Modelling With Quadratic Functions Exam Questions Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)	Attempts $P = 100 - 6.25(15-9)^2$	M1	3.4
	$= -125$ $\therefore$ not sensible as the company would make a loss	A1	2.4
		(2)	
(b)	Uses $P > 80 \Rightarrow (x-9)^2 < 3.2$ or $P = 80 \Rightarrow (x-9)^2 = 3.2$	M1	3.1b
	$\Rightarrow 9 - \sqrt{3.2} < x < 9 + \sqrt{3.2}$	dM1	1.1b
	Minimum Price = £7.22	A1	3.2a
		(3)	
(c)	States (i) maximum profit =£ 100 000 and (ii) selling price £9	B1 B1	3.2a 2.2a
		(2)	
			(7 marks)

Q2

Question	Scheme	Marks	AOs
(a)	117 tonnes	B1	3.4
		(1)	
(b)	1200 tonnes	B1	2.2a
		(1)	
(c)	Attempts $\left\{1200 - 3\times(5-20)^2\right\} - \left\{1200 - 3\times(4-20)^2\right\}$	M1	3.1a
	93 tonnes	A1	1.1b
		(2)	
(d)	States the model is only valid for values of n such that $n \leqslant 20$	B1	3.5b
	States that the total amount mined cannot decrease	B1	2.3
		(2)	
			(6 marks)

Q3

Q	Scheme	Marks
3a	115 (m) is the height of the cliff (as this is the height of the ball when $t = 0$). Accept answer that states 115 (m) is the height of the cliff plus the height of the person who is ready to throw the stone or similar sensible comment.	**B1**
		(1)
3b	Attempt to factorise the – 4.9 out of the first two (or all) terms. $h(t) = -4.9\left(t^2 - 2.5t\right) + 115$ or $h(t) = -4.9\left(t^2 - \frac{5}{2}t\right) + 115$	**M1**
	$h(t) = -4.9\left(t - 1.25\right)^2 - (-4.9)\left(1.25\right)^2 + 115$ or $h(t) = -4.9\left(t - \frac{5}{4}\right)^2 - (-4.9)\left(\frac{5}{4}\right)^2 + 115$	**M1**
	$h(t) = 122.65625 - 4.9\left(t - 1.25\right)^2$ o.e. (N.B. $122.65625 = \frac{3925}{32}$) Accept the first term written to 1, 2, 3 or 4 d.p. or the full answer as shown.	**A1**
		(3)
3ci	Statement that the stone will reach ground level when $h(t) = 0$, or $-4.9t^2 + 12.25t + 115 = 0$ is seen.	**M1**
	Valid attempt to solve quadratic equation (could be using completed square form from part **b**, calculator or formula).	**M1**
	Clearly states that $t = 6.25$ s (accept $t = 6.3$ s) is the answer, or circles that answer and crosses out the other answer, or explains that t must be positive as you cannot have a negative value for time.	**A1**
		(3)
3cii	h_{max} = awrt 123 ft A from part **b**.	**B1ft**
	$t = \frac{5}{4}$ or $t = 1.25$ ft C from part **b**.	**B1ft**
		(2)

Q4

(i)	Sub (0,4) Gives $k = \frac{1}{2}$	M1 A1 **2**	
(ii)	Sub (0, 4) Gives $c = -\frac{1}{4}$	M1 A1 **2**	
(iii)	When $x = 3$ $y = -\frac{1}{4}(3-2)^2(3-4) = 0.25$ for cubic Or when x= 3, y > 0 for cubic John's model is better	B1 DB1 **2**	

Q5

Question	Scheme	Marks	AOs
8 (a) **Way 1**	$H = Ax(40-x)$ {or $H = Ax(x-40)$ }	M1	3.3
	$x = 20,\ H = 12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \frac{3}{100}$	dM1	3.1b
	$H = \frac{3}{100}x(40-x)$ or $H = -\frac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) **Way 2**	$H = 12 - \lambda(x-20)^2$ {or $H = 12 + \lambda(x-20)^2$ }	M1	3.3
	$x = 40,\ H = 0 \Rightarrow 0 = 12 - \lambda(40-20)^2 \Rightarrow \lambda = \frac{3}{100}$	dM1	3.1b
	$H = 12 - \frac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) **Way 3**	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) **Both** $x = 0,\ H = 0 \Rightarrow 0 = 0 + 0 + c \Rightarrow c = 0$ **and either** $x = 40,\ H = 0 \Rightarrow 0 = 1600a + 40b$ **or** $x = 20,\ H = 12 \Rightarrow 12 = 400a + 20b$ **or** $\frac{-b}{2a} = 20$ $\{\Rightarrow b = -40a\}$	M1	3.3
	$b = -40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
		(3)	

(b)	$\{H = 3 \Rightarrow\}\ 3 = \frac{3}{100}x(40 - x) \Rightarrow x^2 - 40x + 100 = 0$ or $\{H = 3 \Rightarrow\}\ 3 = 12 - \frac{3}{100}(x - 20)^2 \Rightarrow (x - 20)^2 = 300$	M1	3.4
	e.g. $x = \frac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\{\text{chooses } 20 + \sqrt{300} \Rightarrow\}$ greatest distance $=$ awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)