Question 1 (Jun 2006, Q12)

Worked Solution

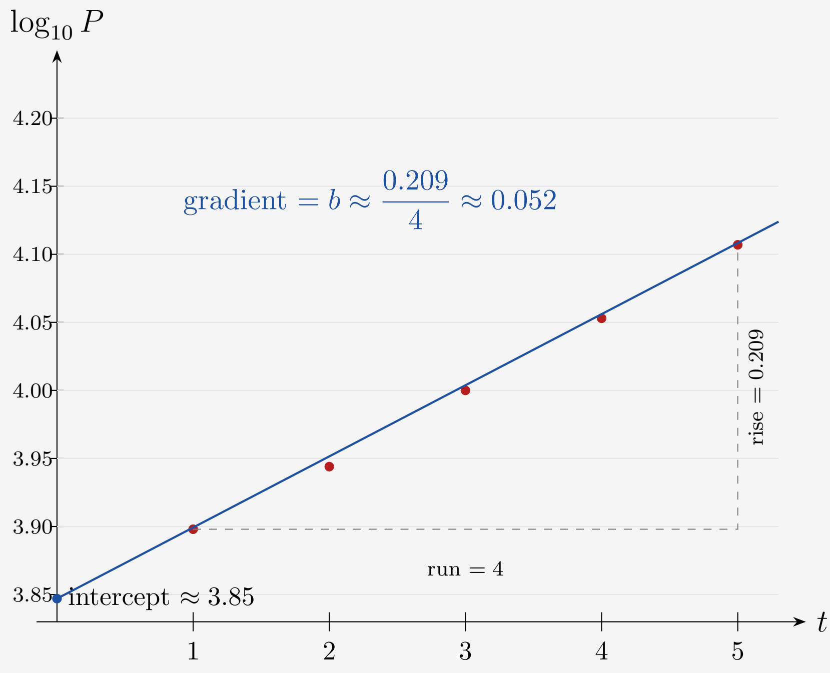
Model: $P = a \times 10^{bt}$.

Part (i): $\log_{10} P = \log_{10} a + bt$. This is linear in t with gradient b and intercept $\log_{10} a$.

Part (ii) & (iii): The $\log_{10} P$ values are:

t	1	2	3	4	5
P	7900	8800	10000	11300	12800
$\log_{10} P$	3.898	3.944	4.000	4.053	4.107

The graph below shows these five points with a line of best fit drawn through them.



$$b \approx 0.052, \qquad \log_{10} a \approx 3.85 \implies a \approx 10^{3.85} \approx 7080.$$

$P \approx 7000 \times 10^{0.05t}$ (accept a between 6760 and 7245; b between 0.04 and 0.06)

Part (iv): $t = 8$: $P = 7080 \times 10^{0.052\times 8} = 7080 \times 10^{0.416} \approx 7080 \times 2.605 \approx 18\,440$.

Predicted population in 2008: $\approx 17\,000$ to $18\,500$

Question 2 (Jun 2008, Q13)

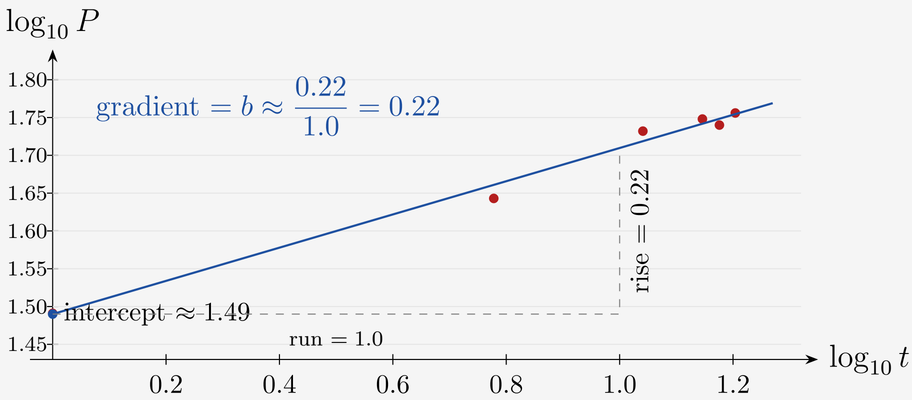
Worked Solution

Model: $P = at^b$ (cinema attendance, t = years after 1985/86).

