Practical Applications of Differentiation

Q1, (OCR 4752, Jan 2008, Q10)

i	$h = 120/x^2$	B1		
	$A = 2x^2 + 4xh$ o.e.	M1		
	completion to given answer	A1	at least one interim step shown	3
ii	$A' = 4x - 480/x^2$ o.e.	2	1 for kx^{-2} o.e. included	
	$A'' = 4 + 960 / x^3$	2	ft their A' only if kx^{-2} seen ; 1 if one error	4
iii	use of $A' = 0$	M1		
	$x = \sqrt[3]{120}$ or 4.9(3..)	A1		
	Test using A' or A'' to confirm minimum	T1		
	Substitution of their x in A	M1	Dependent on previous M1	5
	A = 145.9 to 146	A1		

Q2, (Edexcel 6664, Jun 2015, Q9)

(a)	Either: (Cost of polishing top and bottom (two circles) is)$3\times 2\pi r^2$ **or** (Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1	
	Uses volume to give $(h=)\dfrac{75\pi}{\pi r^2}$ or $(h=)\dfrac{75}{r^2}$ (simplified) (if V is misread – see below)	B1ft	
	$(C) = 6\pi r^2 + 4\pi r\left(\dfrac{75}{r^2}\right)$ — Substitutes expression for h into area or cost expression of form $Ar^2 + Brh$	M1	
	$C = 6\pi r^2 + \dfrac{300\pi}{r}$ *	A1*	(4)
(b)	$\left\{\dfrac{dC}{dr}=\right\}\ 12\pi r - \dfrac{300\pi}{r^2}$ or $12\pi r - 300\pi r^{-2}$ (then isw)	M1 A1 ft	
	$12\pi r - \dfrac{300\pi}{r^2} = 0$ **so** r^k = value where $k = \pm 2, \pm 3, \pm 4$	dM1	
	Use **cube** root to obtain $r = \left(their\ \frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$ - allow $r = 3$, and thus $C =$	ddM1	
	Then C = awrt 483 or 484	A1cao	(5)
(c)	$\left\{\dfrac{d^2C}{dr^2}=\right\}\ 12\pi + \dfrac{600\pi}{r^3} > 0$ **so minimum**	B1ft	(1)
			[10]

Q3, (Edexcel 6664, Jan 2011, Q10)

(a)	$V = 4x(5-x)^2 = 4x(25-10x+x^2)$		
	So, $V = 100x - 40x^2 + 4x^3$	$\pm\alpha x \pm \beta x^2 \pm \gamma x^3$, where $\alpha, \beta, \gamma \neq 0$	M1
		$V = 100x - 40x^2 + 4x^3$	A1
	$\frac{dV}{dx} = 100 - 80x + 12x^2$	At least two of their expanded terms differentiated correctly.	M1
		$100 - 80x + 12x^2$	A1 cao
			(4)
(b)	$100 - 80x + 12x^2 = 0$	Sets their $\frac{dV}{dx}$ from part (a) $= 0$	M1
	$\{\Rightarrow 4(3x^2 - 20x + 25) = 0 \Rightarrow 4(3x-5)(x-5) = 0\}$		
	$\{\text{As } 0 < x < 5\}$ $x = \frac{5}{3}$	$x = \frac{5}{3}$ or $x =$ awrt 1.67	A1
	$x = \frac{5}{3}$, $V = 4(\frac{5}{3})(5 - \frac{5}{3})^2$	Substitute candidate's value of x **where** $0 < x < 5$ into a formula for V.	dM1
	So, $V = \frac{2000}{27} = 74\frac{2}{27} = 74.074...$	Either $\frac{2000}{27}$ or $74\frac{2}{27}$ or awrt 74.1	A1
			(4)
(c)	$\frac{d^2V}{dx^2} = -80 + 24x$	Differentiates their $\frac{dV}{dx}$ **correctly** to give $\frac{d^2V}{dx^2}$.	M1
	When $x = \frac{5}{3}$, $\frac{d^2V}{dx^2} = -80 + 24\left(\frac{5}{3}\right)$		
	$\frac{d^2V}{dx^2} = -40 < 0 \Rightarrow V$ is a maximum	$\underline{\frac{d^2V}{dx^2} = -40}$ and $\underline{< 0 \text{ or negative}}$ and $\underline{\text{maximum}}$.	A1 cso
			(2)
			[10]

Q4, (Edexcel 6664, Jan 2012, Q8)

