Practical Applications of Differentiation

Q1, (OCR 4752, Q10)

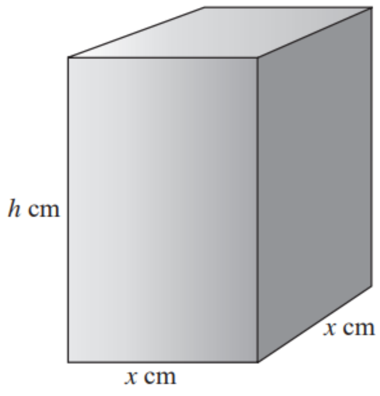

Fig. 10

Fig. 10 shows a solid cuboid with square base of side x cm and height h cm. Its volume is 120 cm^3.

(i) Find h in terms of x. Hence show that the surface area, A cm^2, of the cuboid is given by $A = 2x^2 + \frac{480}{x}$. [3]

(ii) Find $\frac{dA}{dx}$ and $\frac{d^2A}{dx^2}$. [4]

(iii) Hence find the value of x which gives the minimum surface area. Find also the value of the surface area in this case. [5]

Q2, (Edexcel 6664, Jun 2015, Q9)

A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.

The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(4)

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

(1)

Q3, (Edexcel 6664, Jan 2011, Q10)

The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

Q4, (Edexcel 6664, Jan 2012, Q8)

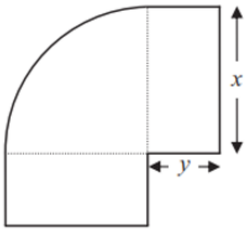

Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2,

(a) show that

$$y = \frac{16 - \pi x^2}{8x}$$

(3)

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

(3)

(c) Use calculus to find the minimum value of P.

(5)

(d) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre.

(2)

Q5, (Edexcel 6664, Jun 2012, Q8)

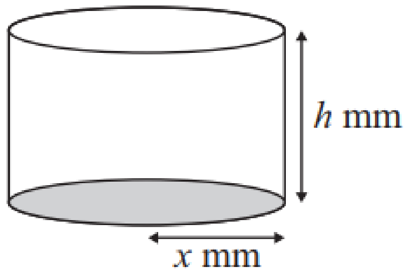

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x,

(1)

(b) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$

(3)

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum.

(5)

(d) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(e) Show that this value of A is a minimum.

(2)

Q6, (OCR 4752, Jun 2011, Q11)

(i) The standard formulae for the volume V and total surface area A of a solid cylinder of radius r and height h are

$$V = \pi r^2 h \qquad \text{and} \qquad A = 2\pi r^2 + 2\pi rh.$$

Use these to show that, for a cylinder with $A = 200$,

$$V = 100r - \pi r^3.$$ **[4]**

(ii) Find $\frac{dV}{dr}$ and $\frac{d^2V}{dr^2}$. **[3]**

(iii) Use calculus to find the value of r that gives a maximum value for V and hence find this maximum value, giving your answers correct to 3 significant figures. **[4]**

Q7, (Edexcel 6664, Jun 2014, Q10)

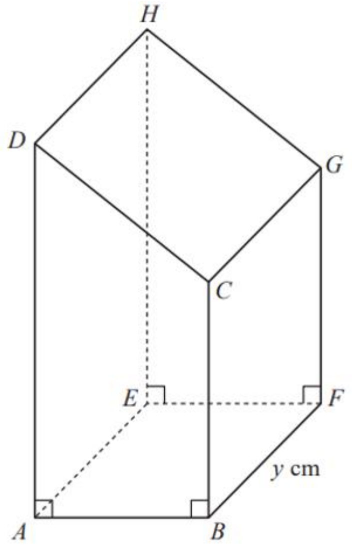

Figure 4

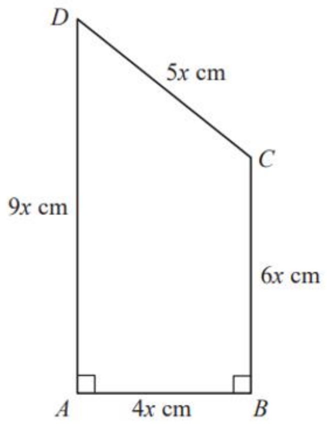

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.
The angle $DAB = 90°$ and the angle $ABC = 90°$.

The volume of the letter box is 9600 cm^3.

(a) Show that

$$y = \frac{320}{x^2}$$

(2)

(b) Hence show that the surface area of the letter box, S cm^2, is given by

$$S = 60x^2 + \frac{7680}{x}$$

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)