Mixed Calculus Exam Questions MS (From OCR 4752)

Q1, (Jan 2007, Q12)

i	$(2x-3)(x-4)$ $x = 4$ or 1.5	M1 A1A1	or $(11 \pm\sqrt{(121-96)})/4$ if M0, then B1 for showing $y = 0$ when $x = 4$ and B2 for x = 1.5	3
ii	$y' = 4x - 11$ = 5 when $x = 4$ c.a.o. grad of normal = −1/their y' $y[-0]$=their −0.2 $(x-4)$ y-intercept for their normal area = ½ × 4 × 0.8 c.a.o.	M1 A1 M1f.t. M1 B1f.t. A1	condone one error or 0 = their (-0.2)x4 + c dep on normal attempt s.o.i. normal must be linear or integrating their f(x) from 0 to 4 M1	6
iii	$\frac{2}{3}x^3 - \frac{11}{2}x^2 + 12x$ attempt difference between value at 4 and value at 1.5 $[-]5\frac{5}{24}$ o.e. or [−]5.2(083..)	M1 M1 A1	condone one error, ignore + c ft their (i), dep on integration attempt. c.a.o.	3

Q2, Jan 2012, Q12)

(i)	$x^2(9-x^2) = 0$ soi	B1	$9\times0^2 - 0^4 = 0$	
	$x = 0$ and ± 3, [so $a = 3$ or $a = -3$]	B1 **[2]**	$9\times3^2 - 3^4 = 0$ *and* $9\times(-3)^2 - (-3)^4 = 0$	
(ii)	$y' = 18x - 4x^3$ $y'' = 18 - 12x^2$ or ft their $y' = 0$ soi $2x(9-2x^2) = 0$ so $x = 0$ oe $x = 0$, $y'' = 18$ cao so minimum $x = \pm\sqrt{4.5}$ oe eg $\pm\frac{3\sqrt{2}}{2}$	B1 B1 M1 A1 B1 A1 **[6]**	 or $18\times0 - 4\times0^3 = 0$ oe or evaluation of y' at $\pm h$ oe where $h < \sqrt{4.5}$ accept 2.12 or better for $\sqrt{4.5}$	
(iii)	$\int_0^3 (9x^2 - x^4)dx$ soi or ft $3x^3 - 0.2x^5$ F[their positive a] [− F[0]] **or** (not **and**) F[0] − F[their negative a] 32.4 oe cao	M1 A1 M1 A1 **[4]**	condone omission of, or wrong limits correct answer implies M1 dependent on at least one term correct	

Q3, (Jan 2011, Q11)

(i) $\frac{x^4}{4} - x^3 - \frac{x^2}{2} + 3x$	**M2**	**M1** if at least two terms correct
their integral at 3 − their integral at 1 [= –2.25 − 1.75]	**M1**	dependent on integration attempted
= –4 isw	**A1**	
represents area between curve and x axis between $x = 1$ and 3	**B1**	
negative since below x-axis	**B1**	
(ii) $y' = 3x^2 - 6x - 1$	**M1**	
their $y' = 0$ soi	**M1**	dependent on differentiation attempted
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 3$, $b = -6$ and $c = -1$ isw	**M1**	or $3(x-1)^2 - 4$ [= 0] or better
$x = \frac{6 \pm \sqrt{48}}{6}$ or better as final answer	**A1**	eg **A1** for $1 \pm \frac{2}{3}\sqrt{3}$
$\frac{6-\sqrt{48}}{6} < x < \frac{6+\sqrt{48}}{6}$ or ft their final answer	**B1**	allow ≤ instead of <

Q4, (Jun 2015, Q10)