(a)	$kr^2 + cxy = 4$ or $kr^2 + c[(x+y)^2 - x^2 - y^2] = 4$	M1	
	$\frac{1}{4}\pi x^2 + 2xy = 4$	A1	
	$y = \dfrac{4 - \frac{1}{4}\pi x^2}{2x} = \dfrac{16 - \pi x^2}{8x}$ *	B1 cso	(3)
(b)	$P = 2x + cy + k\pi r$ where $c = 2$ or 4 and $k = ¼$ or $½$	M1	
	$P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ or $P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{16 - \pi x^2}{8x}\right)$ o.e.	A1	
	$P = \dfrac{\pi x}{2} + 2x + \dfrac{8}{x} - \dfrac{\pi x}{2}$ so $P = \dfrac{8}{x} + 2x$ *	A1	(3)
(c)	$\left(\dfrac{dP}{dx} =\right) -\dfrac{8}{x^2} + 2$	M1 A1	
	$-\dfrac{8}{x^2} + 2 = 0 \Rightarrow x^2 = ..$	M1	
	and so $x = 2$ o.e. (ignore extra answer $x = -2$)	A1	
	$P = 4 + 4 = 8$ (m)	B1	(5)
(d)	$y = \dfrac{4 - \pi}{4}$, (and so width) $= 21$ (cm)	M1, A1	(2)
			13

Q5, (Edexcel 6664, Jun 2012, Q8)

(a)	$(h=)\dfrac{60}{\pi x^2}$ or **equivalent exact** (not decimal) expression e.g. $(h=)60\div\pi x^2$	B1	(1)
(b)	$(A=)2\pi x^2+2\pi xh$ or $(A=)2\pi r^2+2\pi rh$ or $(A=)2\pi r^2+\pi dh$ may not be simplified and may appear on separate lines	B1	
	Either $(A)=2\pi x^2+2\pi x\left(\dfrac{60}{\pi x^2}\right)$ or As $\pi xh=\dfrac{60}{x}$ then $(A=)2\pi x^2+2\left(\dfrac{60}{x}\right)$	M1	
	$A=2\pi x^2+\left(\dfrac{120}{x}\right)$ *	A1 cso	(3)
(c)	$(\dfrac{dA}{dx})=4\pi x-\dfrac{120}{x^2}$ or $=4\pi x-120x^{-2}$	M1 A1	
	$4\pi x-\dfrac{120}{x^2}=0$ implies $x^3=$ (Use of >0 or <0 is M0 then M0A0)	M1	
	$x=\sqrt[3]{\dfrac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)	dM1 A1	(5)
(d)	$A=2\pi(2.12)^2+\dfrac{120}{2.12}$, $=85$ (only ft $x=2$ or 2.1 – both give 85)	M1, A1	(2)
(e)	**Either** $\dfrac{d^2A}{dx^2}=4\pi+\dfrac{240}{x^3}$ and sign considered (May appear in (c)) **Or** *(method 2)* considers gradient to left and right of their 2.12 (e.g at 2 and 2.5) **Or** (*method 3*) considers value of A either side	M1	
	which is >0 and therefore minimum (most substitute 2.12 but it is not essential to see a substitution) (may appear in (c)) Finds numerical values for gradients and observes gradients go from negative to zero to positive so concludes minimum **OR** finds numerical values of A, observing greater than minimum value and draws conclusion	A1	(2)
		13 marks	

Q6, (OCR 4752, Jun 2011, Q11)

(i) $200 - 2\pi r^2 = 2\pi rh$ $h = \dfrac{200 - 2\pi r^2}{2\pi r}$ o.e. substitution of correct h into $V = \pi r^2 h$ $V = 100r - \pi r^3$ convincingly obtained	**M1** **M1** **M1** **A1**	$100 = \pi r^2 + \pi rh$ $100r = \pi r^3 + \pi r^2 h$ $100r = \pi r^3 + V$ $V = 100r - \pi r^3$ or **M1** for $h = \dfrac{V}{\pi r^2}$ **M1** for $200 = 2\pi r^2 + 2\pi r \times \dfrac{V}{\pi r^2}$ **M1** for $200 = 2\pi r^2 + 2\dfrac{V}{r}$ **A1** for $V = 100r - \pi r^3$ convincingly obtained	**sc3** for complete argument working backwards: $V = 100r - \pi r^3$ $\pi r^2 h = 100r - \pi r^3$ $\pi rh = 100 - \pi r^2$ $100 = \pi rh + \pi r^2$ $200 = A = 2\pi rh + 2\pi r^2$ **sc0** if argument is incomplete
(ii) $\dfrac{dV}{dr} = 100 - 3\pi r^2$	**B2**	**B1** for each term	allow $9.42(\ldots.)\, r^2$ or better if decimalised
$\dfrac{d^2V}{dr^2} = -6\pi r$	**B1**		$-18.8(\ldots)\, r$ or better if decimalised
(iii) their $\dfrac{dV}{dr} = 0$ s.o.i.	**M1**	must contain r as the only variable	
$r = 3.26$ c.a.o.	**A2**	**A1** for $r = (\pm)\sqrt{\dfrac{100}{3\pi}}$; may be implied by 3.25…	
$V = 217$ c.a.o.	**A1**	deduct 1 mark only in this part if answers not given to 3 sf,	there must be evidence of use of calculus

Q7, (Edexcel 6664, Jun 2014, Q10)

