Reduction to Linear Form Exam Questions Sheet 2 MS

Q1.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	If $d = kV^n$, then $\log_{10} d = \log_{10} k + n \log_{10} V$	M1	This mark is given for
	Plotting $\log_{10} d$ against $\log_{10} V$ will result in a straight line with gradient n and intercept $\log_{10} k$	A1	This mark is given for an explanation of why the second graph shows that $d = kV^n$
	$\log_{10} k = -1.77$ $k = 10^{-1.77} = 0.01698\ldots \approx 0.017$	A1	This mark is for showing fully that $k \approx 0.017$
(b)	$d = kV^n$ When $V = 30$, $d = 20$ and $k = 0.17$ then $20 = 0.017 \times 30^n$	M1	This mark is given for substituting in the formula as a method to find the value of n
	$n \log 30 = \log\left(\frac{20}{0.017}\right)$	M1	This mark is given for a correct expression for n
	$n = 2.08$ to 3 significant figures $d = 0.017 \times V^{2.08}$	A1	This mark is given for finding a correct value of n to 3 significant figures and writing a complete equation for the model
(c)	$\frac{60}{3600} \times 0.8 \times 1000 = 13.33$ m	M1	This mark is given for a method to find the distance, in metres, covered in the reaction time of 0.8 seconds
	$d = 0.017 \times 60^{2.08} = 84.92$ m	M1	This mark is given for a method to use the formula to find the stopping distance
	13.33 m + 84.92 m = 98.25 m Sean will be able to stop before reaching the puddle	A1	This mark is given for finding a correct value of the total stopping distance and giving a valid conclusion
			(Total 9 marks)

Q2.

Question	Scheme		Marks	AOs
(a)	$\log_{10} h = 2.25 - 0.235\log_{10} m$ $\Rightarrow h = 10^{2.25-0.235\log_{10} m}$ $\Rightarrow h = 10^{2.25} \times m^{-0.235}$	$h = pm^q$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^q$ $\Rightarrow \log_{10} h = \log_{10} p + q\log_{10} m$	M1	1.1b
	Either one of $p = 10^{2.25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178" \times 5^{"-0.235"}$	$\log_{10} h = "2.25" - "0.235"\log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
			(7 marks)	

Notes

(a)

M1: Establishes a link between $h = pm^q$ and $\log_{10} h = 2.25 - 0.235\log_{10} m$.
May be implied by a correct equation in p or q

A1: For a correct equation in p or q

A1: $p = 178$ and $q = -0.235$

(b)

M1: Uses either model to set up an equation in h (or m)

A1: h = awrt 122. Condone h = awrt 122 bpm

A1ft: Comments on the suitability of the model. Follow through on their answer.

Requires a comment consistent with their answer from using the model.

E.g. It is a suitable model as it is only "3" bpm away from the real value ✓
Do not allow an argument stating that it should be the same.
It is an unsuitable model as "122" bpm is not equal to 119 bpm ✗

(c)

B1: "p" would be the (resting) heart rate of a mammal with a mass of 1 kg

Q3.

Question	Scheme	Marks	AOs
(a)	For a correct equation in p or q $\quad p=10^{4.8}$ or $q=10^{0.05}$	M1	1.1b
	For $p=$ awrt 63100 or $q=$ awrt 1.122	A1	1.1b
	For correct equations in p and q $\quad p=10^{4.8}$ and $q=10^{0.05}$	dM1	3.1a
	For $p=$ awrt 63100 and $q=$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V=63100\times1.122^{30}$ or $\log V=0.05\times30+4.8$ leading to $V=$	M1	3.4
	$=$ awrt $(£)2000000$	A1	1.1b
		(2)	
			(8 marks)

Notes

(a)

M1: For a correct equation in p or q This is usually $p=10^{4.8}$ or $q=10^{0.05}$ but may be $\log q=0.05$ or $\log p=4.8$

A1: For $p=$ awrt 63100 or $q=$ awrt 1.122

M1: For linking the two equations and forming correct equations in p and q. This is usually $p=10^{4.8}$ and $q=10^{0.05}$ but may be $\log q=0.05$ and $\log p=4.8$

A1: For $p=$ awrt 63100 and $q=$ awrt 1.122 Both these values implies M1 M1

ALT I(a)

M1: Substitutes $t=0$ and states that $\log p=4.8$

A1: $p=$ awrt 63100

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p=$ awrt 63100 and $q=$ awrt 1.122

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled "p is........... " and "q is ...."

(c)

M1: For substituting $t=30$ into $V=pq^t$ using their values for p and q or substituting $t=30$ into $\log_{10}V=0.05t+4.8$ and proceeds to V

A1: For awrt either £1.99 million or £2.00 million. Condone the omission of the £ sign.

