Practical Applications of Differentiation Exam Questions Sheet 2

Q1.

The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \qquad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

(Total 10 marks)

Q2.

A solid right circular cylinder has radius r cm and height h cm.
The total surface area of the cylinder is 800 cm^2.
(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400\text{r} - \pi r^3.$$

(4)

Given that r varies,
(b) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)

(Total 12 marks)

Q3.

Figure 4

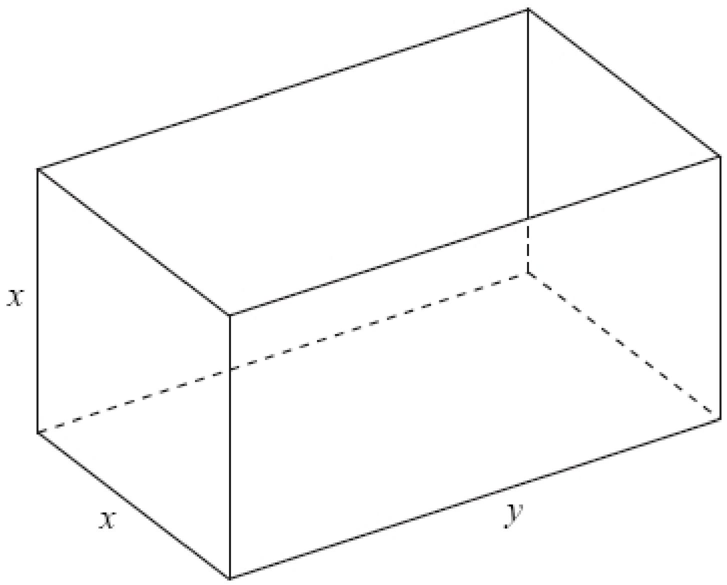

Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)

(Total 12 marks)

Q4.

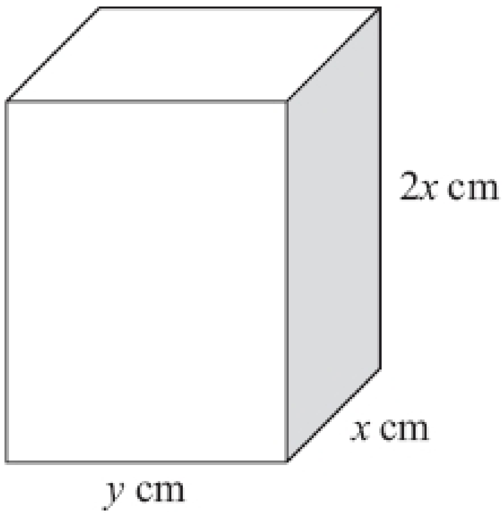

Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(c) Justify that the value of V you have found is a maximum.

(2)

(Total 11 marks)

Q5.

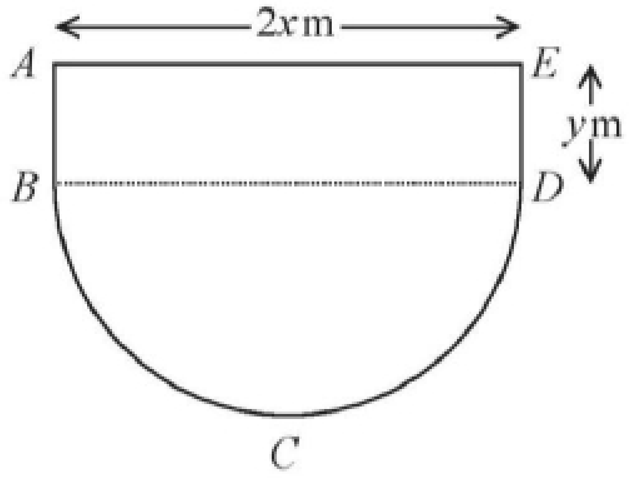

Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is 250 m^2,

(a) show that the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(4)

(Total for question = 10 marks)

Q6.

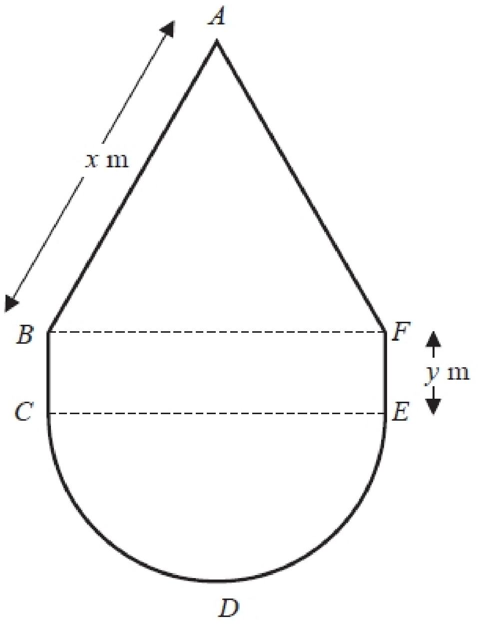

Figure 4

Figure 4 shows the plan of a pool.

The shape of the pool *ABCDEFA* consists of a rectangle *BCEF* joined to an equilateral triangle *BFA* and a semi-circle *CDE*, as shown in Figure 4.

Given that $AB = x$ metres, $EF = y$ metres, and the area of the pool is 50 m^2,

(a) show that

$$y = \frac{50}{x} - \frac{x}{8}(\pi + 2\sqrt{3})$$

(3)

(b) Hence show that the perimeter, P metres, of the pool is given by

$$P = \frac{100}{x} + \frac{x}{4}(\pi + 8 - 2\sqrt{3})$$

(3)

(c) Use calculus to find the minimum value of P, giving your answer to 3 significant figures.

(5)

(d) Justify, by further differentiation, that the value of P that you have found is a minimum.

(2)

(Total 13 marks)