Polynomial Division and the Factor Theorem Exam Questions Sheet 2 MS

Q1.

Question Number	Scheme		Marks
	If there is no labelling, mark (a) and (b) in that order		
(a)	$f(x)=2x^3-7x^2+4x+4$		
	$f(2)=2(2)^3-7(2)^2+4(2)+4$	Attempts $f(2)$ or $f(-2)$	M1
	$=0$, and so $(x-2)$ is a factor.	$f(2)=0$ with no sign or substitution errors ($2(2)^3-7(2)^2+4(2)+4=0$ is sufficient) **and for conclusion.** Stating "hence factor" or "it is a factor" or a "tick" or "QED" or "no remainder" or "as required" are fine for the conclusion **but not = 0 just underlined and not hence (2 or f(2)) is a factor.** Note also that a conclusion can be implied from a preamble, eg: "If $f(2)=0$, $(x-2)$ is a factor…."	A1
	Note: Long division scores no marks in part (a). The factor theorem is required.		
			[2]
(b)	$f(x)=\{(x-2)\}(2x^2-3x-2)$	M1: Attempts long division by $(x-2)$ or other method using $(x-2)$, to obtain $(2x^2\pm ax\pm b)$, $a\neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection."	M1 A1
		A1: $(2x^2-3x-2)$	
	$=(x-2)(x-2)(2x+1)$ or $(x-2)^2(2x+1)$ or equivalent e.g. $=2(x-2)(x-2)(x+\frac{1}{2})$ or $2(x-2)^2(x+\frac{1}{2})$	dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic in the General Principles for Core Maths Marking). This is **dependent** on the previous method mark being awarded but there must have been no remainder. Allow an attempt to solve the quadratic to determine the factors.	dM1 A1
		A1: cao – needs all three factors **on one line**. Ignore following work (such as a solution to a quadratic equation.)	
	Note $=(x-2)(\frac{1}{2}x-1)(4x+2)$ would lose the last mark as it is not fully factorised		
	For correct answers only award full marks in (b)		
			[4]
			Total 6

Q2.

Question number	Scheme	Marks
	(a) f(2) = 24 − 20 − 32 + 12 = −16 (M: Attempt f(2) or f(−2)) (If continues to say 'remainder = 16', isw) Answer must be seen in part (a), not part (b). (b) $(x+2)(3x^2-11x+6)$ $(x+2)(3x-2)(x-3)$ (If continues to 'solve an equation', isw)	M1 A1 (2) M1 A1 M1 A1 (4) **6**
	(a) Answer only (if correct) scores both marks. (16 as 'answer only' is M0 A0). <u>Alternative (long division):</u> Divide by $(x-2)$ to get $(3x^2+ax+b)$, $a \neq 0, b \neq 0$. [M1] $(3x^2+x-14)$, and − 16 seen. [A1] (If continues to say 'remainder = 16', isw) (b) First M requires division by $(x+2)$ to get $(3x^2+ax+b)$, $a \neq 0, b \neq 0$. Second M for attempt to factorise <u>their</u> quadratic, even if wrongly obtained, perhaps with a remainder from their division. Usual rule: $(kx^2+ax+b)=(px+c)(qx+d)$, where $\lvert pq\rvert = \lvert k\rvert$ and $\lvert cd\rvert = \lvert b\rvert$. Just solving their quadratic by the formula is M0. "Combining" all 3 factors is <u>not</u> required. <u>Alternative (first 2 marks):</u> $(x+2)(3x^2+ax+b)=3x^3+(6+a)x^2+(2a+b)x+2b=0$, then compare coefficients to find <u>values</u> of a and b. [M1] $a=-11,\ b=6$ [A1] <u>Alternative:</u> Factor theorem: Finding that $f(3)=0$ ∴ factor is, $(x-3)$ [M1, A1] Finding that $f\left(\frac{2}{3}\right)=0$ ∴ factor is, $(3x-2)$ [M1, A1] If just one of these is found, score the <u>first 2 marks</u> M1 A1 M0 A0. <u>Losing a factor of 3:</u> $(x+2)\left(x-\frac{2}{3}\right)(x-3)$ scores M1 A1 M1 A0. <u>Answer only, one sign wrong:</u> e.g. $(x+2)(3x-2)(x+3)$ scores M1 A1 M1 A0.	

Q3.

Question	Scheme	Marks	AOs
(a)	States or uses $f(+3)=0$	M1	1.1b
	$4(3)^3-12(3)^2+2(3)-6=108-108+6-6=0$ and so $(x-3)$ is a factor	A1	1.1b
		(2)	
(b)	Begins division or factorisation so $4x^3-12x^2+2x-6=(x-3)(4x^2+...)$	M1	2.1
	$4x^3-12x^2+2x-6=(x-3)(4x^2+2)$	A1	1.1b
	Considers the roots of their quadratic function using completion of square or discriminant	M1	2.1
	$(4x^2+2)=0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2+2>0$ for all x) So $x=3$ is the only real root of $f(x)=0$ *	A1*	2.4
		(4)	
			(6 marks)

Notes

(a) M1: States or uses $f(+3)=0$

A1: See correct work evaluating and achieving zero, together with correct conclusion

(b) M1: Needs to have $(x-3)$ and first term of quadratic correct

A1: Must be correct – may further factorise to $2(x-3)(2x^2+1)$

M1: Considers **their** quadratic for no real roots by use of completion of the square or consideration of discriminant then

A1*: a correct explanation.

