Proof Exam Questions MS (From OCR 4751 unless otherwise stated)

Q1, (Jun 2006, Q4)

(i) P $\Leftarrow$ Q (ii) P $\Leftrightarrow$ Q	1 1	condone omission of P and Q	2

Q2, (Jun 2007, Q3)

'If $2n$ is an even integer, then n is an odd integer'	1	or: $2n$ an even integer $\Rightarrow$ n an odd integer	
showing wrong eg 'if n is an even integer, $2n$ is an even integer'	1	or counterexample eg $n = 2$ and $2n = 4$ seen [in either order]	2

Q3, (Jun 2011, Q10)

$n(n+1)(n+2)$	M1	condone division by n and then $(n+1)(n+2)$ seen, or separate factors shown after factor theorem used;
argument from general consecutive numbers leading to:		
at least one must be even	A1	or divisible by 2;
[exactly] one must be multiple of 3	A1	if M0: allow SC1 for showing given expression always even

Q4, (Jan 2012, Q9)

(i)		'if n even then n^3 even, so $n^3 + 1$ odd' oe	B1	must mention n^3 is even or even^3 is even or even × even = even
		$\Leftarrow$ with if $n^3 + 1$ odd then n^3 even but if n^3 is even, n is not necessarily an integer or $\Leftrightarrow$ with '$n^3 + 1$ odd then n^3 even so n even', [assuming n is an integer]	B1 **[2]**	or '$\Leftrightarrow$ with if n is odd, n^3 is odd, so $n^3 + 1$ is even' if 0 in question, allow SC1 for $\Leftrightarrow$ or $\Leftarrow$ and attempt at using general odd/even in explanation
(ii)		showing $\Leftarrow$ is true	B1	eg when $x > 3$, +ve × +ve > 0
		$\Leftarrow$ chosen and showing that $\Rightarrow$ [and therefore $\Leftrightarrow$] is/ are not true	B1 **[2]**	stating that true when $x < 2$ or giving a counterexample such as 1, 0 or a negative number [to show quadratic inequality also true for this number] allow B2 for $\Leftarrow$ and $x > 3$ and $x < 2$ shown/stated as soln or sketch showing two solns of $x^2 - 5x + 6 > 0$

Q5, (Jun 2013, Q9)

(i)		$3n$ isw	1 **[1]**	accept equivalent general explanation
(ii)		at least one of $(n - 1)^2$ and $(n + 1)^2$ correctly expanded	M1	must be seen
		$3n^2 + 2$	B1	
		comment eg $3n^2$ is always a multiple of 3 so remainder after dividing by 3 is always 2	B1 **[3]**	dep on previous B1 B0 for just saying that 2 is not divisible by 3 – must comment on $3n^2$ term as well allow B1 for $\frac{3n^2+2}{3} = n^2 + \frac{2}{3}$

Q6, (Jun 2014, Q9)

(i)		$3n^2 + 6n + 5$ isw	B2 **[2]**	M1 for a correct expansion of at least one of $(n+1)^2$ and $(n+2)^2$
(ii)		odd numbers with valid explanation	B2 **[2]**	marks dep on 9(i) correct or starting again for B2 must see at least odd × odd = odd [for $3n^2$] (or when n is odd, [3]n^2 is odd) and odd [+ even] + odd = even soi, condone lack of odd × even = even for $6n$; condone no consideration of n being even or B2 for deductive argument such as: $6n$ is always even [and 5 is odd] so $3n^2$ must be odd so n is odd B1 for odd numbers with a correct partial explanation or a partially correct explanation or B1 for an otherwise fully correct argument for odd numbers but with conclusion positive odd numbers or conclusion negative odd numbers B0 for just a few trials and conclusion

Q7, (OCR 4753, Jun 2006, Q5)

5(i)	$a^2 + b^2 = (2t)^2 + (t^2 - 1)^2$ $= 4t^2 + t^4 - 2t^2 + 1$ $= t^4 + 2t^2 + 1$ $= (t^2 + 1)^2 = c^2$	M1 M1 E1	substituting for a, b and c in terms of t Expanding brackets correctly www
(ii)	$c = \sqrt{(20^2 + 21^2)} = 29$ For example: $2t = 20 \Rightarrow t = 10$ $\Rightarrow$ $t^2 - 1 = 99$ which is not consistent with 21	B1 M1 E1 [6]	 Attempt to find t Any valid argument or E2 'none of 20, 21, 29 differ by two'.

Q8, (OCR 4753, Jun 2007, Q5)

$n = 1, n^2 + 3n + 1 = 5$ prime $n = 2, n^2 + 3n + 1 = 11$ prime $n = 3, n^2 + 3n + 1 = 19$ prime $n = 4, n^2 + 3n + 1 = 29$ prime $n = 5, n^2 + 3n + 1 = 41$ prime $n = 6, n^2 + 3n + 1 = 55$ not prime so statement is false	M1 E1 [2]	One or more trials shown finding a counter-example – must state that it is not prime.

Q9, (OCR 4753, Jun 2009, Q7)

7(i) (*A*) $(x-y)(x^2+xy+y^2)$ $= x^3 + x^2y + xy^2 - yx^2 - xy^2 - y^3$ $= x^3 - y^3$ *	M1 E1	expanding - allow tabulation www
(*B*) $(x + ½ y)^2 + ¾ y^2$ $= x^2 + xy + ¼ y^2 + ¾ y^2$ $= x^2 + xy + y^2$	M1 E1 [4]	$(x + ½ y)^2 = x^2 + ½ xy + ½ xy + ¼ y^2$ o.e. cao www
(ii) $x^3 - y^3 = (x - y)[(x + ½ y)^2 + ¾ y^2]$	M1	substituting results of (i)
$(x + ½ y)^2 + ¾ y^2 > 0$ [as squares ≥ 0]	M1	
$\Rightarrow$ if $x - y > 0$ then $x^3 - y^3 > 0$ $\Rightarrow$ if $x > y$ then $x^3 > y^3$ *	E1 [3]	

Q10, (OCR 4753, Jun 2011, Q7)

(i) $(3^n + 1)(3^n - 1) = (3^n)^2 - 1$ or $3^{2n} - 1$	B1 [1]	mark final answer
(ii) 3^n is odd $\Rightarrow 3^n + 1$ and $3^n - 1$ both even As consecutive even nos, one must be divisible by 4, so product is divisible by 8.	M1 M1 A1 [3]	3^n is odd $\Rightarrow 3^n + 1$ and $3^n - 1$ both even completion

Q11, (OCR 4753, Jan 2013, Q7)

(i)		$3^5 + 2 = 245$ [which is not prime]	M1 A1 **[2]**	Attempt to find counter-example correct counter-example identified
(ii)		$(3^0 = 1)$, $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, …	M1	Evaluate 3^n for $n = 0$ to 4 or 1 to 5
		so units digits cycle through 1, 3, 9, 7, 1, 3, … so cannot be a '5'.	A1	
		OR		
		3^n is not divisible by 5	B1	
		all numbers ending in '5' are divisible by 5.	B1	
		so its last digit cannot be a '5'		must state conclusion for B2
			[2]	

Q12, (OCR 4753, Jan 2012, Q4)

Cubes are 1, 8, 27, 64, 125, 216, 343, 512 [so false as] $8^3 = 512$	M1 A1 **[2]**	Attempt to find counter example counter-example identified (e.g. underlining, circling) [counter-examples all have 8 as units digit]