Question 1 *(Jun 2006, Q3)*

Worked Solution

Part (i): $\log_6 36$

$$\log_6 36 = \log_6 6^2 = 2$$

$\log_6 36 = 2$

Part (ii): Express $2\log_a 3 + \log_a 11$ as a single logarithm.

Apply the power law first, then the addition law:

$$2\log_a 3 + \log_a 11 = \log_a 3^2 + \log_a 11 = \log_a 9 + \log_a 11 = \log_a(9 \times 11)$$

$\log_a 99$

Question 2 *(Jan 2008, Q5)*

Worked Solution

$a = 3b$ and $\log_3 a + \log_3 b = 2$. Find exact values of a and b.

Step 1: Apply the addition law to the log equation:

$$\log_3(ab) = 2 \implies ab = 3^2 = 9$$

Step 2: Substitute $a = 3b$:

$$3b \cdot b = 9 \implies 3b^2 = 9 \implies b^2 = 3 \implies b = \sqrt{3}$$

(taking positive value since b is a positive constant)

Step 3: Find a:

$$a = 3b = 3\sqrt{3}$$

$b = \sqrt{3}, \quad a = 3\sqrt{3}$

Question 3 *(Jan 2009, Q4)*

Worked Solution

Given $0 < x < 4$ and $\log_5(4-x) - 2\log_5 x = 1$, find x.

Step 1: Apply the power law:

$$\log_5(4-x) - \log_5 x^2 = 1$$

Step 2: Apply the subtraction law:

$$\log_5\left(\frac{4-x}{x^2}\right) = 1$$

Step 3: Remove the log (base 5):

$$\frac{4-x}{x^2} = 5 \implies 4 - x = 5x^2 \implies 5x^2 + x - 4 = 0$$

Step 4: Factorise and solve:

$$(5x-4)(x+1) = 0 \implies x = \frac{4}{5} \quad \text{or} \quad x = -1$$

Since $0 < x < 4$, reject $x = -1$:

$$x = \frac{4}{5}$$

Question 4 *(Jun 2009, Q8)*

Worked Solution

Part (a): $\log_2 y = -3$

$$y = 2^{-3} = \frac{1}{8}$$

$y = \dfrac{1}{8}$

Part (b): $\dfrac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$

Step 1: Evaluate the numerator:

$$\log_2 32 = 5, \quad \log_2 16 = 4 \implies \log_2 32 + \log_2 16 = 9$$

Step 2: The equation becomes:

$$\frac{9}{\log_2 x} = \log_2 x \implies (\log_2 x)^2 = 9 \implies \log_2 x = \pm 3$$

Step 3: Solve each case:

$$\log_2 x = 3 \implies x = 2^3 = 8$$

$$\log_2 x = -3 \implies x = 2^{-3} = \frac{1}{8}$$

$x = 8 \quad \text{or} \quad x = \dfrac{1}{8}$

Question 5 *(Jun 2012, Q2)*

Worked Solution

$2\log_3 x - \log_3(x-2) = 2$

Step 1: Apply the power law:

$$\log_3 x^2 - \log_3(x-2) = 2$$

Step 2: Apply the subtraction law:

$$\log_3\left(\frac{x^2}{x-2}\right) = 2$$

Step 3: Remove the log:

$$\frac{x^2}{x-2} = 9 \implies x^2 = 9(x-2) \implies x^2 - 9x + 18 = 0$$

Step 4: Factorise:

$$(x-3)(x-6) = 0 \implies x = 3 \quad \text{or} \quad x = 6$$

Both values give $x > 2$ so both are valid.

$x = 3 \quad \text{or} \quad x = 6$

Question 6 *(Jan 2010, Q5)*

Worked Solution

Part (a): $\log_x 64 = 2$

$$x^2 = 64 \implies x = 8$$

$x = 8$

Part (b): $\log_2(11 - 6x) = 2\log_2(x-1) + 3$

Step 1: Apply the power law to the right-hand side:

$$\log_2(11 - 6x) = \log_2(x-1)^2 + 3$$

Step 2: Write $3 = \log_2 8$ and apply the addition law:

$$\log_2(11 - 6x) = \log_2\left[8(x-1)^2\right]$$

Step 3: Remove the logs:

$$11 - 6x = 8(x-1)^2 = 8(x^2 - 2x + 1)$$

$$11 - 6x = 8x^2 - 16x + 8 \implies 0 = 8x^2 - 10x - 3$$

Step 4: Factorise:

$$(4x+1)(2x-3) = 0 \implies x = \frac{3}{2} \quad \text{or} \quad x = -\frac{1}{4}$$

Check validity ($x - 1 > 0$ requires $x > 1$): reject $x = -\frac{1}{4}$.

$x = \dfrac{3}{2}$

Question 7 *(Jun 2010, Q7)*

Worked Solution

Part (a): Show $2\log_3(x-5) - \log_3(2x-13) = 1$ leads to $x^2 - 16x + 64 = 0$.

