Proof Exam Questions ?S (Mark Scheme)

Q1

$N = 3k + 1$ or $N = 3k + 2$ (where k is an integer)	M1	3.1a	One of these. Allow without "N = "	Any letter other than p
$(3k + 1)^2$ $= 9k^2 + 6k + 1$; $(3k + 2)^2$ $= 9k^2 + 12k + 4$	M1 A1	1.1 2.1	Attempt one of these Both correct	Allow p Allow p
$= 3(3k^2 + 2k) + 1$ or $= 3(3k^2 + 4k + 1) + 1$	A1	2.4	Or $9k^2+6k$ div by 3 or $9k^2+12k+3$ div by3 One of these	or similar in words. Allow p
Both these are of form $3p + 1$, p an integer	E1	2.2a	Must say p is integer or $3k^2 + 2k$ and $3k^2 + 4k + 1$ are integers Similar marks for method using $N = 3k + 1$ & $N = 3k - 1$	Dep on M1M1A1A1 $N = 3p + 1$: max M0M1A1A1E0
	[5]			

Q2

Question	Scheme	Marks	AOs
2(i)	$x^2 - 8x + 17 = (x-4)^2 - 16 + 17$	M1	3.1a
	$= (x-4)^2 + 1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2 \geqslant 0 \Rightarrow (x-4)^2 + 1 \geqslant 1$ hence $x^2 - 8x + 17 > 0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x = -5$ $(-5+3)^2 = 4$ whereas $(-5)^2 = 25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x = 5$ $(5+3)^2 = 64$ whereas $(5)^2 = 25$ True When $x = -5$ $(-5+3)^2 = 4$ whereas $(-5)^2 = 25$ Not true	A1	2.4
		(2)	
			(5 marks)

Q3

(i)		$P \Rightarrow Q$	B1 [1]	1.1
(ii)		$P \Leftrightarrow Q$	B1 [1]	1.1
(iii)		$P \Rightarrow Q$	B1 [1]	1.1

Q4

Logical approach	States that if n is odd, n^3 is odd	M1	2.1
	so n^3+2 is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if n is even, n^3 is a multiple of 8	M1	2.1
	so n^3+2 cannot be a multiple of 8 So (Given $n\in\mathbb{N}$), n^3+2 is not divisible by 8	A1	2.2a
		(4)	
			4 marks

Algebraic approach	(If n is even,) $n=2k$ and $n^3+2=(2k)^3+2=8k^3+2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8 , $8k^3+2$ isn't'	A1	2.2a
	(If n is odd,) $n=2k+1$ and $n^3+2=(2k+1)^3+2$	M1	2.1
	$=\underline{\underline{8k^3+12k^2+6k+3}}$ which is an even number add 3, therefore odd. Hence it is not divisible by 8 So (given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n=2k$ and $\frac{n^3+2}{8}=\frac{(2k)^3+2}{8}=\frac{8k^3+2}{8}$	M1	2.1
	$=k^3+\frac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n=2k+1$ and $\frac{n^3+2}{8}=\frac{(2k+1)^3+2}{8}$	M1	2.1
	$=\frac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $\underline{\underline{8k^3+12k^2+6k+3}}$ is an even number +3 hence not divisible by 8 So (Given $n\in\mathbb{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	

Q5

$(2n+1)^2-(2n-1)^2$ oe	B1	2.1		OR $P=Q+2$ $P^2-Q^2=(Q+2)^2-Q^2$ B1
$4n^2+4n+1-(4n^2-4n+1)$	M1	1.1	Allow one slip eg sign error	$=4(Q+1)$ (factorized) M1 $(Q+1)$ divisible by 2 so
$=8n$ (so multiple of 8)	A1	2.4	Note: Numerical verification 0	$4(Q+1)$ is multiple of 8 A1 OR $Q=P-2$
	[3]			

Q6

LHS is $(\sqrt{9}\times\sqrt{2})=\sqrt{18}$	B1	2.1	OR LHS squared is 18	No calculator. No decimal values allowed.
RHS is $(\sqrt{4}\times\sqrt{3})=\sqrt{12}$	B1	1.1	RHS squared is 12	
$\sqrt{18}>\sqrt{12}$ oe (so $3\sqrt{2}>2\sqrt{3}$)	E1	2.4	AG OR eg $\sqrt{3}\times\sqrt{3}\times\sqrt{2}>\sqrt{2}\times\sqrt{2}\times\sqrt{3}$ $\sqrt{3}>\sqrt{2}$, which is true	Allow proof that starts with answer, and shows it must be true.
	[3]			

Q7

	Marking instructions	AO	Marks	Typical solution
(a)	Explains that only 11 cases have been checked so this cannot be exhaustive	AO2.3	E1	Proof by exhaustion requires all cases to be checked and the student has only checked 11, which is not exhaustive.
	Total		**1**	
(b)	Substitutes n = 29 into the expression	AO1.1a	M1	$2\times29^2+29=1711$ $=59\times29$ $\therefore n = 29$ generates a composite number.
	Completes rigorous argument showing clearly that this leads to a composite number	AO2.1	R1	
	Total		**2**	
(c)	Makes a substitution that is a multiple of c	AO3.1a	M1	$n=c\Rightarrow$ $\mathrm{p}(c)=ac^2+bc+c$ $=c(ac+b+1)$ This is a composite number with factors c and $ac+b+1$, provided $c\neq1$ In the case where c = 1 , p(0)=1 which is not prime
	Completes rigorous argument to explain why a composite number can generally be found	AO2.1	R1	
	Completes proof by considering c = 1	AO2.1	R1	
	Total		**3**	

Q8

(a)		eg $9^2 + 4 = 85$ and 85 is multiple of 5 or $85 = 5\times17$ or $85 \div 5 = 17$ or 85 has a factor of 5 (or 17)	**B1** **[1]**	**1.1**	oe Not just "and 85 is not prime" One correct example and one incorrect B1	Condone eg $9^2+4 = 85 \div 5 = 17$
(b)		$(2k)^2 + 1 = 4k^2 + 1$ (k an integer) or eg $(2k + 2)^2 + 1 = 4k^2 + 8k + 5$ which is not a multiple of 4	**B1**	**2.2a**	Allow any letter, even n	Allow omission of "(k an integer)" in both places
		$(2k + 1)^2 + 1 = 4k^2 + 4k + 1 + 1$ (k an integer) $= 4(k^2 + k) + 2$ or $4k^2 + 4k + 2$	**M1** **A1***	**1.1** **1.1**	Attempt expand & add 1, eg $4k^2+1+1$: M1 Must see one of these forms	Same marks for $(2k - 1)^2 + 1$
		Any sensible explanation why this is not mult of 4 eg This is of the form $4 \times$ integer + 2 $4k^2$ & $4k$ are mults of 4 but when + 2, not mult of 4	**A1** dep*	**2.2a**	eg $4(k^2 + k)$ is a multiple of 4 2 is not a multiple of 4 $k^2 + k + \frac{1}{2}$ not an integer Other correct methods may be seen	Not just "This is not a mult of 4" Not "This is mult of 2, not of 4" Numerical examples: no mks
		Alternative method n even n^2 is mult of 4, hence $n^2 + 1$ is not mult of 4 or n^2 is even, so $n^2 + 1$ is odd so not mult of 4	**B1**			
		n even, $n + 1$ is odd. $(n + 1)^2 + 1$ $= n^2 + 2n + 2$ n^2 and $2n$ are mults of 4, hence $n^2 + 2n + 2$ is not	 **M1** **A1** **A1** **[4]**			