Part (i): $\log_{10} P = \log_{10} a + b \log_{10} t$ — straight line vs $\log_{10} t$, gradient b, intercept $\log_{10} a$.

Part (ii) & (iii): Completed table and graph:

t	1	6	11	14	15	16
P	31	44	54	56	55	57
$\log_{10} t$	0	0.778	1.041	1.146	1.176	1.204
$\log_{10} P$	1.491	1.643	1.732	1.748	1.740	1.756



$b \approx 0.22$; $a = 10^{1.49} \approx 31$.

$P \approx 31\, t^{0.22}$ (accept b between 0.22 and 0.23; a between 30 and 32)

Part (iv): $t = 22$: $P = 31 \times 22^{0.22} \approx 31 \times 1.97 \approx 61\%$.

Predicted ≈ 60 to 63% of the adult population

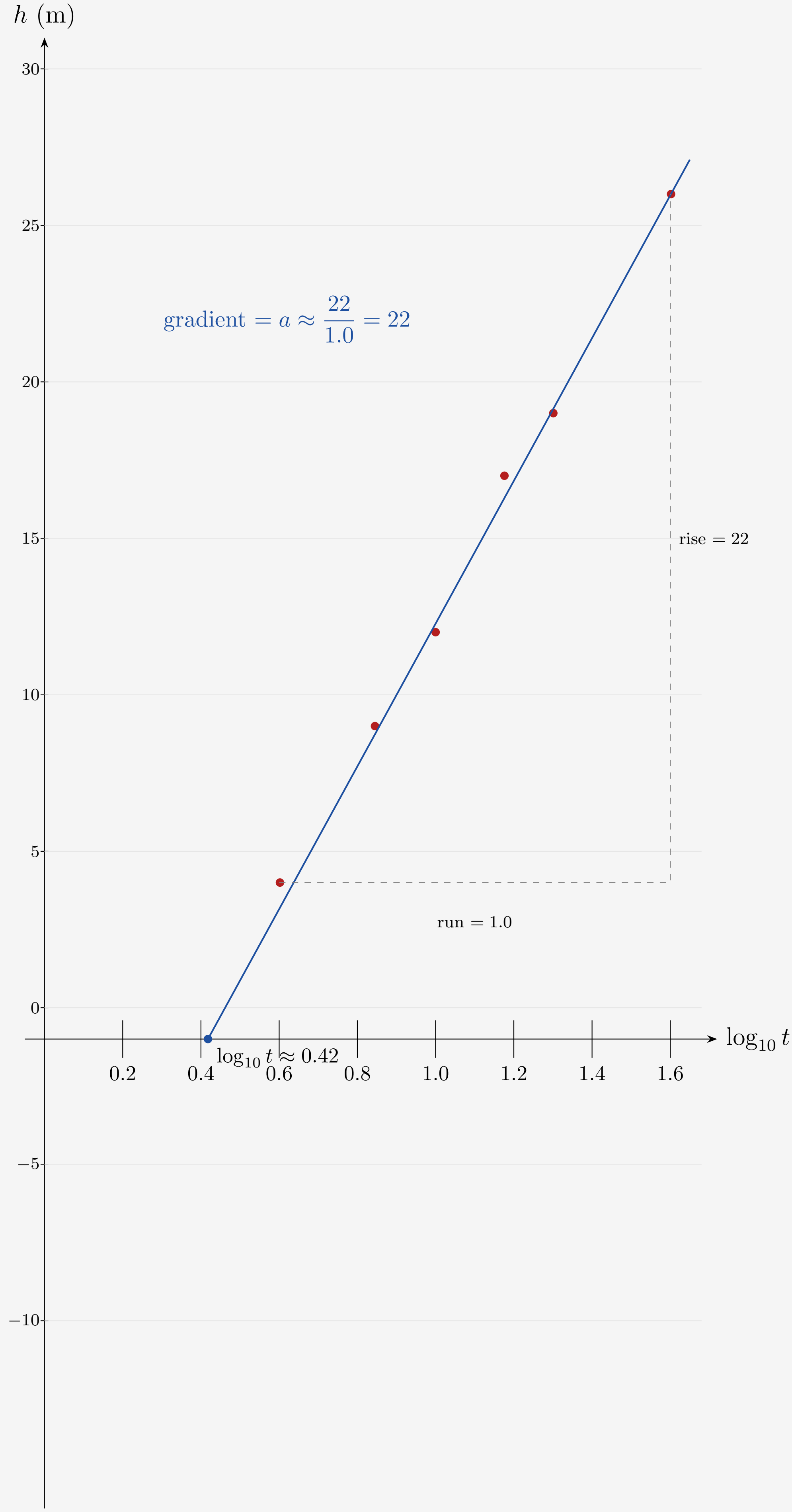
Question 3 (Jun 2009, Q10)

