Laws of Logarithms and Logarithmic Equations Exam Questions Sheet 2 MS

Q1.

Question number	Scheme	Marks
	$2\log x = \log x^2$	B1
	$\log_3 x^2 - \log_3(x-2) = \log_3 \frac{x^2}{x-2}$	M1
	$\frac{x^2}{x-2} = 9$	A1 o.e.
	Solves $x^2 - 9x + 18 = 0$ to give $x = ...$	M1
	$x = 3$, $x = 6$	A1
		Total 5
Notes	**B1** for **this** correct use of power rule (may be implied) **M1**: for **correct** use of subtraction rule (or addition rule) for logs N.B. $2\log_3 x - \log_3(x-2) = 2\log_3 \frac{x}{x-2}$ is **M0** **A1**. for correct equation without logs (Allow any correct equivalent including 3^2 instead of 9.) **M1** for attempting to solve $x^2 - 9x + 18 = 0$ to give $x =$ (see notes on marking quadratics) **A1** for these **two** correct answers	
Alternative Method	$\log_3 x^2 = 2 + \log_3(x-2)$ is B1, *so* $x^2 = 3^{2+\log_3(x-2)}$ needs to be followed by $(x^2) = 9(x-2)$ for M1 A1 Here M1 is for **complete** method i.e.correct use of powers after logs are used correctly	
Common Slips	$2\log x - \log x + \log 2 = 2$ may obtain B1 if $\log x^2$ appears but **the statement is M0** and so leads to no further marks $2\log_3 x - \log_3(x-2) = 2$ *so* $\log_3 x - \log_3(x-2) = 1$ and $\log_3 \frac{x}{x-2} = 1$ can earn M1 for *correct* subtraction rule following error, but no other marks	
Special Case	$\frac{\log x^2}{\log(x-2)} = 2$ leading to $\frac{x^2}{x-2} = 9$ and then to x=3, x=6, usually earns B1M0A0, but may then earn M1A1 (special case) so 3/5 [This *recovery* after uncorrected error is very common] Trial and error, Use of a table or just stating answer **with both** x=3 and x=6 should be awarded B0M0A0 then final M1A1 i.e. 2/5	

Q2.

Question Number	Scheme	Marks
(a)	Way 1: $\log_3(9x) = \log_3 9 + \log_3 x$ or Way 2: $\log_3(9x) = \log_3 3^{a+2}$	M1
	$= 2 + a$	A1 (2)
(b)	Way 1: $\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81$ or Way 2 $= \log_3 \frac{3^{5a}}{3^4}$	M1
	$\log x^5 = 5\log x$ or log81=4log3 or log81=4 / $= \log_3 3^{5a-4}$	M1
	$= 5a - 4$	A1 cso (3)
(c)	$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$	
	Method 1 / Method 2	
	Method 1: $\Rightarrow 2 + a + 5a - 4 = 3$ / Method 2: $\log_3\left(9x.\frac{x^5}{81}\right) = (3 \text{ or } \log 27)$	M1
	Method 1: $\Rightarrow a = \frac{5}{6}$ / Method 2: $\log_3\left(\frac{x^6}{9}\right) = 3 \text{ or } \log 27$	A1
	Method 1: $\Rightarrow x = 3^{\frac{5}{6}}$ or $\log_{10} x = a\log_{10} 3$ so $x =$ / Method 2: $\Rightarrow \frac{x^6}{9} = 3^3 \Rightarrow x^6 = 3^5 \Rightarrow x =$	M1
	Method 1: $x = 2.498$ **or awrt** / Method 2: $x = 2.498$ **or awrt**	A1 (4)
	If x = -2.498 appears as well or instead this is A0	**Total 9**

Notes for Question

(a)	Way 1: M1: Use of log(ab) = log(a) + log(b) A1: must be $a + 2$ or $2 + a$ **Way 2:** Uses $x = 3^a$ to give $\log_3(9x) = \log_3 3^{a+2}$. A1 for $a + 2$ or $2 + a$
(b)	Way 1: M1: Use of log(a/b) = log(a) – log(b) M1: Use of nlog(a) = log(a)^n Way 2: M1 Use of correct powers of 3 in numerator and denominator M1: Subtracts powers A1: No errors seen
(c)	**Method 1**: M1: Uses (a) and (b) results to form an equation in a (may not be linear) A1: a = awrt 0.833 M1: Finds x by use of 3 to a power, or change of base performed correctly A1: $x = 2.498$ (accept answer which round to this value from 2.498049533…) **Method 2:** M1: Use of log(ab) = log(a) + log(b) in an equation (RHS may be wrong) A1: Equation correct and simplified M1: Tries to undo log by 3 to power correctly, and uses root to obtain x A1: $x = 2.498$ (accept answer which round to this value from 2.498049533…) Lose this mark if negative answer is given as well as or instead of positive answer.

