Question 1

Solve the simultaneous equations

$$x + y = 2, \qquad 4y^2 - x^2 = 11$$

Worked Solution

From the linear equation: $y = 2 - x$.

Substitute into the quadratic:

$$4(2 - x)^2 - x^2 = 11$$

$$4(4 - 4x + x^2) - x^2 = 11$$

$$16 - 16x + 4x^2 - x^2 = 11$$

$$3x^2 - 16x + 5 = 0$$

$$(3x - 1)(x - 5) = 0$$

$$x = \tfrac{1}{3} \quad \text{or} \quad x = 5$$

Back-substitute $y = 2 - x$:

$$x = \tfrac{1}{3} \Rightarrow y = \tfrac{5}{3}; \qquad x = 5 \Rightarrow y = -3$$

$$x = \frac{1}{3},\ y = \frac{5}{3} \quad \text{and} \quad x = 5,\ y = -3$$

Question 2

Solve the simultaneous equations

$$y - 3x + 2 = 0, \qquad y^2 - x - 6x^2 = 0$$

Worked Solution

From the linear equation: $y = 3x - 2$.

Substitute into the quadratic:

$$(3x - 2)^2 - x - 6x^2 = 0$$

$$9x^2 - 12x + 4 - x - 6x^2 = 0$$

$$3x^2 - 13x + 4 = 0$$

$$(3x - 1)(x - 4) = 0$$

$$x = \tfrac{1}{3} \quad \text{or} \quad x = 4$$

Back-substitute $y = 3x - 2$:

$$x = \tfrac{1}{3} \Rightarrow y = -1; \qquad x = 4 \Rightarrow y = 10$$

$$x = \frac{1}{3},\ y = -1 \quad \text{and} \quad x = 4,\ y = 10$$

Question 3

Solve the simultaneous equations

$$y + 4x + 1 = 0, \qquad y^2 + 5x^2 + 2x = 0$$

Worked Solution

From the linear equation: $y = -4x - 1$.

Substitute into the quadratic:

$$(-4x - 1)^2 + 5x^2 + 2x = 0$$

$$16x^2 + 8x + 1 + 5x^2 + 2x = 0$$

$$21x^2 + 10x + 1 = 0$$

$$(7x + 1)(3x + 1) = 0$$

$$x = -\tfrac{1}{7} \quad \text{or} \quad x = -\tfrac{1}{3}$$

Back-substitute $y = -4x - 1$:

$$x = -\tfrac{1}{7} \Rightarrow y = \tfrac{4}{7} - 1 = -\tfrac{3}{7}; \qquad x = -\tfrac{1}{3} \Rightarrow y = \tfrac{4}{3} - 1 = \tfrac{1}{3}$$

$$x = -\frac{1}{7},\ y = -\frac{3}{7} \quad \text{and} \quad x = -\frac{1}{3},\ y = \frac{1}{3}$$

Question 4

Solve the simultaneous equations

$$y - 2x - 4 = 0, \qquad 4x^2 + y^2 + 20x = 0$$

Worked Solution

From the linear equation: $y = 2x + 4$.

Substitute into the quadratic:

$$4x^2 + (2x + 4)^2 + 20x = 0$$

$$4x^2 + 4x^2 + 16x + 16 + 20x = 0$$

$$8x^2 + 36x + 16 = 0$$

$$2x^2 + 9x + 4 = 0$$

$$(2x + 1)(x + 4) = 0$$

$$x = -\tfrac{1}{2} \quad \text{or} \quad x = -4$$

Back-substitute $y = 2x + 4$:

$$x = -\tfrac{1}{2} \Rightarrow y = 3; \qquad x = -4 \Rightarrow y = -4$$

$$x = -\frac{1}{2},\ y = 3 \quad \text{and} \quad x = -4,\ y = -4$$

Question 5

(a) By eliminating y from

$$y = x - 4, \qquad 2x^2 - xy = 8,$$

show that $x^2 + 4x - 8 = 0$.

(b) Hence solve the simultaneous equations, giving answers in the form $a \pm b\sqrt{3}$.

Worked Solution

Part (a):

Substitute $y = x - 4$ into $2x^2 - xy = 8$:

$$2x^2 - x(x - 4) = 8$$

$$2x^2 - x^2 + 4x = 8$$

$$x^2 + 4x - 8 = 0 \qquad \checkmark$$

Part (b):

Apply the quadratic formula to $x^2 + 4x - 8 = 0$:

$$x = \frac{-4 \pm \sqrt{16 + 32}}{2} = \frac{-4 \pm \sqrt{48}}{2} = \frac{-4 \pm 4\sqrt{3}}{2} = -2 \pm 2\sqrt{3}$$

Find y using $y = x - 4$:

$$y = (-2 \pm 2\sqrt{3}) - 4 = -6 \pm 2\sqrt{3}$$

$x = -2 + 2\sqrt{3},\ y = -6 + 2\sqrt{3}$ and $x = -2 - 2\sqrt{3},\ y = -6 - 2\sqrt{3}$

Question 6

The line $y = x + 2$ meets the curve $x^2 + 4y^2 - 2x = 35$ at points A and B.

(a) Find the coordinates of A and B.

(b) Find the distance AB in the form $r\sqrt{2}$ where r is rational.

Worked Solution

Part (a):

Substitute $y = x + 2$ into $x^2 + 4y^2 - 2x = 35$:

$$x^2 + 4(x+2)^2 - 2x = 35$$

$$x^2 + 4(x^2 + 4x + 4) - 2x = 35$$

$$x^2 + 4x^2 + 16x + 16 - 2x = 35$$

$$5x^2 + 14x - 19 = 0$$

$$(5x + 19)(x - 1) = 0$$

$$x = -\tfrac{19}{5} \quad \text{or} \quad x = 1$$

Using $y = x + 2$:

$$x = -\tfrac{19}{5} \Rightarrow y = -\tfrac{9}{5}; \qquad x = 1 \Rightarrow y = 3$$

$$A = (1,\ 3) \quad \text{and} \quad B = \left(-\frac{19}{5},\ -\frac{9}{5}\right)$$

Part (b):

$$d^2 = \left(1 - \left(-\tfrac{19}{5}\right)\right)^2 + \left(3 - \left(-\tfrac{9}{5}\right)\right)^2 = \left(\tfrac{24}{5}\right)^2 + \left(\tfrac{24}{5}\right)^2 = 2 \times \left(\tfrac{24}{5}\right)^2$$

$$d = \tfrac{24}{5}\sqrt{2}$$

$$AB = \frac{24}{5}\sqrt{2}$$