Worked Solution

Model: $h = a\log_{10} t + b$ (ash tree height).

Part (i): Completed table:

t (years)	4	7	10	15	20	40
$\log_{10} t$	0.602	0.845	1.000	1.176	1.301	1.602
h (m)	4	9	12	17	19	26


h (m)
30
25
20
15
10
5
0
−5
−10
gradient = a ≈ 22/1.0 = 22
rise = 22
run = 1.0
log10 t ≈ 0.42
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
log10 t

Gradient $a \approx 22$; intercept: $b = 4 - 22 \times 0.6 \approx -9.2$.

Part (ii): $h \approx 22 \log_{10} t - 9$.

$h \approx 22 \log_{10} t - 9$ (accept a between 21 and 23.5; b between -11 and -8)

Part (iii): $t = 100$: $h = 22 \log_{10} 100 - 9 = 22 \times 2 - 9 = 35\,\text{m}$.

$h \approx 35\,\text{m}$ at age 100 years

Part (iv): $29 = 22 \log_{10} t - 9 \Rightarrow \log_{10} t = \frac{38}{22} \approx 1.727 \Rightarrow t \approx 53$ years.

$t \approx 55$ years (accept 53 to 59 depending on line of best fit)

Part (v): For small t, the model predicts $h \leq 0$ (zero at $t \approx 2.75$ years), which is impossible for a real tree.

For very young trees, the model predicts zero or negative height — unsuitable.

Question 4 (Jun 2014, Q13)

