \qquad 10x + 2y + 11 = 0$$

**(ii)** What can you deduce about the curve $y = 2x^2 - 3x - 5$ and the line $10x + 2y + 11 = 0$?

### Worked Solution

**Part (i):**

From the linear equation: $y = \dfrac{-10x - 11}{2}$.

Substitute into $y = 2x^2 - 3x - 5$:

$$2x^2 - 3x - 5 = \frac{-10x - 11}{2}$$

$$4x^2 - 6x - 10 = -10x - 11$$

$$4x^2 + 4x + 1 = 0$$

$$(2x + 1)^2 = 0$$

$$x = -\tfrac{1}{2} \quad \text{(repeated root)}$$

Back-substitute: $y = \dfrac{-10(-\frac{1}{2}) - 11}{2} = \dfrac{5 - 11}{2} = -3$

$x = -\dfrac{1}{2}, \; y = -3$

**Part (ii):**

Since the quadratic has a repeated root, the line is a **tangent** to the curve.

## Question 4 (Jun 2015, Q6)

Solve the simultaneous equations

$$2x + y - 5 = 0, \qquad x^2 - y^2 = 3$$

**Worked Solution**

From the linear equation: $y = 5 - 2x$.

Substitute into $x^2 - y^2 = 3$:

$$x^2 - (5 - 2x)^2 = 3$$
$$x^2 - (25 - 20x + 4x^2) = 3$$
$$x^2 - 25 + 20x - 4x^2 = 3$$
$$-3x^2 + 20x - 28 = 0$$
$$3x^2 - 20x + 28 = 0$$
$$(3x - 14)(x - 2) = 0$$
$$x = \tfrac{14}{3} \quad \text{or} \quad x = 2$$

Back-substitute $y = 5 - 2x$:

$$x = \tfrac{14}{3} \Rightarrow y = 5 - \tfrac{28}{3} = -\tfrac{13}{3}; \qquad x = 2 \Rightarrow y = 1$$

$$x = \frac{14}{3},\ y = -\frac{13}{3} \quad \text{and} \quad x = 2,\ y = 1$$

## Question 5 (Jun 2016, Q3)

Solve the simultaneous equations

$$x^2 + y^2 = 34, \qquad 3x - y + 4 = 0$$

**Worked Solution**

From the linear equation: $y = 3x + 4$.

Substitute into $x^2 + y^2 = 34$:

$$x^2 + (3x + 4)^2 = 34$$

$$x^2 + 9x^2 + 24x + 16 = 34$$

$$10x^2 + 24x - 18 = 0$$

$$5x^2 + 12x - 9 = 0$$

$$(5x - 3)(x + 3) = 0$$

$$x = \tfrac{3}{5} \quad \text{or} \quad x = -3$$

Back-substitute $y = 3x + 4$:

$$x = \tfrac{3}{5} \Rightarrow y = \tfrac{9}{5} + 4 = \tfrac{29}{5}; \qquad x = -3 \Rightarrow y = -5$$

$$x = \frac{3}{5},\ y = \frac{29}{5} \quad \text{and} \quad x = -3,\ y = -5$$

End of Worked Solutions