

Laws of Logarithms and Logarithmic Equations (From Edexcel 6664)

Q1, (Jun 2006, Q3)

(i) 2	B1	(1)
(ii) $2\log 3 = \log 3^2$ (or $2\log p = \log p^2$)	B1	
$\log_a p + \log_a 11 = \log_a 11p, = \log_a 99$ (Allow e.g. $\log_a(3^2 \times 11)$)	M1, A1	(3)
		4
(ii) Ignore 'missing base' or wrong base. The correct answer with no working scores full marks. $\log_a 9 \times \log_a 11 = \log_a 99$, or similar mistakes, score M0 A0.		

Q2, (Jan 2008, Q5)

<u>Method 1</u> (Substituting $a = 3b$ into second equation at some stage)		
Using a law of logs correctly (anywhere)	e.g. $\log_3 ab = 2$	M1
Substitution of $3b$ for a (or $a/3$ for b)	e.g. $\log_3 3b^2 = 2$	M1
Using base correctly on correctly derived $\log_3 p = q$	e.g. $3b^2 = 3^2$	M1
First correct value	$b = \sqrt{3}$ (allow $3^{1/2}$)	A1
Correct method to find other value (dep. on at least first M mark)		
Second answer	$a = 3b = 3\sqrt{3}$ or $\sqrt{27}$	A1
<u>Method 2</u> (Working with two equations in $\log_3 a$ and $\log_3 b$)		
* "Taking logs" of first equation and " separating"	$\log_3 a = \log_3 3 + \log_3 b$ (= $1 + \log_3 b$)	M1
Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$		
[$\log_3 a = 1\frac{1}{2}, \log_3 b = \frac{1}{2}$]		
Using base correctly to find a or b		
Correct value for a or b	$a = 3\sqrt{3}$ or $b = \sqrt{3}$	A1
Correct method for second answer, dep. on first M; correct second answer [Ignore negative values]		
		M1;A1[6]

Q3, (Jan 2009, Q4)

$2\log_5 x = \log_5(x^2),$	$\log_5(4-x) - \log_5(x^2) = \log_5 \frac{4-x}{x^2}$	B1, M1
$\log\left(\frac{4-x}{x^2}\right) = \log 5$	$5x^2 + x - 4 = 0$ or $5x^2 + x = 4$ o.e.	M1 A1
$(5x-4)(x+1) = 0$	$x = \frac{4}{5}$ $(x = -1)$	dM1 A1
		(6) [6]

B1 is awarded for $2\log x = \log x^2$ anywhere.

M1 for correct use of $\log A - \log B = \log \frac{A}{B}$

M1 for replacing 1 by $\log_k k$. **A1** for correct quadratic

$(\log(4-x) - \log x^2 = \log 5 \Rightarrow 4-x-x^2 = 5$ is **B1M0M1A0 M0A0**)

dM1 for attempt to solve quadratic with usual conventions. (Only award if previous two **M** marks have been awarded)

A1 for 4/5 or 0.8 or equivalent (Ignore extra answer).

Q4, (Jun 2009, Q8)

(a)	$\log_2 y = -3 \Rightarrow y = 2^{-3}$ $y = \frac{1}{8}$ or 0.125	M1 A1	(2)
(b)	$32 = 2^5$ or $16 = 2^4$ or $512 = 2^9$ [or $\log_2 32 = 5\log_2 2$ or $\log_2 16 = 4\log_2 2$ or $\log_2 512 = 9\log_2 2$] [or $\log_2 32 = \frac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \frac{\log_{10} 512}{\log_{10} 2}$] $\log_2 32 + \log_2 16 = 9$ $(\log x)^2 = \dots$ or $(\log x)(\log x) = \dots$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2) $\log_2 x = 3 \Rightarrow x = 2^3 = 8$ $\log_2 x = -3 \Rightarrow x = 2^{-3} = \frac{1}{8}$	M1 A1 M1 A1 A1ft	(5) [7]

$$2 \log x = \log x^2$$

$$\log_3 x^2 - \log_3 (x-2) = \log_3 \frac{x^2}{x-2}$$

$$\frac{x^2}{x-2} = 9$$

Solves $x^2 - 9x + 18 = 0$ to give $x = \dots$

$$x = 3, x = 6$$

B1

M1

A1 o.e.

M1

A1

Total 5

Q6, (Jan 2010, Q5)

(a) $\log_x 64 = 2 \Rightarrow 64 = x^2$

So $x = 8$

(b) $\log_2 (11 - 6x) = \log_2 (x - 1)^2 + 3$

$$\log_2 \left[\frac{11 - 6x}{(x - 1)^2} \right] = 3$$

$$\frac{11 - 6x}{(x - 1)^2} = 2^3$$

$$\{11 - 6x = 8(x^2 - 2x + 1)\} \text{ and so } 0 = 8x^2 - 10x - 3$$

$$0 = (4x + 1)(2x - 3) \Rightarrow x = \dots$$

$$x = \frac{3}{2}, \left[-\frac{1}{4} \right]$$

M1

A1 (2)

M1

M1

M1

A1

dM1

A1 (6)

[8]

Q7, (Jun 2010, Q7)