(i)		$\left[\frac{dy}{dx}=\right] 4 \times 2 + 3$ or 11 isw	**M1***	
		$9 =$ their $(4 \times 2 + 3) \times 2 + c$	**M1dep***	or $y - 9 =$ their $(4 \times 2 + 3) \times (x - 2)$
		$y = 11x - 13$ or $y = 11x + c$ and $c = -13$ stated isw	**A1**	or $y - 9 = 11(x - 2)$ isw
			[3]	
(ii)		$\frac{4x^2}{2} + 3x$	**M1***	
		$[y =]\ 2x^2 + 3x + c$	**A1**	must see "2" and " + c"; may be earned later eg after attempt to find c
		$9 = 2\times2^2 + 3\times2 + c$	**M1dep***	must include constant, which may be implied by answer
		$y = 2x^2 + 3x - 5$ cao	**A1**	allow first 4 marks for $y = 2x^2 + 3x + c$ and $c = -5$ stated
		$(1, 0)$ *and* $(-2.5, 0)$ oe cao	**B1**	or for $x = 1, y = 0$ and $x = -2.5, y = 0$
		$x = -\frac{3}{4}$	**B1**	
		$y = -\frac{49}{8}$	**B1**	−6.125 or − 6⅛
			[7]	
(iii)		substitution to obtain $[y =]$ f(2x) in polynomial form	**M1**	f(x) must be the quadratic in x with linear and constant term obtained in part (ii), may be in factorised form
		$y = (2x - 1)(4x + 5)$ or $y = 8x^2 + 6x - 5$ or $y = 2\left(2x + \frac{3}{4}\right)^2 - \frac{49}{8}$	**A1FT**	must be simplified to one of these forms, **FT** their quadratic in x with linear and constant term obtained in part (ii)
		$\left(-\frac{3}{8}, -\frac{49}{8}\right)$ oe	**B1**	or **FT** their (both non-zero) co-ordinates for minimum point or their quadratic in x with linear and constant term obtained in part (ii)
			[3]	

Q5, (Jan 2013, Q10)

(i)		at A $y = 3$	B1	
		$\frac{dy}{dx} = 2x - 4$	B1	
		their $\frac{dy}{dx} = 2 \times 4 - 4$	M1*	must follow from attempt at differentiation
		grad of normal $= {}^{-1}/_{\text{their 4}}$	M1dep*	
		$y - 3 = ({}^{-1}/_{4}) \times (x - 4)$ oe isw	A1	
		substitution of $y = 0$ and completion to given result with at least 1 correct interim step www	A1	or substitution of $x = 16$ to obtain $y = 0$
			[6]	
(ii)		at B, $x = 3$	B1	may be embedded
		$F[x] = \frac{x^3}{3} - \frac{4x^2}{2} + 3x$	M1*	condone one error, must be three terms, ignore + c
		F[4] – F[their 3]	M1* dep	dependent on integration attempted
		area of triangle = 18 soi	B1	
		area of region = $19\frac{1}{3}$ oe isw	A1	19.3 or better
			[5]	

Q6, (Jan 2009, Q10)

i	$7 - 2x$	M1		
	$x = 2$, gradient $= 3$	A1	differentiation must be used	
	$x = 2, y = 4$	B1		
	y – their 4 = their grad $(x - 2)$	M1	or use of $y =$ their $mx + c$ and subst (2, their 4), dependent on diffn	
	subst $y = 0$ in their linear eqn	M1	seen	
	completion to $x = \frac{2}{3}$ (ans given)	A1		6
ii	f(1) = 0 or factorising to $(x - 1)(6 - x)$ or $(x - 1)(x - 6)$	1	or using quadratic formula correctly to obtain $x = 1$	
	6 www	1		2
iii	$\frac{7}{2}x^2 - \frac{1}{3}x^3 - 6x$	M1	for two terms correct; ignore $+c$	
	value at 2 − value at 1	M1	ft attempt at integration only	
	$2\frac{1}{6}$ or 2.16 to 2.17	A1		
	$\frac{1}{2} \times \frac{4}{3} \times 4$ − their integral	M1		
	0.5 o.e.	A1		5