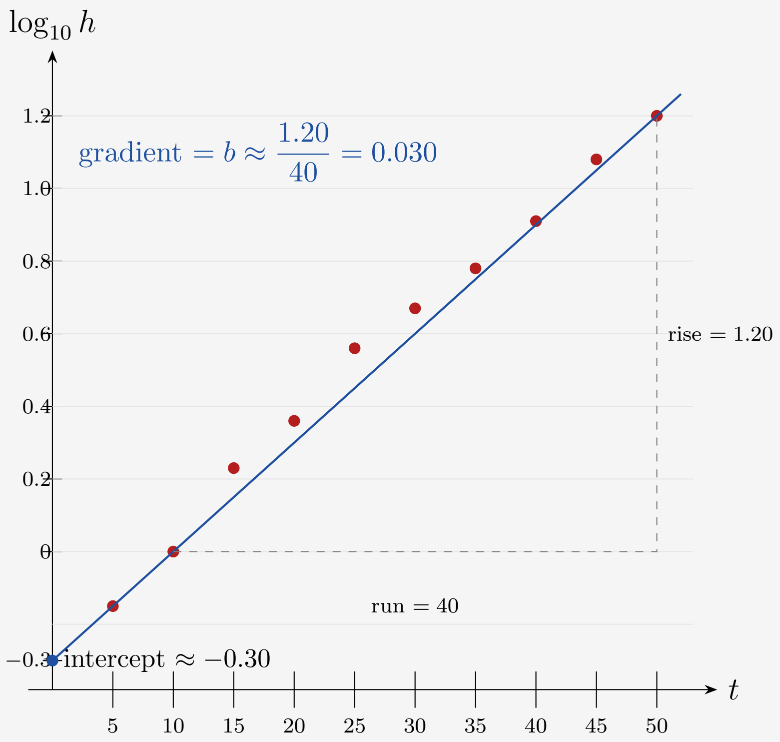
Worked Solution

Model: $h = a \times 10^{bt}$ (glacier thickness reduction).

Part (i): $\log_{10} h = bt + \log_{10} a$, so $m = b$ and $c = \log_{10} a$.

Part (ii) & (iii): Completed table:

t	5	10	15	20	25	30	35	40	45	50
h	0.7	1.0	1.7	2.3	3.6	4.7	6.0	8.2	12	15.9
$\log_{10} h$	−0.15	0.00	0.23	0.36	0.56	0.67	0.78	0.91	1.08	1.20



$b \approx 0.030$; $\log_{10} a \approx -0.30 \Rightarrow a \approx 10^{-0.30} \approx 0.50$.

$h \approx 0.50 \times 10^{0.030t}$ (accept a between 0.50 and 0.603; b between 0.028 and 0.032)

Part (iv): Reduction 2010 to 2020 ($t = 50$ to $t = 60$): $h(60) - h(50) = 0.50 \times 10^{1.8} - 0.50 \times 10^{1.5} \approx 31.6 - 15.8 \approx 15.8\,\text{m}$.

Reduction in thickness ≈ 8 to 26 m (depending on line of best fit values)

Part (v): The model predicts the glacier keeps thinning indefinitely — eventually past the point where it has fully melted, which is physically impossible.

The model predicts continuous reduction in thickness even after the glacier would have fully melted.

Question 5 (Jun 2016, Q11)

