Question 1

Annual profit model: $P = 100 - 6.25(x-9)^2$, where P is profit (£ thousands) and x is selling price (£).

Worked Solution

Part (a): Is £15 a sensible selling price?

$P = 100 - 6.25(15-9)^2 = 100 - 6.25(36) = 100 - 225 = -125$

$P = -125 < 0$, so the company would make a **loss** of £125 000. Not sensible.

Part (b): Given profit > £80 000 (i.e. $P > 80$):

$$100 - 6.25(x-9)^2 > 80$$

$$6.25(x-9)^2 < 20$$
$$(x-9)^2 < 3.2$$
$$|x-9| < \sqrt{3.2}$$
$$9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$$

The least possible (positive) selling price is $x = 9 - \sqrt{3.2} \approx £7.22$.

Minimum selling price $= 9 - \sqrt{3.2} \approx £7.22$

Part (c):

The model $P = 100 - 6.25(x-9)^2$ has its maximum when $(x-9)^2 = 0$, i.e. $x = 9$.

(i) Maximum profit $= 100\,000$.

(ii) Selling price that maximises profit $= £9$.

Question 2

Tin mining model: $T = 1200 - 3(n-20)^2$, where T tonnes is total mass mined in n years.

Worked Solution

Part (a): Mass mined up to 1 Jan 2020 ($n = 1$):

$$T = 1200 - 3(1-20)^2 = 1200 - 3(361) = 1200 - 1083 = 117 \text{ tonnes}$$

Part (b): Maximum total mass:

The maximum of T occurs when $(n-20)^2 = 0$, i.e. $n = 20$. Maximum $T = 1200$ tonnes.

Part (c): Mass mined *in* 2023, i.e. during year 5 (n goes from 4 to 5):

$$T(5) - T(4) = \left[1200 - 3(5-20)^2\right] - \left[1200 - 3(4-20)^2\right]$$

$$= [1200 - 675] - [1200 - 768] = 525 - 432 = 93 \text{ tonnes}$$

Part (d): Limitation on values of n:

The model gives T increasing then decreasing. Since the total mass mined cannot decrease (you cannot un-mine tin), the model is only valid for $n \leq 20$. Also $n \geq 0$.

Question 3

Ball thrown from cliff top: $h(t) = 115 + 12.25t - 4.9t^2$.

Worked Solution

Part (a):

When $t = 0$, $h = 115\,\text{m}$. This is the height of the ball at the moment of release, i.e. **the height of the cliff top above the ground** ($115\,\text{m}$).

Part (b): Complete the square:

$$h(t) = -4.9(t^2 - 2.5t) + 115 = -4.9\,(t - 1.25)^2 + 4.9(1.25)^2 + 115$$

$$= -4.9(t - 1.25)^2 + 4.9(1.5625) + 115 = -4.9(t - 1.25)^2 + 7.65625 + 115$$

$h(t) = 122.65625 - 4.9(t - 1.25)^2$ (so $A = 122.65625$, $B = 4.9$, $C = 1.25$)

Part (c)(i): Ball reaches ground when $h(t) = 0$:

$$122.65625 - 4.9(t - 1.25)^2 = 0$$

$$(t - 1.25)^2 = \frac{122.65625}{4.9} = 25.03...$$

$$t - 1.25 = \pm\sqrt{25.03...} = \pm 5.003...$$

$$t = 1.25 + 5.003... \approx 6.25 \text{ s} \quad (\text{taking the positive root as } t > 0)$$

$t \approx 6.25\,\text{s}$

Part (c)(ii): Maximum height occurs at $t = 1.25\,\text{s}$ (the vertex):

$$h_{\text{max}} = 122.65625 \approx 122.7\,\text{m}$$

Maximum height $\approx 123\,\text{m}$ at $t = 1.25\,\text{s}$

Question 4

Data: $(0,4)$, $(2,0)$, $(3,0.25)$, $(4,0)$. Paul's model: $y = k(x-2)(x-4)$. John's model: $y = c(x-2)^2(x-4)$.

Worked Solution

Part (i): Sub $(0,4)$ into Paul's model:

$$4 = k(0-2)(0-4) = k \cdot 8$$

$$k = \tfrac{1}{2}$$

Part (ii): Sub $(0,4)$ into John's model:

$$4 = c(0-2)^2(0-4) = c \cdot 4 \cdot (-4) = -16c$$

$$c = -\tfrac{1}{4}$$

Part (iii): Check which model fits the data point $(3, 0.25)$.

Paul's model: $y = \frac{1}{2}(3-2)(3-4) = \frac{1}{2}(1)(-1) = -0.5 \neq 0.25$.