Q3.

Question Number	Scheme	Marks
(i)	Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$	M1
	Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$ Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$	M1
	$(x=)\ 4a^3 - a$ (depends on previous M's and must be this expression or equivalent)	A1cao **(3)**
(ii) Way 1	$\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ — Applies quotient law of logarithms	M1
	$\dfrac{(9y+b)}{(2y-b)} = 3^2$ — Uses $\log_3 3^2 = 2$	M1
	$(9y+b) = 9(2y-b) \Rightarrow y =$ — Multiplies across and makes y the subject	M1
	$y = \frac{10}{9}b$	A1cso **(4)**
Way 2	Or : $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ — 2nd M mark	M1
	$\log_3(9y+b) = \log_3 9(2y-b)$ — 1st M mark	M1
	$(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ — Multiplies across and makes y the subject	M1 A1cso **(4)** **[7]**

	Notes
(i)	1st M1: Applies power law of logarithms correctly to one side of the equation M1: Correct log work in correct order. If they square and obtain a quadratic the algebra should be correct. The marks is for $x + a = \sqrt{16a^6}$ isw so allow $x + a = \pm\ 4a^3$ for Method mark. Also allow $x + a = 4a^4$ or $x + a = \pm\ 4a^{5.5}$ or even $x + a = 16a^3$ as there is evidence of attempted square root. May see the correct $x + a = 10^{(\log 4 + 3\log a)}$ so $x = -a + 10^{(\log 4 + 3\log a)}$ which gains M1A0 unless followed by the answer in the scheme. A1: Do not allow $x = \pm\ 4a^3 - a$ for accuracy mark. You may see the factorised $a(2a+1)(2a-1)$ o.e.
(ii)	M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term in y M1: Uses $\log_3 3^2 = 2$ 3rd M1: Obtains **correct** linear equation in y usually the one in the scheme and attempts $y =$ A1cso: $y = \frac{10}{9}b$ or correct equivalent after **completely correct** work. **Special case:** $\dfrac{\log_3(9y+b)}{\log_3(2y-b)} = 2$ is M0 unless clearly crossed out and replaced by the correct $\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ Candidates may then write $\dfrac{(9y+b)}{(2y-b)} = 3^2$ and proceed to the *correct* answer – allow M0M1M1A0 as the answer requires a completely correct solution.

Q4.

Scheme		Marks
Two Ways of answering the question are given in part (i)		
$\log_3\left(\frac{3b+1}{a-2}\right) = -1$ or $\log_3\left(\frac{a-2}{3b+1}\right) = 1$	Applying the subtraction law of logarithms	M1
$\frac{3b+1}{a-2} = 3^{-1}\left\{= \frac{1}{3}\right\}$ or $\left(\frac{a-2}{3b+1}\right) = 3$	Making a correct connection between log base 3 and 3 to a power.	M1
$\{9b+3 = a-2 \Rightarrow\}\ b = \frac{1}{9}a - \frac{5}{9}$	$b = \frac{1}{9}a - \frac{5}{9}$ or $b = \frac{a-5}{9}$	A1 oe
		[3]
In **Way 2** a correct connection between log base 3 and "3 to a power" is used before applying the subtraction or addition law of logs		
Either $\log_3(3b+1) - \log_3(a-2) = -\log_3 3$ or $\log_3(3b+1) + \log_3 3 = \log_3(a-2)$		2nd M1
$\log_3(3b+1) = \log_3(a-2) - \log_3 3 = \log_3\left(\frac{a-2}{3}\right)$ or $\log_3 3(3b+1) = \log_3(a-2)$		1st M1
$\{3b+1 = \frac{a-2}{3}\}\ b = \frac{1}{9}a - \frac{5}{9}$		A1
		[3]
Five Ways of answering the question are given in part (ii)		

Q5.

$\log_2(11y-3) - \log_2 3 - 2\log_2 y = 1$	
$\log_2(11y-3) - \log_2 3 - \log_2 y^2 = 1$	M1
$\log_2\frac{(11y-3)}{3y^2} = 1$ or $\log_2\frac{(11y-3)}{y^2} = 1 + \log_2 3 = 2.58496501$	dM1
$\log_2\frac{(11y-3)}{3y^2} = \log_2 2$ or $\log_2\frac{(11y-3)}{y^2} = \log_2 6$ (allow awrt 6 if replaced by 6 later)	B1
Obtains $6y^2 - 11y + 3 = 0$ o.e. i.e. $6y^2 = 11y - 3$ for example	A1
Solves quadratic to give $y =$	ddM1
$y = \frac{1}{3}$ and $\frac{3}{2}$ (need both- one should not be rejected)	A1
	(6)
	[9]

Q6.