Q4.

Question	Scheme	Marks	AOs
	Sets $f(-2)=0 \Rightarrow (-2-4)\left((-2)^2-3\times-2+k\right)-42=0$	M1	3.1a
	$-6(k+10)=42 \Rightarrow k=...$	M1	1.1b
	$k=-17$	A1	1.1b
		(3)	
			(3 marks)

Notes:

Q5.

Part	Working or answer an examiner might expect to see	Mark	Notes
	$f(-3) = 3 \times (-3)^2 + 2a \times (-3)^2 - 4 \times -3 + 5a = 0$	M1	This mark is given for a method to set $f(-3) = 0$
	$f(-3) = 23a - 69 = 0$ $23a = 69$	M1	This mark is given for finding an equation to solve for a
	$a = 3$	A1	This mark is given for finding the correct value of a

Q6.

Question Number	Scheme	Marks
a)i) ii) (b)	$f(3) = 3^3 - 2 \times 3^2 - 4 \times 3 + 8$; $= 5$ $f(-2) = (-8 - 8 + 8 + 8) = 0$ (B1 on Epen, but A1 in fact) M1 is for attempt at **either** f(3) or f(–3) in (i) **or** f(–2) or f(2) in (ii). $[(x+2)](x^2 - 4x + 4)$ (= 0 not required) [must be seen or used in (b)] $(x+2)(x-2)^2$ (= 0) (can imply previous 2 marks) Solutions: $x = 2$ or -2 (both) or (–2, 2, 2) **[no wrong working seen]**	M1; A1 B1 (3) M1 A1 M1 A1 (4) **[7]**
Notes: (a) (b)	**No** working seen: Both answers correct scores full marks One correct ;M1 then A1B0 or A0B1, whichever appropriate. <u>Alternative (Long division)</u> Divide by (x - 3) OR (x + 2) to get $x^2 + ax + b$, a may be zero [M1] $x^2 + x - 1$ and $+5$ seen i.s.w. (or "remainder = 5") [A1] $x^2 - 4x + 4$ and 0 seen (or "no remainder") [B1] First M1 requires division by a **found factor** ; e.g $(x+2)$, $(x-2)$ or what candidate thinks is a **factor** to get $(x^2 + ax + b)$, a may be zero. First A1 for $[(x+2)](x^2 - 4x + 4)$ or $(x-2)(x^2 - 4)$ Second M1:attempt to factorise **their** found quadratic. (or use formula correctly) [Usual rule: $x^2 + ax + b = (x+c)(x+d)$, *where* $\lvert cd \rvert = \lvert b \rvert$.] **N.B. Second A1 is for solutions, not factors** **SC:** (i) Answers only: Both correct, and no wrong, award M0A1M0A1 (as if B1,B1) One correct, (even if 3 different answers) award M0A1M0A0 (as if B1) (ii) Factor theorem used to find two correct factors, award M1A1, then M0, A1 if both correct solutions given. (-2,2,2 would earn all marks) (iii) If in (a) candidate has $(x+2)(x^2-4)$ B0, but then repeats in (b), can score M1A0M1(if goes on to factorise)A0 (answers fortuitous) *<u>Alternative (first two marks)</u>* $(x+2)(x^2 + bx + c) = x^3 + (2+b)x^2 + (2b+c)x + 2c = 0$ and then compare with $x^3 - 2x^2 - 4x + 8 = 0$ to find b and c. [M1] $b = -4$, $c = 4$ [A1] *<u>Method of grouping</u>* $x^3 - 2x^2 - 4x + 8 = x^2(x-2)$,$4(x \pm 2)$ M1; $= x^2(x-2) - 4(x-2)$ A1 $[= (x^2 - 4)(x-2)] = (x+2)(x-2)^2$ M1 Solutions: $x = 2$, $x = -2$ both A1	

Q7.

Question	Scheme	Marks	AOs
(a)	$(\mathrm{g}(-2)) = 4\times -8 - 12\times 4 - 15\times -2 + 50$	M1	1.1b
	$\mathrm{g}(-2) = 0 \Rightarrow (x+2)$ is a factor	A1	2.4
		(2)	
(b)	$4x^3 - 12x^2 - 15x + 50 = (x+2)(4x^2 - 20x + 25)$	M1 A1	1.1b 1.1b
	$= (x+2)(2x-5)^2$	M1 A1	1.1b 1.1b
		(4)	
(c)	(i) $x \leqslant -2,\ x = 2.5$	M1 A1ft	1.1b 1.1b
	(ii) $x = -1,\ x = 1.25$	B1ft	2.2a
		(3)	
			(9 marks)