John's model: $y = -\frac{1}{4}(3-2)^2(3-4) = -\frac{1}{4}(1)(-1) = 0.25$

John's model is better, as it correctly predicts $y = 0.25$ when $x = 3$.

Question 5

Rugby ball trajectory: max height 12 m, hits ground at $x = 40$ m. Starts at $x = 0$, $H = 0$.

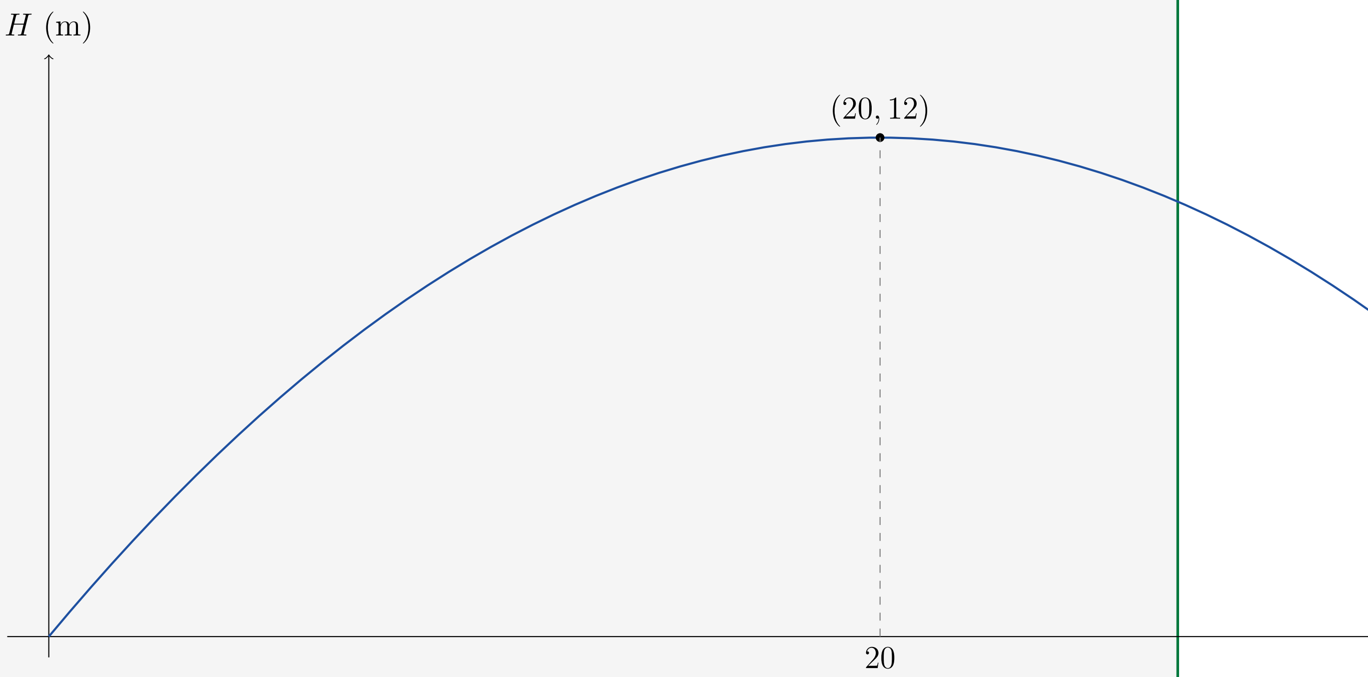
Worked Solution

Part (a):

Since $H = 0$ at $x = 0$ and $x = 40$, use $H = Ax(40 - x)$.

At maximum height $H = 12$ when $x = 20$ (midpoint by symmetry):

$$12 = A(20)(20) = 400A \implies A = \frac{3}{100}$$



$$H = \frac{3}{100}x(40 - x)$$

Part (b): Bar is at $H = 3$. Find the greatest x where $H \geq 3$:

$$\frac{3}{100}x(40 - x) = 3$$

$$x(40 - x) = 100$$

$$x^2 - 40x + 100 = 0$$

$$x = \frac{40 \pm \sqrt{1600 - 400}}{2} = \frac{40 \pm \sqrt{1200}}{2} = 20 \pm \sqrt{300}$$

The greatest distance is $x = 20 + \sqrt{300} \approx 37.3$ m.

Greatest horizontal distance $= 20 + \sqrt{300} \approx 37.3\,\text{m}$

Part (c): One limitation of the model (any sensible answer, e.g.):

The model assumes the ball travels in a perfect parabola, ignoring air resistance and spin.

End of Worked Solutions