Question 1

Worked Solution

$2\log_3 x - \log_3(x-2) = 2$

Step 1: Apply the power law:

$$\log_3 x^2 - \log_3(x-2) = 2$$

Step 2: Apply the subtraction law:

$$\log_3\left(\frac{x^2}{x-2}\right) = 2$$

Step 3: Remove the log (base 3, RHS $= 3^2 = 9$):

$$\frac{x^2}{x-2} = 9 \implies x^2 = 9(x-2) \implies x^2 - 9x + 18 = 0$$

Step 4: Factorise and solve:

$$(x-3)(x-6) = 0 \implies x = 3 \quad \text{or} \quad x = 6$$

Both give $x > 2$ so both are valid.

$x = 3 \quad \text{or} \quad x = 6$

Question 2

Worked Solution

Given $\log_3 x = a$.

Part (a): $\log_3(9x)$

$$\log_3(9x) = \log_3 9 + \log_3 x = 2 + a$$

$2 + a$

Part (b): $\log_3\left(\frac{x^5}{81}\right)$

$$\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81 = 5\log_3 x - 4 = 5a - 4$$

$5a - 4$

Part (c): Solve $\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$.

Using parts (a) and (b):

$$(2 + a) + (5a - 4) = 3 \implies 6a - 2 = 3 \implies a = \frac{5}{6}$$

Since $\log_3 x = \frac{5}{6}$, use $\log_{10} x = \frac{5}{6}\log_{10} 3$:

$$x = 3^{5/6} \approx 2.498$$

$x = 2.498$ (to 4 s.f.)

Question 3

Worked Solution

Part (i): $2\log(x+a) = \log(16a^6)$

Step 1: Apply the power law to the left side:

$$\log(x+a)^2 = \log(16a^6)$$

Step 2: Remove the logs:

$$(x+a)^2 = 16a^6$$

Step 3: Take the square root (positive since $a > 0$ and $x + a > 0$):

$$x + a = 4a^3 \implies x = 4a^3 - a$$

$x = 4a^3 - a$

Part (ii): $\log_3(9y+b) - \log_3(2y-b) = 2$

Step 1: Apply the subtraction law:

$$\log_3\left(\frac{9y+b}{2y-b}\right) = 2$$

Step 2: Use $3^2 = 9$ to remove the log:

$$\frac{9y+b}{2y-b} = 9$$

Step 3: Multiply out and solve for y:

$$9y + b = 9(2y-b) = 18y - 9b \implies 10b = 9y \implies y = \frac{10b}{9}$$

$y = \frac{10b}{9}$

Question 4

Worked Solution

$\log_3(3b+1) - \log_3(a-2) = -1$, $a > 2$. Express b in terms of a.

Step 1: Apply the subtraction law:

$$\log_3\left(\frac{3b+1}{a-2}\right) = -1$$

Step 2: Use $3^{-1} = \frac{1}{3}$ to remove the log:

$$\frac{3b+1}{a-2} = \frac{1}{3}$$

Step 3: Solve for b:

$$3(3b+1) = a-2 \implies 9b+3 = a-2 \implies 9b = a-5 \implies b = \frac{a-5}{9}$$

$$b = \frac{a-5}{9}$$

Question 5

Worked Solution

$\log_2(11y-3) - \log_2 3 - 2\log_2 y = 1, \quad y > \frac{3}{11}$

Step 1: Apply the power law:

$$\log_2(11y-3) - \log_2 3 - \log_2 y^2 = 1$$

Step 2: Combine using subtraction law:

$$\log_2\left(\frac{11y-3}{3y^2}\right) = 1$$

Step 3: Use $2^1 = 2$ to remove the log:

$$\frac{11y-3}{3y^2} = 2 \implies 11y - 3 = 6y^2 \implies 6y^2 - 11y + 3 = 0$$

Step 4: Factorise:

$$(3y-1)(2y-3) = 0 \implies y = \frac{1}{3} \quad \text{or} \quad y = \frac{3}{2}$$

Both satisfy $y > \frac{3}{11}$ and give positive arguments for all logs.

$y = \frac{1}{3} \quad \text{or} \quad y = \frac{3}{2}$

Question 6

Worked Solution

$\log_2(x+15) - 4 = \frac{1}{2}\log_2 x$

Step 1: Apply the power law to the right side:

$$\log_2(x+15) - 4 = \log_2 x^{1/2} = \log_2 \sqrt{x}$$

Step 2: Write $4 = \log_2 16$ and rearrange:

$$\log_2(x+15) - \log_2 16 = \log_2 \sqrt{x}$$

$$\log_2\left(\frac{x+15}{16}\right) = \log_2 \sqrt{x}$$

Step 3: Remove the logs:

$$\frac{x+15}{16} = \sqrt{x}$$

Step 4: Let $u = \sqrt{x}$, so $x = u^2$:

$$\frac{u^2+15}{16} = u \implies u^2 - 16u + 15 = 0 \implies (u-1)(u-15) = 0$$

$$\sqrt{x} = 1 \implies x = 1 \qquad \text{or} \qquad \sqrt{x} = 15 \implies x = 225$$

Both are positive so both are valid.

$x = 1$ or $x = 225$

Question 7

Worked Solution

Part (a): $\log_3(x-2) = -1$

$$x - 2 = 3^{-1} = \frac{1}{3} \implies x = 2\frac{1}{3}$$

$x = 2\frac{1}{3}$

Part (b): $2\log_3 x - \log_3 7x = 1$

Step 1: Apply the power law:

$$\log_3 x^2 - \log_3 7x = 1$$

Step 2: Apply the subtraction law:

$$\log_3\left(\frac{x^2}{7x}\right) = 1 \implies \log_3\left(\frac{x}{7}\right) = 1$$

Step 3: Remove the log:

$$\frac{x}{7} = 3 \implies x = 21$$

$x = 21$

Question 8

Worked Solution

The student attempts to solve $2\log_2 x - \log_2 \sqrt{x} = 3$.

Part (a): Identify the two errors.

Error 1: The student applied the subtraction law directly to $2\log_2 x - \log_2 \sqrt{x}$, writing it as $2\log_2\left(\frac{x}{\sqrt{x}}\right)$. This is wrong because the subtraction law $\log A - \log B = \log\left(\frac{A}{B}\right)$ requires the coefficients of both log terms to be 1. The coefficient of the first term is 2, so the power law must be applied first.

Error 2: The student concluded from $\log_2 x = 3$ that $x = 3^2 = 9$. This is wrong: $\log_2 x = 3$ means $x = 2^3 = 8$, not 3^2.

Part (b): Correct solution.

Step 1: Apply the power law to the first term:

$$\log_2 x^2 - \log_2 x^{1/2} = 3$$

Step 2: Apply the subtraction law:

$$\log_2\left(\frac{x^2}{x^{1/2}}\right) = 3 \implies \log_2 x^{3/2} = 3$$

Step 3: Remove the log:

$$x^{3/2} = 2^3 = 8 \implies x = 8^{2/3} = (2^3)^{2/3} = 2^2 = 4$$

$x = 4$

End of Worked Solutions — Year 1 Pure: Laws of Logarithms (Sheet 2)