Remember to isw after a correct answer

Q4.

Question	Scheme		Marks	AOs
(a)	$\log_{10} P = mt + c$		M1	1.1b
	$\log_{10} P = \frac{1}{200}t + 5$		A1	1.1b
			(2)	
(b)	Way 1: As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$	Way 2: As $\log_{10} P = \frac{t}{200} + 5$ then $P = 10^{\left(\frac{t}{200}+5\right)} = 10^5 10^{\left(\frac{t}{200}\right)}$	M1	2.1
	$\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$	$a = 10^5$ or $b = 10^{\left(\frac{1}{200}\right)}$	M1	1.1b
	so a = 100 000 **or** b = 1.0116		A1	1.1b
	both a = 100 000 **and** b = 1.0116 (awrt 1.01)		A1	1.1b
			(4)	
(c)	(i) The initial population		B1	3.4
	(ii) The proportional increase of population each year		B1	3.4
			(2)	
(d)	(i) 300000 to nearest hundred thousand		B1	3.4
	(ii) Uses $200000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give $t =$		M1	3.4
	60.2 years to 3sf		A1ft	1.1b
			(3)	
(e)	Any two valid reasons- e.g. • 100 years is a long time and population may be affected by wars and disease • Inaccuracies in measuring gradient may result in widely different estimates • Population growth may not be proportional to population size • The model predicts unlimited growth		B2	3.5b
			(2)	
				(13 marks)

Notes

(a) M1: Uses a linear equation to relate log P and t
A1: Correct use of gradient and intercept to give a correct line equation
(b) M1: Way 1: Uses logs correctly to give log equation; Way 2 Uses powers correctly to "undo" log equation and expresses as product of two powers
M1: Way 1: Identifies log b or log a or both; Way 2: identifies a or b as powers of 10
A1: Correct value for a or b
A1: Correct values for both
(c) (i) B1: Accept equivalent answers e.g. The population at $t = 0$
(ii) B1: So accept rate at which the population is increasing each year or scale factor 1.01 or increase of 1% per year
(d) (i) B1: cao
(ii) M1: as in the scheme A1ft: on their values of a and b with correct log work
(e) As given in the scheme – any two valid reasons

Q5.

Question	Scheme	Marks	AOs
(a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1	2.1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1	1.1b
		(2)	
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** $b =$ gradient	M1	3.1b
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** $b =$ gradient	M1	1.1b
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1	3.1b
	Number of microbes ≈ 800	A1	1.1b
		(4)	
(c)	$N = 1000000 \Rightarrow \log_{10} N = 6$	M1	3.4
	We cannot 'extrapolate' the graph and assume that the model still holds	A1	3.5b
		(2)	
(d)	States that 'a' is the number of microbes 1 day after the start of the experiment	B1	3.2a
		(1)	
		(9 marks)	

Notes:
(a) **M1:** Takes logs of both sides and shows the addition law **M1:** Uses the power law, writes $\log_{10} N = \log_{10} a + b\log_{10} T$ and states $m = b$ **and** $c = \log_{10} a$
(b) **M1:** Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** $b =$ gradient. This would be implied by the sight of $b = 2.3$ **or** $a = 10^{1.8} \approx 63$ **M1:** Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** $b =$ gradient. This would be implied by the sight of $b = 2.3$ **and** $a = 10^{1.8} \approx 63$ **M1:** Uses $T = 3 \Rightarrow N = aT^b$ with their a and b. This is implied by an attempt at $63 \times 3^{2.3}$ **A1:** Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work. There is an alternative to this using a graphical approach. **M1:** Finds the value of $\log_{10} T$ from T=3. Accept as $T = 3 \Rightarrow \log_{10} T \approx 0.48$ **M1:** Then using the line of best fit finds the value of $\log_{10} N$ from their "0.48" Accept $\log_{10} N \approx 2.9$ **M1:** Finds the value of N from their value of $\log_{10} N$ $\log_{10} N \approx 2.9 \Rightarrow N = 10^{'2.9'}$ **A1:** Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work

(c)	
M1	For using $N = 1000000$ and stating that $\log_{10} N = 6$
A1:	Statement to the effect that "we only have information for values of $\log N$ between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate" There is an alternative approach that uses the formula.
M1:	Use $N = 1000000$ in their $N = 63 \times T^{2.3} \Rightarrow \log_{10} T = \dfrac{\log_{10}\left(\dfrac{1000000}{63}\right)}{2.3} \approx 1.83$.
A1:	The reason would be similar to the main scheme as we only have $\log_{10} T$ values from 0 to 1.2. We cannot 'extrapolate' the graph and assume that the model still holds
(d)	
B1:	Allow a numerical explanation $T = 1 \Rightarrow N = a1^b \Rightarrow N = a$ giving a is the value of N at $T = 1$