Proof Exam Questions Sheet 2

Q1 (OCR A, AS Maths, Jun 2018, Paper 1, Q5)

N is an integer that is not divisible by 3. Prove that N^2 is of the form $3p+1$, where p is an integer. **[5]**

Q2 (Edexcel, AS Maths, Jun 2018, Paper 1, Q2)

(i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

Q3 (OCR A, AS Maths, Jun 2018, Paper 2, Q3)

[Part (iii) requires knowledge of logarithms – leave (iii) out if not yet covered]

In each of the following cases choose one of the statements

$$P \Rightarrow Q \qquad P \Leftarrow Q \qquad P \Leftrightarrow Q$$

to describe the relationship between P and Q.

(i) P: $y = 3x^5 - 4x^2 + 12x$

Q: $\frac{dy}{dx} = 15x^4 - 8x + 12$ **[1]**

(ii) P: $x^5 - 32 = 0$ where x is real

Q: $x = 2$ **[1]**

(iii) P: $\ln y < 0$

Q: $y < 1$ **[1]**

Q4 (Edexcel, AS Maths, Jun 2019, Paper 1, Q15)

Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(4)

Q5 (OCR B, AS Maths, Jun 2018, Paper 2, Q3)

P and Q are consecutive **odd** positive integers such that $P > Q$.

Prove that $P^2 - Q^2$ is a multiple of 8. **[3]**

Q6 (OCR B, AS Maths, Jun 2019, Paper 2, Q3)

Without using a calculator, prove that $3\sqrt{2} > 2\sqrt{3}$. **[3]**

Q7 (AQA, A Level Maths, Practice Set 2, Paper 2, Q7)

Arthur uses the quadratic expression

$$2n^2 + 29$$

to generate the first few terms of a sequence by substituting non-negative integers for n

$n = 0$	$2 \times 0^2 + 29 = 29$
$n = 1$	$2 \times 1^2 + 29 = 31$
$n = 2$	$2 \times 2^2 + 29 = 37$
$n = 3$	$2 \times 3^2 + 29 = 47$

Arthur notices that all the terms are prime and attempts to prove that he has found an expression that will always generate prime numbers.

He continues to check examples:

$n = 4$	$2 \times 4^2 + 29 = 61$
$n = 5$	$2 \times 5^2 + 29 = 79$
$n = 6$	$2 \times 6^2 + 29 = 101$
$n = 7$	$2 \times 7^2 + 29 = 127$
$n = 8$	$2 \times 8^2 + 29 = 157$
$n = 9$	$2 \times 9^2 + 29 = 191$
$n = 10$	$2 \times 10^2 + 29 = 229$

Having checked that all the integers from 0 to 10 give prime numbers, Arthur declares that he has proved by exhaustion that his expression will always generate prime numbers.

(a) Explain why Arthur has not completed a proof by exhaustion.

[1 mark]

(b) Use n = 29 as a counter example to show that the expression does not always generate a prime number.

[2 marks]

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(c) $p(n) = an^2 + bn + c$

where a, b, c are positive integers

Prove that the statement

$p(n)$ is prime for every non-negative integer, n

is false for all possible values of a, b and c

[3 marks]

Q8, (OCR A, AS Maths, Jun 2019,Paper 1,Q5)

(a) Prove that the following statement is **not** true.

m is an odd number greater than 1 $\Rightarrow m^2 + 4$ is prime. **[1]**

(b) By considering separately the case when n is odd and the case when n is even, prove that the following statement is true.

n is a positive integer $\Rightarrow n^2 + 1$ is not a multiple of 4. **[4]**
