

Polynomial Division and Factor Theorem Exam Questions MS (from OCR 4722)

Q1, (Jan 2006, Q8i)

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| <p>(i) $-2 + k + 1 + 6 = 0 \Rightarrow k = -5$</p> <p>OR</p> <p>OR</p> <p><i>EITHER:</i> $(x+1)(2x^2 - 7x + 6)$</p> <p style="margin-left: 40px;">$= (x+1)(x-2)(2x-3)$</p> <p><i>OR:</i> $f(2) = 16 - 20 - 2 + 6 = 0$ Hence $(x-2)$ is a factor Third factor is $(2x-3)$ Hence $f(x) = (x+1)(x-2)(2x-3)$</p> | <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B2</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> | <p>For attempting $f(-1)$</p> <p>For equating $f(-1)$ to 0 and deducing the correct value of k AG</p> <p>Match coefficients and attempt k</p> <p>Show $k = -5$</p> <p>Following division, state remainder is 0, hence $(x+1)$ is a factor, hence $k = -5$</p> <p>For correct leading term $2x^2$</p> <p>For attempt at complete division by $f(x)$ by $(x+1)$ or equiv.</p> <p>For completely correct quadratic factor</p> <p>For all three factors correct</p> <p>For further relevant use of the factor theorem</p> <p>For correct identification of factor $(x-2)$</p> <p>For any method for the remaining factor</p> <p>For all three factors correct</p> |
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Q2, (Jan 2007, Q8)

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| <p>(i) $-8 - 36 - 14 + 33 = -25$</p> <p>(ii) $27 - 81 + 21 + 33 = 0$ A.G.</p> <p>(iii) $x = 3$ $f(x) = (x-3)(x^2 - 6x - 11)$</p> <p style="margin-left: 40px;">$x = \frac{6 \pm \sqrt{36 + 44}}{2}$ $= 3 \pm 2\sqrt{5}$ or $3 \pm \sqrt{20}$</p> | <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> | <p>Substitute $x = -2$, or attempt complete division by $(x+2)$</p> <p>Obtain -25, as final answer</p> <p>Confirm $f(3) = 0$, or equiv using division</p> <p>State $x = 3$ as a root at any point</p> <p>Attempt complete division by $(x-3)$ or equiv</p> <p>Obtain $x^2 - 6x + k$</p> <p>Obtain completely correct quotient</p> <p>Attempt use of quadratic formula, or equiv, to find roots</p> <p>Obtain $3 \pm 2\sqrt{5}$ or $3 \pm \sqrt{20}$</p> |
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Q3 (Jun 2007, Q9i,ii)

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| <p>(i) (a) $f(-1) = -1 + 6 - 1 - 4 = 0$</p> <p>(b) $x = -1$ $f(x) = (x+1)(x^2 + 5x - 4)$</p> <p style="margin-left: 40px;">$x = \frac{-5 \pm \sqrt{25 + 16}}{2}$</p> <p style="margin-left: 40px;">$x = \frac{1}{2}(-5 \pm \sqrt{41})$</p> | <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> | <p>Confirm $f(-1) = 0$, through any method</p> <p>State $x = -1$ at any point</p> <p>Attempt complete division by $(x+1)$, or equiv</p> <p>Obtain $x^2 + 5x + k$</p> <p>Obtain completely correct quotient</p> <p>Attempt use of quadratic formula, or equiv, find roots</p> <p>Obtain $\frac{1}{2}(-5 \pm \sqrt{41})$</p> |
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Q4 (Jun 2008, Q4)

(i) $f(3) = 27a - 36 - 21a + 12 = 0$
 $6a = 24$
 $a = 4$

OR

M1* Attempt $f(3)$
M1d* Equate attempt at $f(3)$ to 0 and attempt to solve
A1 Obtain $a = 4$

M1* Attempt complete division / matching coeffs
M1d* Equate remainder to 0
A1 Obtain $a = 4$

3

(ii) $f(-2) = -32 - 16 + 56 + 12$
 $= 20$

M1 Attempt $f(-2)$
A1√ Obtain 20 (or $6a - 4$, following their a)