(a)	$\frac{1}{2}(9x+6x)4x$ or $2x\times 15x$ or $\left(\frac{1}{2}4x\times(9x-6x)+6x\times 4x\right)$ or $6x^2+24x^2$ or $\left(9x\times 4x-\frac{1}{2}4x\times(9x-6x)\right)$ or $36x^2-6x^2$	M1: Correct attempt at the area of a trapezium. Note that $30x^2$ on its own or $30x^2$ from incorrect work e.g. $5x\times 6x$ is M0. If there is a clear intention to find the area of the trapezium correctly allow the M1 but the A1 can be withheld if there are any slips.	M1A1**cso**
	$\Rightarrow 30x^2y=9600\Rightarrow y=\frac{9600}{30x^2}\Rightarrow y=\frac{320}{x^2}$ *	A1: Correct proof with at least one intermediate step and no errors seen. **" y =" is required.**	
			[2]
(b)	$(S=)\frac{1}{2}(9x+6x)4x+\frac{1}{2}(9x+6x)4x+6xy+9xy+5xy+4xy$		M1A1
	M1: An attempt to find the area of **six** faces of the prism. The 2 trapezia may be combined as $(9x+6x)4x$ or $60x^2$ and the 4 other faces may be combined as $24xy$ but all six faces must be included. There must be attempt at the areas of two trapezia that are dimensionally correct. A1: Correct expression in any form. Allow just $(S=)\ 60x^2+24xy$ for M1A1		
	$y=\frac{320}{x^2}\Rightarrow(S=)\ 30x^2+30x^2+24x\left(\frac{320}{x^2}\right)$		M1
	Substitutes $y=\frac{320}{x^2}$ into their expression for S (may be done earlier). S should have at least one x^2 term and one xy term but there may be other terms which may be dimensionally incorrect.		
	So, $(S=)\ 60x^2+\frac{7680}{x}$ *	Correct solution only. " S = " is **not** required here.	A1* **cso**
			[4]

(c)	$\frac{dS}{dx} = 120x - 7680x^{-2}$ $\left\{= 120x - \frac{7680}{x^2}\right\}$	M1: Either $60x^2 \to 120x$ or $\frac{7680}{x} \to \frac{\pm\lambda}{x^2}$	M1
		A1: Correct differentiation (need not be simplified).	A1 aef
	$120x - \frac{7680}{x^2} = 0$ $\Rightarrow x^3 = \frac{7680}{120}; = 64 \Rightarrow x = 4$	M1: $S' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value" Setting their $\frac{dS}{dx} = 0$ and "candidate's ft ***correct*** power of x = a value". **The power of *x* must be consistent with their differentiation**. If inequalities are used this mark cannot be gained until candidate states value of x or S from their x without inequalities. $S' = 0$ can be implied by $120x = \frac{7680}{x^2}$. Some may spot that $x = 4$ gives $S' = 0$ and provided they clearly show $S'(4) = 0$ allow this mark as long as S' is correct. (If S' is incorrect this method is allowed if their derivative is clearly zero for their value of x)	M1A1**cso**
		A1: $x = 4$ **only** ($x^3 = 64 \Rightarrow x = \pm 4$ scores A0) Note that the value of x is not explicitly required so the use of $x = \sqrt[3]{64}$ to give $S = 2880$ would imply this mark.	
	Note some candidates stop here and do not go on to find *S* – maximum mark is 4/6		
	$\{x = 4,\}$ $S = 60(4)^2 + \frac{7680}{4} = 2880 \text{ (cm}^2)$	Substitute candidate's value of x ($\neq 0$) into a formula for S. **Dependent on both previous M marks.**	**dd**M1
		2880 **cso (Must come from correct work)**	A1 **cao and cso**
			[6]
(d)	$\frac{d^2S}{dx^2} = 120 + \frac{15360}{x^3} > 0$ $\Rightarrow$ Minimum	M1: Attempt S'' $(x^n \to x^{n-1})$ **and** considers sign. This mark requires an attempt at the second derivative and **some consideration of its sign**. There does not necessarily need to be any substitution. An attempt to solve $S'' = 0$ is M0	M1A1ft
		A1: $120 + \frac{15360}{x^3}$ and > 0 **and** conclusion. Requires a <u>**correct**</u> second derivative of $120 + \frac{15360}{x^3}$ (need not be simplified) <u>**and**</u> a valid reason (e.g. > 0), <u>**and**</u> conclusion. Only follow through a correct second derivative i.e. x may be incorrect **but must be positive** and/or S'' may have been <u>evaluated</u> incorrectly.	
	A correct S'' followed by $S''(\text{"4"}) = \text{"360"}$ therefore minimum would score no marks in (d) **A correct S'' followed by $S''(\text{"4"}) = \text{"360"}$ which is positive therefore minimum would score both marks**		
			[2]
	Note parts (c) and (d) can be marked together.		
			Total 14