$\log_2(x+15) - 4 = \log_2 x^{\frac{1}{2}}$	Applies the power law of logarithms seen **at any point in their working**	M1
$\log_2\left(\frac{x+15}{x^{\frac{1}{2}}}\right) = 4$	Applies the subtraction or addition law of logarithms **at any point in their working**	M1
$\left(\frac{x+15}{x^{\frac{1}{2}}}\right) = 2^4$	Obtains a correct expression with logs removed and no errors	M1
$x - 16x^{\frac{1}{2}} + 15 = 0$ or e.g. $x^2 + 225 = 226x$	Correct **three** term quadratic in any form	A1
$(\sqrt{x}-1)(\sqrt{x}-15) = 0 \Rightarrow \sqrt{x} = ...$	A **valid** attempt to factorise or solve their **three term quadratic** to obtain $\sqrt{x} = ...$ or $x = ...$ Dependent on all previous method marks.	dddM1
$\{\sqrt{x} = 1, 15\}$		
$x = 1, 225$	**Both $x = 1$ and $x = 225$ (If both are seen, ignore any other values of $x \le 0$ from an otherwise correct solution)**	A1
		[6]
		Total 8

Alternative:		
$2\log_2(x+15) - 8 = \log_2 x$		
$\log_2(x+15)^2 - 8 = \log_2 x$	Applies the power law of logarithms	M1
$\log_2\left(\frac{(x+15)^2}{x}\right) = 8$	Applies the subtraction law of logarithms	M1
$\frac{(x+15)^2}{x} = 2^8$	Obtains a correct expression with logs removed	M1
$x^2 + 30x + 225 = 256x$		
$x^2 - 226x + 225 = 0$	Correct three term quadratic in any form	A1
$(x-1)(x-225) = 0 \Rightarrow x = ...$	A valid attempt to factorise or solve their 3TQ to obtain $x = ...$ Dependent on all previous method marks.	dddM1
$x = 1, 225$	**Both $x = 1$ and $x = 225$ (If both are seen, ignore any other values of $x \le 0$ from an otherwise correct solution)**	A1

Q7.

(a)

$(x-2)=3^{-1}$	$(x-2)=3^{-1}$ or $\frac{1}{3}$	M1 oe
$x\left\{=\frac{1}{3}+2\right\}=2\frac{1}{3}$	$2\frac{1}{3}$ or $\frac{7}{3}$ or $2.\dot{3}$ or awrt 2.33	A1
		[2] 4

(b)

(b) $2\log x=\log x^2$	B1
$\log x^2-\log 7x=\log\dfrac{x^2}{7x}$	M1
"Remove logs" to form equation in x, using the base correctly: $\dfrac{x^2}{7x}=3$	M1
$x=21$ (Ignore $x=0$, if seen)	A1cso (4) 6

Q8.

Question	Scheme		Marks	AOs
(a)	Identifies one of the two errors "You cannot use the subtraction law without dealing with the 2 first" " They undo the logs incorrectly. It should be $x=2^3=8$ "		B1	2.3
	Identifies both errors. See above.		B1	2.3
			(2)	
(b)	$\log_2\left(\dfrac{x^2}{\sqrt{x}}\right)=3$	$\dfrac{3}{2}\log_2(x)=3$	M1	1.1b
	$x^{\frac{3}{2}}=2^3$ or $\dfrac{x^2}{\sqrt{x}}=2^3$	$x=2^2$	M1	1.1b
	$x=\left(2^3\right)^{\frac{2}{3}}=4$	$x=4$	A1	1.1b
			(3)	
				(5 marks)

(a)
B1: States one of the two errors.
Error One: Either in words states 'They cannot use the subtraction law without dealing with the 2 first' or writes ' that line 2 should be $\log_2\left(\frac{x^2}{\sqrt{x}}\right)$ $(=3)$' If they rewrite line two it must be correct. Allow 'the coefficient of each log term is different so we cannot use the subtraction law'
Allow responses such as 'it must be $\log x^2$ before subtracting the logs'
Do not accept an incomplete response such as "the student ignored the 2". **There must be some reference to the subtraction law as well.**
Error Two: Either in words states 'They undo the log incorrectly' or writes that 'if $\log_2 x = 3$ then $x = 2^3 = 8$' If it is rewritten it must be correct. Eg $x = \log_2 9$ is B0
B1: States both of the two errors. (See above)

(b)
M1: Uses a correct method of combining the two log terms. Either uses both the power law and the subtraction law to reach a form $\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$ oe. Or uses both the power law and subtraction to reach $\frac{3}{2}\log_2(x) = 3$
M1: Uses correct work to "undo" the log. Eg moves from $\log_2(Ax^n) = b \Rightarrow Ax^n = 2^b$
This is independent of the previous mark so allow following earlier error.
A1: cso $x = 4$ achieved with at least one intermediate step shown. Extra solutions would be A0
SC: If the "answer" rather than the "solution" is given score 1,0,0.