<p>(a) $2 \log_3(x-5) = \log_3(x-5)^2$</p> <p>$\log_3(x-5)^2 - \log_3(2x-13) = \log_3 \frac{(x-5)^2}{2x-13}$</p> <p>$\log_3 3 = 1$ seen or used correctly</p> <p>$\log_3\left(\frac{P}{Q}\right) = 1 \Rightarrow P = 3Q \quad \left\{ \frac{(x-5)^2}{2x-13} = 3 \Rightarrow (x-5)^2 = 3(2x-13) \right\}$</p> <p style="text-align: center;">$x^2 - 16x + 64 = 0$ (*)</p>	<p>B1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>A1 cso</p>	<p>(5)</p>
<p>(b) $(x-8)(x-8) = 0 \Rightarrow x = 8$ <u>Must</u> be seen in part (b).</p> <p>Or: Substitute $x = 8$ into original equation and verify.</p> <p>Having additional solution(s) such as $x = -8$ loses the A mark.</p> <p>$x = 8$ with no working scores both marks.</p>	<p>M1 A1</p>	<p>(2)</p> <p>7</p>

Q8, (Jan 2012, Q4)

<p>(a)</p> <p>$\log_3 3x^2 = \log_3 3 + \log_3 x^2$ or $\log y - \log x^2 = \log 3$ or</p> <p>$\log y - \log 3 = \log x^2$</p> <p>$\log_3 x^2 = 2 \log_3 x$</p> <p>Using $\log_3 3 = 1$</p>	<p>B1</p> <p>B1</p> <p>B1</p>	<p>(3)</p>
<p>(b)</p> <p>$3x^2 = 28x - 9$</p> <p>Solves $3x^2 - 28x + 9 = 0$ to give $x = \frac{1}{3}$ or $x = 9$</p>	<p>M1</p> <p>M1 A1</p>	<p>(3)</p> <p>6</p>

Q9, (Jan 2013, Q6)

(a)	$2 \log(x+15) = \log(x+15)^2$		B1
	$\log(x+15)^2 - \log x = \log \frac{(x+15)^2}{x}$	Correct use of $\log a - \log b = \log \frac{a}{b}$	M1
	$2^6 = 64$ or $\log_2 64 = 6$	64 used in the correct context	B1
	$\log_2 \frac{(x+15)^2}{x} = 6 \Rightarrow \frac{(x+15)^2}{x} = 64$	Removes logs correctly	M1
	$\Rightarrow x^2 + 30x + 225 = 64x$ or $x + 30 + 225x^{-1} = 64$	Must see expansion of $(x+15)^2$ to score the final mark.	
	$\therefore x^2 - 34x + 225 = 0$ *		A1
			(5)
(b)	$(x-25)(x-9) = 0 \Rightarrow x = 25$ or $x = 9$	M1: Correct attempt to solve the given quadratic as far as $x = \dots$ A1: Both 25 and 9	M1 A1
			(2)
			[7]

Q10, Jun 2013, Q7)

7. (i) Method 1	$\log_2 \left(\frac{2x}{5x+4} \right) = -3$ or $\log_2 \left(\frac{5x+4}{2x} \right) = 3$, or $\log_2 \left(\frac{5x+4}{x} \right) = 4$ (see special case 2)		M1
	$\left(\frac{2x}{5x+4} \right) = 2^{-3}$ or $\left(\frac{5x+4}{2x} \right) = 2^3$ or $\left(\frac{5x+4}{x} \right) = 2^4$ or $\left(\log_2 \left(\frac{2x}{5x+4} \right) \right) = \log_2 \left(\frac{1}{8} \right)$		M1
	$16x = 5x + 4 \Rightarrow x =$ (depends on previous Ms and must be this equation or equivalent)		dM1
	$x = \frac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work		A1 cs (4)
7(i) Method 2	$\log_2(2x) + 3 = \log_2(5x + 4)$ So $\log_2(2x) + \log_2(8) = \log_2(5x + 4)$ (3 replaced by $\log_2 8$) Then $\log_2(16x) = \log_2(5x + 4)$ (addition law of logs) Then final M1 A1 as before		2 nd M1 1 st M1 dM1A1
(ii)	$\log_a y + \log_a 2^3 = 5$ $\log_a 8y = 5$ $y = \frac{1}{8} a^5$	Applies product law of logarithms. $y = \frac{1}{8} a^5$	M1 dM1 A1cao (3) [7]

Q11, (Jun 2013(R), Q6)

i.(a)	Way 1: $\log_3(9x) = \log_3 9 + \log_3 x$ $= 2 + a$	or Way 2: $\log_3(9x) = \log_3 3^{a+2}$ $= 2 + a$	M1 A1 (2)
(b)	Way 1: $\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81$ $\log x^5 = 5 \log x$ or $\log 81 = 4 \log 3$ or $\log 81 = 4$ $= 5a - 4$	or Way 2 $= \log_3 \frac{3^{5a}}{3^4}$ $= \log_3 3^{5a-4}$	M1 M1 A1 cso (3)
(c)	$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$ Method 1 $\Rightarrow 2 + a + 5a - 4 = 3$ $\Rightarrow a = \frac{5}{6}$ $\Rightarrow x = 3^{\frac{5}{6}}$ or $\log_{10} x = a \log_{10} 3$ so $x =$ $x = 2.498$ or awrt If $x = -2.498$ appears as well or instead this is A0	Method 2 $\log_3\left(9x \cdot \frac{x^5}{81}\right) = (3 \text{ or } \log 27)$ $\log_3\left(\frac{x^6}{9}\right) = 3 \text{ or } \log 27$ $\Rightarrow \frac{x^6}{9} = 3^3 \Rightarrow x^6 = 3^5 \Rightarrow x =$ $x = 2.498$ or awrt	M1 A1 M1 A1 (4) Total 9