Question 1

Worked Solution

Model: $d = kV^n$.

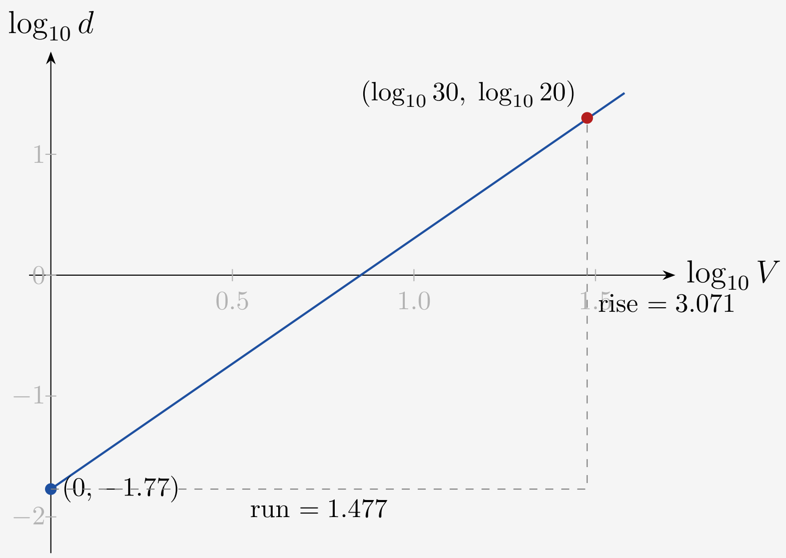
Part (a): Take $\log_{10}$ of $d = kV^n$:

$$\log_{10} d = \log_{10} k + n \log_{10} V.$$

This is linear in $\log_{10} V$, so Figure 6 being a straight line confirms the power-law model. The intercept is -1.77:

$$\log_{10} k = -1.77 \implies k = 10^{-1.77} \approx 0.017. \quad \square$$

Part (b): The line in Figure 6 passes through the intercept $(0, -1.77)$ and through the data point $(\log_{10} 30, \log_{10} 20) = (1.477, 1.301)$:



$$n = \frac{\log_{10} 20 - (-1.77)}{\log_{10} 30 - 0} = \frac{1.301 + 1.77}{1.477} = \frac{3.071}{1.477} \approx 2.08.$$

$n \approx 2.08$ (to 3 s.f.), giving $d = 0.017\,V^{2.08}$

Part (c): Reaction distance at 60 km/h for 0.8 s: $\frac{60}{3.6} \times 0.8 \approx 13.3$ m. Braking distance: $0.017 \times 60^{2.08} \approx 84.9$ m. Total ≈ 98.3 m < 100 m.

Total stopping distance ≈ 98.3 m < 100 m — Sean **will stop** before the puddle.

Question 2

Worked Solution

Model: $h = pm^q$. Line: y-intercept 2.25, gradient -0.235.

Part (a): $\log_{10} h = \log_{10} p + q \log_{10} m$, so $q = -0.235$ and $p = 10^{2.25} \approx 178$.

$p = 178$ (to 3 s.f.), $q = -0.235$

Part (b): $h = 178 \times 5^{-0.235} \approx 122$ bpm; actual is 119 bpm. Only 3 bpm out — the model is reasonably suitable.

Model predicts ≈ 122 bpm vs actual 119 bpm. Close (2.5% error) — reasonably suitable.

Part (c): When $m = 1$: $h = p \times 1 = p$.

p is the resting heart rate (bpm) of a mammal with mass 1 kg.

Question 3

Worked Solution

Model: $V = pq^t$. Given line: $\log_{10} V = 0.05t + 4.8$.

Part (a): $\log_{10} q = 0.05 \Rightarrow q = 10^{0.05} \approx 1.122$; $\log_{10} p = 4.8 \Rightarrow p = 10^{4.8} \approx 63100$.

$p = 63100$ (to 4 s.f.), $q = 1.122$ (to 4 s.f.)

Part (b)(i): $p \approx £63\,100$ is the initial value of the painting on 1st January 1980.

Part (b)(ii): $q \approx 1.122$ is the annual multiplier — value rises $\approx 12.2\%$ per year.

Part (c): $t = 30$: $\log_{10} V = 0.05 \times 30 + 4.8 = 6.3 \Rightarrow V = 10^{6.3} \approx £2\,000\,000$.

$V \approx £2\,000\,000$ (to the nearest hundred thousand)

Question 4

Worked Solution

Model: $P = ab^t$. Line l: intercept $(0, 5)$, gradient $\frac{1}{200}$.

Part (a): $\log_{10} P = \frac{1}{200}t + 5$.

Part (b): $\log_{10} b = \frac{1}{200} \Rightarrow b = 10^{1/200} \approx 1.0116$; $\quad a = 10^5 = 100\,000$.

$a = 100\,000, \quad b \approx 1.0116$

Part (c)(i): a is the initial population ($t = 0$). **(ii):** $b \approx 1.0116$ — population grows $\approx 1.16\%$ per year.

Part (d)(i): $t = 100$: $\log_{10} P = 5.5 \Rightarrow P \approx 316\,000 \approx$ **300 000**.

Part (d)(ii): $\log_{10} 200\,000 = \frac{t}{200} + 5 \Rightarrow t = 200 \times 0.301 \approx 60.2$ years.

(i) $\approx 300\,000$ (nearest hundred thousand) (ii) ≈ 60.2 years

Part (e): (1) Model predicts unlimited growth; in reality population is limited by resources, disease, war. (2) The proportional growth rate is unlikely to stay constant over 100 years.

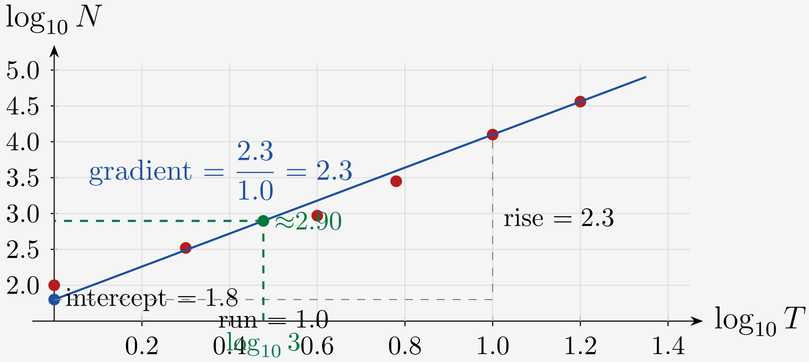
Question 5

Worked Solution

Model: $N = aT^b$.

Part (a): $\log_{10} N = \log_{10} a + b\log_{10} T$, so $m = b$ and $c = \log_{10} a$.

Part (b): The graph below shows the data from Figure 3 and the line of best fit. The line passes through (0, 1.8) and (1.0, 4.1).



Gradient $b \approx 2.3$; intercept $c = 1.8 \Rightarrow a = 10^{1.8} \approx 63$.

When $T = 3$: $\log_{10} T = 0.477$, reading off $\log_{10} N \approx 2.90$.

$$N = 10^{2.90} \approx 800.$$

$N \approx 800$ microbes

Part (c): $N = 10^6$ requires $\log_{10} T \approx 1.83$, far beyond the data range of 0 to 1.2 — we cannot extrapolate with confidence.

Required $\log_{10} T \approx 1.83$ lies outside the observed data range; extrapolation is unreliable.

Part (d): $T = 1 \Rightarrow N = a \times 1 = a$, so a is the number of microbes on day 1.

a is the number of microbes present 1 day after the start of the experiment.

End of Worked Solutions