2

Q5 (Jun 2009, Q7)

(i) $f(-2) = -16 + 36 - 22 - 8$
 $= -10$

M1 Attempt $f(-2)$, or equiv
A1 2 Obtain -10

(ii) $f(\frac{1}{2}) = \frac{1}{4} + 2\frac{1}{4} + 5\frac{1}{2} - 8 = 0$ AG

M1 Attempt $f(\frac{1}{2})$ (no other method allowed)
A1 2 Confirm $f(\frac{1}{2}) = 0$, extra line of working required

(iii) $f(x) = (2x - 1)(x^2 + 5x + 8)$

M1 Attempt complete division by $(2x - 1)$ or $(x - \frac{1}{2})$ or equiv
A1 Obtain $x^2 + 5x + c$ or $2x^2 + 10x + c$
A1 3 State $(2x - 1)(x^2 + 5x + 8)$ or $(x - \frac{1}{2})(2x^2 + 10x + 16)$

(iv) $f(x)$ has one real root ($x = \frac{1}{2}$)
 because $b^2 - 4ac = 25 - 32 = -7$
 hence quadratic has no real roots as $-7 < 0$,

B1√ State 1 root, following their quotient, ignore reason
B1√ 2 Correct calculation, eg discriminant or quadratic formula, following their quotient, or cubic has max at $(-2.15, -9.9)$

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Q6 (Jan 2012, Q5)

(i) $f(3) = 54 + 27 - 51 + 6$
 $= 36$

M1 Attempt $f(3)$
A1 Obtain 36
[2]

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| (ii) | $f(x) = (x - 2)(2x^2 + 7x - 3)$ | B1 | State or imply that $(x - 2)$ is a factor |
| | | M1 | Attempt full division, or equiv, by $(x \pm 2)$ |
| | | A1 | Obtain $2x^2$ and at least one other correct term |
| | | A1 [4] | Obtain $(x - 2)(2x^2 + 7x - 3)$ |
| (iii) | $b^2 - 4ac = 73$ > 0 hence 3 roots | M1 | Attempt explicit numerical calculation to find number of roots of quadratic |
| | | A1ft | State 3 roots ($\sqrt{\text{their quotient}}$) Condone no explicit check for repeated roots |
| | | [2] | |

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| (iii) | $2\cos\theta + 1 = 0$ | $\cos\theta + 1 = 0$ | M1* | Identify relationship between factors of $f(\cos\theta)$ and factors of $f(x)$ |
| | $2\cos\theta - 3 = 0$ | | | |
| | $\cos\theta = -1/2$ | $\cos\theta = -1$ | M1d* | Attempt to solve $\cos\theta = k$ at least once |
| | $\cos\theta = 3/2$ | | | |
| | $\theta = 120, 240$ | 180 | A1 | Obtain at least 2 correct angles |
| | | | A1 | Obtain all 3 correct angles |

Q8 (Jun 2014, Q7)

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| (i) | $f(-2) = 12 - 22(-2) + 9(-2)^2 - (-2)^3$ | M1 | Attempt $f(-2)$ or equiv |
| | $= 12 + 44 + 36 + 8$ | | |
| | $= 100$ | A1 | Obtain 100 |
| | | [2] | |
| (ii) | $f(3) = 12 - 66 + 81 - 27 = 0$ | B1 | Attempt $f(3)$, and show $= 0$ |

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| (iii) | $f(x) = (3 - x)(x^2 - 6x + 4)$ | M1 | Attempt complete division by $(3 - x)$ or $(x - 3)$, or equiv |
| | | A1 | Obtain $x^2 - 6x + 4$ or $-x^2 + 6x - 4$ |
| | | A1 | Obtain $(3 - x)(x^2 - 6x + 4)$ or $(x - 3)(-x^2 + 6x - 4)$ |
| | | [3] | |
| (iv) | $x = 3$ $x = 3 \pm \sqrt{5}$ | B1 | State $x = 3$ |
| | | M1 | Attempt to find roots of quadratic quotient |
| | | A1 | Obtain $x = 3 \pm \sqrt{5}$ |
| | | [3] | |