Step 1: Apply the power law:

$$\log_3(x-5)^2 - \log_3(2x-13) = 1$$

Step 2: Apply the subtraction law:

$$\log_3\left(\frac{(x-5)^2}{2x-13}\right) = 1$$

Step 3: Use $\log_3 3 = 1$, so remove the log:

$$\frac{(x-5)^2}{2x-13} = 3 \implies (x-5)^2 = 3(2x-13)$$

$$x^2 - 10x + 25 = 6x - 39 \implies x^2 - 16x + 64 = 0 \quad \checkmark$$

Part (b): Solve $x^2 - 16x + 64 = 0$.

$$(x-8)^2 = 0 \implies x = 8$$

Check: $x - 5 = 3 > 0$ and $2x - 13 = 3 > 0$. ✓

$x = 8$

Question 8 *(Jan 2012, Q4)*

Worked Solution

Given $y = 3x^2$.

Part (a): Show $\log_3 y = 1 + 2\log_3 x$.

$$\log_3 y = \log_3(3x^2) = \log_3 3 + \log_3 x^2 = 1 + 2\log_3 x \quad \checkmark$$

Part (b): Solve $1 + 2\log_3 x = \log_3(28x - 9)$.

Using part (a), $1 + 2\log_3 x = \log_3(3x^2)$, so:

$$\log_3(3x^2) = \log_3(28x - 9) \implies 3x^2 = 28x - 9$$

$$3x^2 - 28x + 9 = 0 \implies (3x - 1)(x - 9) = 0$$

$$x = \frac{1}{3} \quad \text{or} \quad x = 9$$

Check $28x - 9 > 0$: both values satisfy this.

$$x = \frac{1}{3} \quad \text{or} \quad x = 9$$

Question 9 *(Jan 2013, Q6)*

Worked Solution

$2\log_2(x+15) - \log_2 x = 6$

Part (a): Show $x^2 - 34x + 225 = 0$.

Step 1: Apply the power law:

$$\log_2(x+15)^2 - \log_2 x = 6$$

Step 2: Apply the subtraction law:

$$\log_2\left(\frac{(x+15)^2}{x}\right) = 6$$

Step 3: Use $2^6 = 64$:

$$\frac{(x+15)^2}{x} = 64 \implies (x+15)^2 = 64x$$

$$x^2 + 30x + 225 = 64x \implies x^2 - 34x + 225 = 0 \quad \checkmark$$

Part (b): Solve $x^2 - 34x + 225 = 0$.

$$(x-25)(x-9) = 0 \implies x = 25 \quad \text{or} \quad x = 9$$

Both are positive so both are valid.

$x = 25 \quad \text{or} \quad x = 9$

Question 10 *(Jun 2013, Q7)*

Worked Solution

Part (i): $\log_2(2x) = \log_2(5x+4) - 3$

Step 1: Write $3 = \log_2 8$ and move to the right:

$$\log_2(2x) + \log_2 8 = \log_2(5x+4)$$

Step 2: Apply the addition law:

$$\log_2(16x) = \log_2(5x+4)$$

Step 3: Remove the logs:

$$16x = 5x + 4 \implies 11x = 4 \implies x = \frac{4}{11}$$

$$x = \frac{4}{11}$$

Part (ii): $\log_a y + 3\log_a 2 = 5$. Express y in terms of a.

Step 1: Apply the power law:

$$\log_a y + \log_a 2^3 = 5 \implies \log_a(8y) = 5$$

Step 2: Remove the log:

$$8y = a^5 \implies y = \frac{a^5}{8}$$

$$y = \frac{a^5}{8}$$

Question 11 *(Jun 2013(R), Q6)*

Worked Solution

Given $\log_3 x = a$.

Part (a): $\log_3(9x)$

$$\log_3(9x) = \log_3 9 + \log_3 x = 2 + a$$

$2 + a$

Part (b): $\log_3\left(\dfrac{x^5}{81}\right)$

$$\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81 = 5\log_3 x - 4 = 5a - 4$$

$5a - 4$

Part (c): Solve $\log_3(9x) + \log_3\left(\dfrac{x^5}{81}\right) = 3$.

Using parts (a) and (b):

$$(2 + a) + (5a - 4) = 3 \implies 6a - 2 = 3 \implies a = \frac{5}{6}$$

Since $\log_3 x = a = \dfrac{5}{6}$:

$$x = 3^{5/6} = (3^5)^{1/6} = 243^{1/6}$$

Using $\log_{10} x = \dfrac{5}{6}\log_{10} 3$:

$$x = 10^{\frac{5}{6}\log_{10} 3} \approx 2.498$$

$x = 2.498$ (to 4 s.f.)

End of Worked Solutions — Year 1 Pure: Laws of Logarithms (Sheet 1)