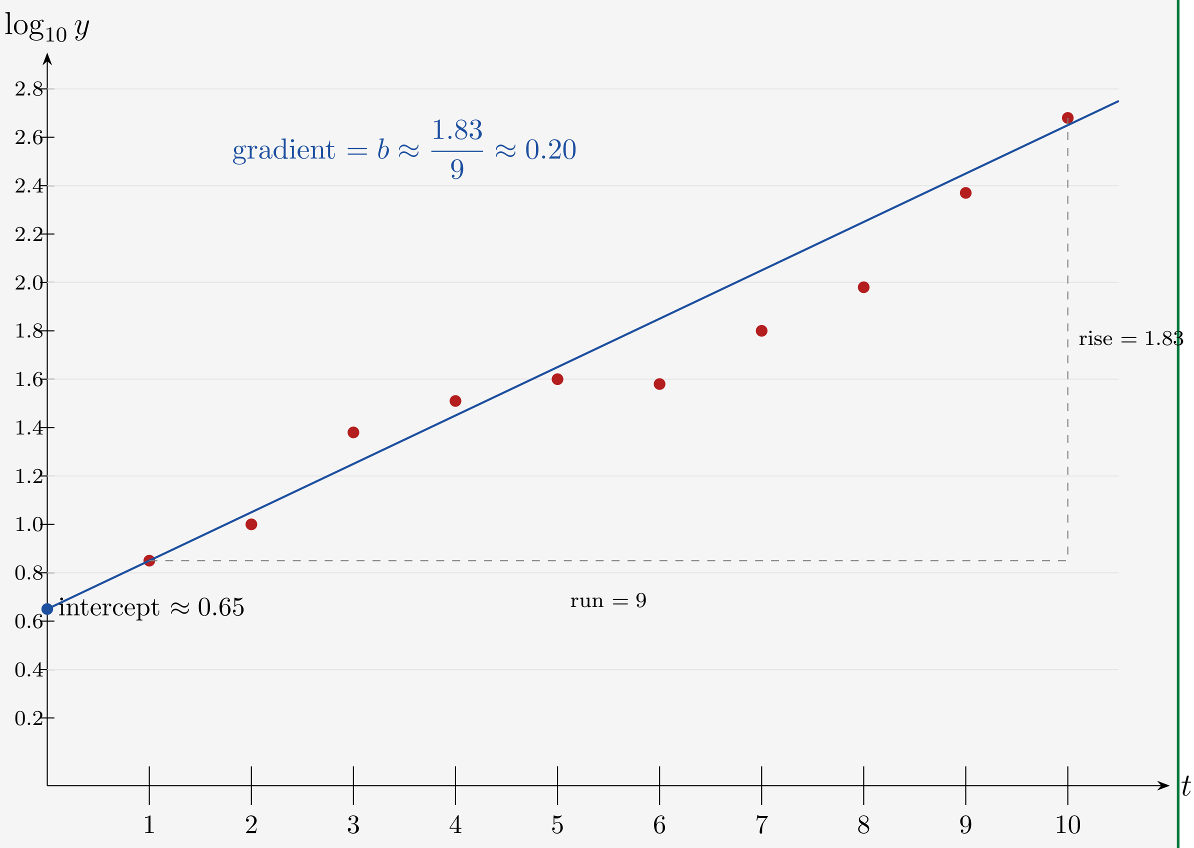
Worked Solution

Model: $y = a \times 10^{bt}$ (flu viruses).

Part (i): $\log_{10} y = \log_{10} a + bt$ — straight line in t, gradient b, intercept $\log_{10} a$.

Part (ii): Completed $\log_{10} y$ values and line of best fit:

t	1	2	3	4	5	6	7	8	9	10
y	7	10	24	32	40	38	63	96	234	480
$\log_{10} y$	0.85	1.00	1.38	1.51	1.60	1.58	1.80	1.98	2.37	2.68



$b \approx 0.20$; $\log_{10} a \approx 0.65 \Rightarrow a \approx 10^{0.65} \approx 4.5$.

$y \approx 4.5 \times 10^{0.20t}$ (accept b between 0.14 and 0.24; $\log_{10} a$ between 0.4 and 0.8)

Part (iii): Decline model $y = 921 \times 10^{-0.137w}$ at $w = 4$: $y = 921 \times 10^{-0.548} \approx 921 \times 0.283 \approx 261$.

$y \approx 260$ or 261 flu viruses detected in week 4 of the decline

Question 6 (Jan 2006, Q9)

Worked Solution

$\log_{10} y$ against x is a straight line through $(0, 3)$ and $(4, 5)$.

Gradient $= \dfrac{5-3}{4-0} = 0.5$; intercept $= 3$.

Part (i):

$$\log_{10} y = 0.5x + 3$$

Part (ii): $y = 10^{0.5x+3} = 10^3 \times 10^{0.5x} = 1000 \times 10^{0.5x}$.

$$y = 1000 \times 10^{0.5x}$$

Question 7 (Jun 2012, Q7)

Worked Solution

$\log_{10} y$ against $\log_{10} x$ is a straight line through $(1, 5)$ and $(5, 17)$.

Gradient $= \dfrac{17-5}{5-1} = \dfrac{12}{4} = 3$.

Using $(1, 5)$: $\log_{10} y - 5 = 3(\log_{10} x - 1) \Rightarrow \log_{10} y = 3\log_{10} x + 2 = \log_{10} x^3 + \log_{10} 100$.

$y = 100x^3$

Question 8 (Jun 2015, Q8)

Worked Solution

$\log_{10} y$ against $\log_{10} x$ is a straight line through $(0, 2)$ and $(2, 8)$.

Gradient $= \dfrac{8-2}{2-0} = 3$.

$\log_{10} y = 3\log_{10} x + 2 = \log_{10}(100x^3)$.

$\log_{10} y = 3\log_{10} x + 2, \quad \text{hence } y = 100x^3$

End of Worked Solutions